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AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 998.

A RATIONAL FORM OF STIFFENED SUSPENSION BRIDGE.*

By Gustav Lindenthal, M. Am. Soc. C. E.

With Discussion by
Messrs. W. Hildenbrand, Joseph Mayer, R. S. Buck, W. W. Crehore, Theodore Cooper, C. C. Schneider, Oswald Erlinghagen, Henry W. Hodge, F. Schüle, J. Melan, Leon S. Moisseiff, A. Rieppel and Gustav Lindenthal.

That a suspension bridge can be made as rigid as any form of metal bridge, be it arch, cantilever, or truss, is now no longer disputed, and the old prejudice against its supposed unfitness for concentrated loads may be said to have died out.

All the older suspension bridges are limber and undulating structures, without adequate provision against deformation under moving loads. The belief became general that they could not be made as rigid as other bridge systems, and hence they acquired a bad reputation for railroad purposes.

Therefore, when the writer, more than sixteen years ago, proposed a stiffened suspension bridge, of 3 100 ft. span, over the North River, for fast railroad trains, the proposition was received with much doubt and criticism. This, however, had the good effect of inducing discussion and investigation, and of gradually clearing away much of the fog which surrounded the theory of rigidity.

* Presented at the meeting of September 21st, 1904.

Rankine was the first mathematician (in 1869) to publish a rational theory for the adequate dimensioning of stiffening trusses. His formulas were at first believed to give too heavy trusses, as compared with the makeshifts in vogue, and for that reason they were rarely used. For instance, they were not used in the plans for the Brooklyn Bridge.

Theoretical refinement has led to somewhat lighter stiffening trusses than were required originally by his theory, which had ignored that portion of the live load absorbed by the deformation of the cable. The saving in weight is not great enough, however, to justify the belief that a properly designed suspension bridge of that kind represents a striking economy over the cantilever, arch and truss, except for very long spans (say of 2 000 ft. and more), when the dead load of the structure is several times greater than the live load.

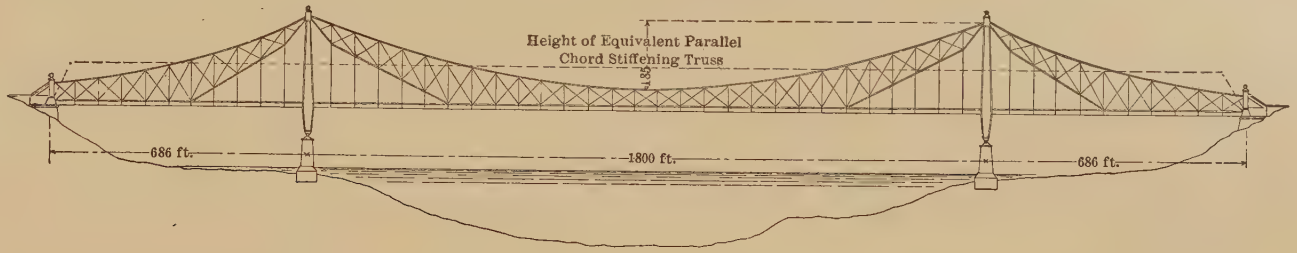
The term "properly designed" is used advisedly, for in no other bridge system is it possible to use such attenuated dimensions, to have so much deflection up and down, and to commit such errors of design, as in the suspension system, without inviting instant collapse of the structure.

As an illustration in point, compare the cantilever bridge over the Forth in Scotland, having two main spans, each of 1 700 ft., with the Brooklyn Suspension Bridge, having one main span of 1 600 ft. If one of the main spans of the Forth Bridge were loaded on both tracks with locomotives and the other spans were not loaded, the structure would still be perfectly safe. If the main span of the Brooklyn Suspension Bridge were loaded on its four tracks with the full cars in daily use, close together, and the side spans were not loaded, then the cables, otherwise strong enough, would sag so much that they would wreck the stiffening trusses, if not the entire structure.

Another prejudice, originating with weak and inadequately stiffened suspension bridges, is the rule prohibiting marching in step, on the ground that the rhythmical cumulative vibrations might eventually break down the structure, whether weak or strong. The fabled fiddler would have to be longer lived than iron and steel efficiently protected against corrosion, before he would succeed with a modern railroad bridge, or equally well designed highway bridge.

A suspension structure, to be comparable with other bridge systems, therefore, must be dimensioned on the same conditions of strength and stability. When that is done, its vaunted economy vanishes, except for very long spans, as before mentioned.

STIFFENED RAILROAD SUSPENSION BRIDGE
OVER THE ST. LAWRENCE RIVER AT QUEBEC, PROPOSED IN 1898.



DESIGN PROPOSED FOR
MANHATTAN BRIDGE OVER THE EAST RIVER, NEW YORK.

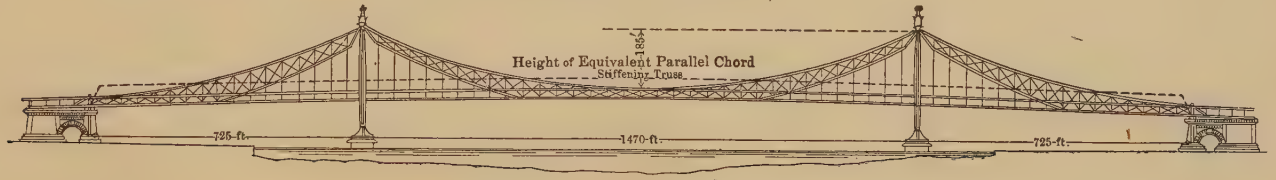


FIG. 1.

The only recommendation of stiffening trusses is that of a convenient construction independent of the cable, which can be made more or less efficient or costly as may suit the fancy of the designer, or the financial resources of the owners, frequently on the principle that "half a loaf is better than no bread."

From a scientific point of view, the stiffening truss is a crude device, as compared with other modes of stiffening, which do not permit of the same laxity in dimensioning.

This appears most strikingly if that method of stiffening be applied to an erect arch. Small arches of that kind, in combination with trusses, had been built of wood and cast iron many years ago, when the art of bridge designing was yet in its infancy. But no one would think of applying that system to erect arches at this day, because better and more effective systems of arch bracing have been worked out since. As every suspension bridge is theoretically an inverted arch bridge, so, analogically, all the different systems of bracing in erect arches are applicable to suspended arches, and any system of bracing considered poor and inefficient in an erect arch is so, also, when used in a suspended arch. The principal difference is that the erect arch is in unstable equilibrium and therefore requires strong lateral cross-bracing, to keep it in position, while in the suspended arch, little or no such bracing is required. The center line of gravity in the suspended arch is far below the points of support on the towers, and the structure is in stable equilibrium. To that condition many badly designed suspension bridges owe their life; it covers a multitude of sins against good engineering.

The merit and serviceableness of a bridge structure, other things being equal, can be gauged by no other criterion so reliably as by the degree of its rigidity. The theory of cumulative vibration might find other fruitful fields of application. It has none in a bridge, dimensioned to present standards of safety. Absence of sufficient rigidity marks the inferior bridge. This, for instance, is one of the reasons why, for smaller spans, stone bridges, where practicable, will always be preferred to metal bridges, in spite of their greater cost. High metal trusses, the American practice, are preferred to low trusses, the English practice. Not only are such trusses stiffer, but also cheaper, a point which found its confirmation among others in the competition for the Atbara Bridge.

The same considerations apply to arches, erect and suspended.

PLATE I.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 998.
LINDENTHAL ON
STIFFENED SUSPENSION BRIDGES.

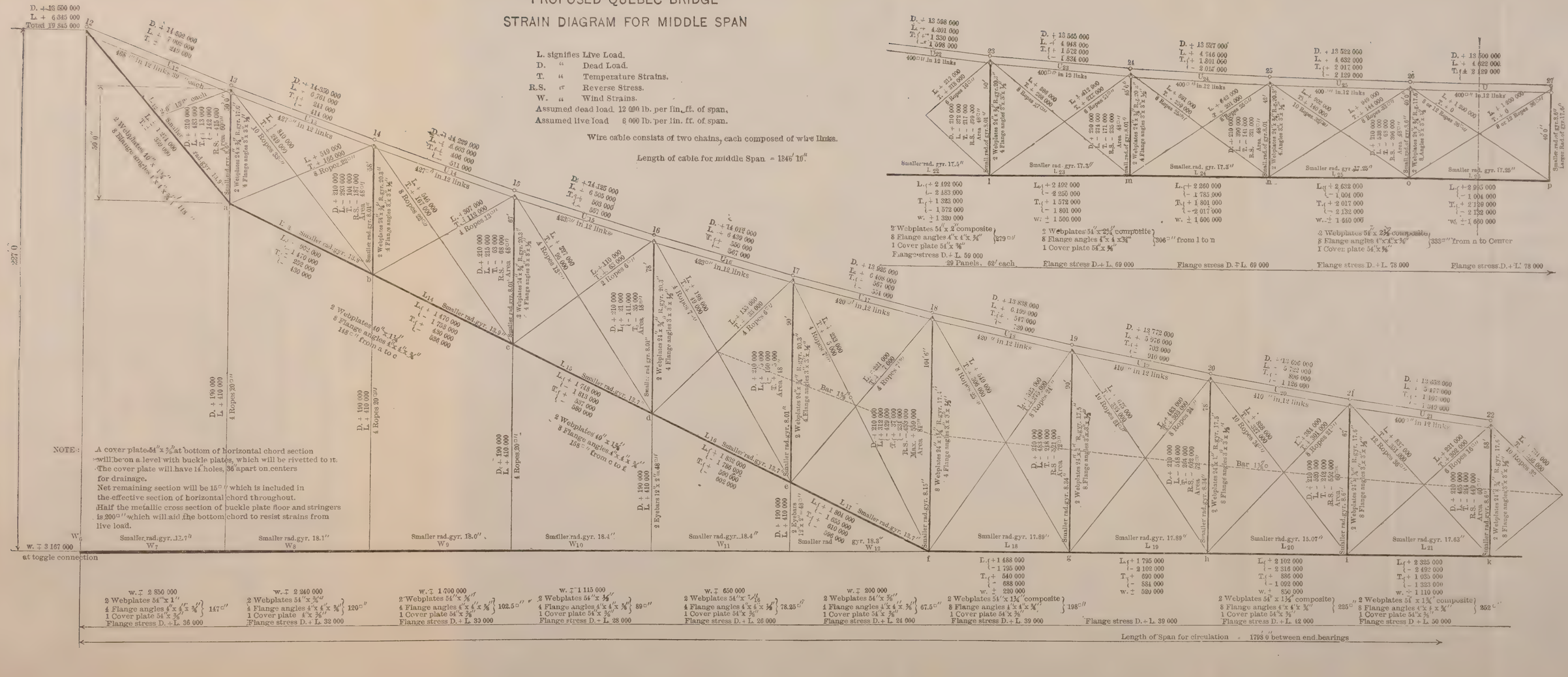
PROPOSED QUEBEC BRIDGE

STRAIN DIAGRAM FOR MIDDLE SPAN

L. signifies Live Load.
D. " Dead Load.
T. " Temperature Strains.
R.S. " Reverse Stress.
W. " Wind Strains.
Assumed dead load 12 000 lb. per lin. ft. of span.
Assumed live load 8 000 lb. per lin. ft. of span.

Wire cable consists of two chains, each composed of wire links.

Length of cable for middle Span = 1846′ 10″.


Length of Span for circulation = 1798′ 0″ between end bearings
Flange stress D.+L. 39 000
Flange stress D.+L. 69 000
Flange stress D.+L. 78 000
Flange stress D.+L. 78 000
Flange stress D.+L. 69 000
Flange stress D.+L. 59 000
Flange stress D.+L. 50 000
Flange stress D.+L. 42 000
Flange stress D.+L. 39 000
Flange stress D.+L. 36 000
Flange stress D.+L. 32 000
Flange stress D.+L. 33 000
Flange stress D.+L. 28 000
Flange stress D.+L. 26 000
Flange stress D.+L. 24 000
Smaller rad. gyr. 12.7″
Smaller rad. gyr. 18.1″
Smaller rad. gyr. 18.0″
Smaller rad. gyr. 18.1″
Smaller rad. gyr. 18.4″
Smaller rad. gyr. 18.3″
Smaller rad. gyr. 17.89″
Smaller rad. gyr. 17.89″
Smaller rad. gyr. 15.07″
Smaller rad. gyr. 17.63″
Smaller rad. gyr. 17.5″
Smaller rad. gyr. 17.3″
Smaller rad. gyr. 17.3″
Smaller rad. gyr. 17.25″
Smaller rad. gyr. 17.25″
at toggle connection
from a to c
from c to e
from 1 to n
from n to Center
Bar 1⅞″ ø
Bar 1¾″ ø
Bar 1⅝″ ø
Bar 1½″ ø
Bar 1⅜″ ø
Bar 1¼″ ø
Bar 1⅛″ ø
Bar 1″ ø
NOTE: A cover plate 54″ x ⅜″ at bottom of horizontal chord section will be on a level with buckle plates, which will be rivetted to it. The cover plate will have 14″ holes, 36″ apart on centers for drainage. Net remaining section will be 15□′ which is included in the effective section of horizontal chord throughout. Half the metallic cross section of buckle plate floor and stringers is 200□″ which will aid the bottom chord to resist strains from live load.

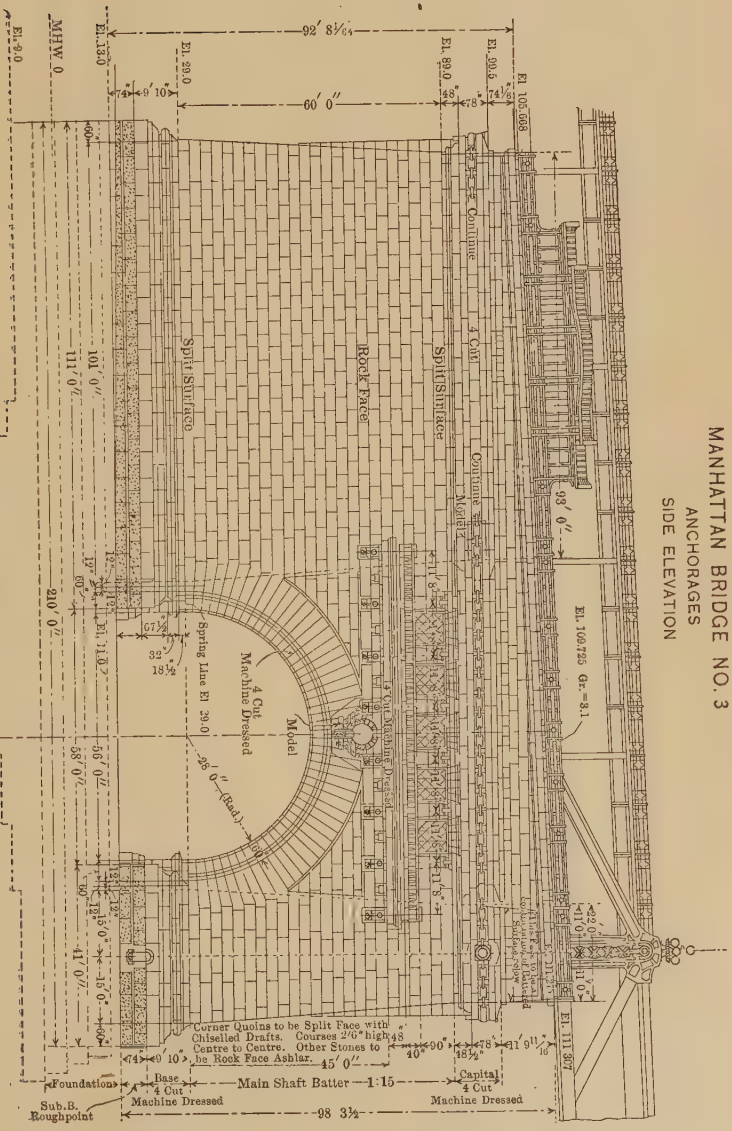

PROPOSED
MANHATTAN BRIDGE NO. 3
ANCHORAGES
SIDE ELEVATION
92′ 8 1/64
60′ 0″
El. 9.0
MHW 0
El. 13.0
El. 29.0
El. 89.0
El. 99.5
El. 105.668
Split Surface
Rock Face
Split Surface
Continue
4 Cut
Continue
Model
93′ 0″
El. 109.725 Gr. = 3.1
101′ 0″
111′ 0″
210′ 0″
El. 11.0
Spring Line El 29.0
4 Cut
Machine Dressed
Model
28′ 0″ (Rad.)
58′ 0″
56′ 0″
41′ 0″
Corner Quoins to be Split Face with Chiselled Drafts. Courses 2′6″ high Centre to Centre. Other Stones to be Rock Face Ashlar.
45′ 0″
Foundation
Base
4 Cut
Machine Dressed
Main Shaft Batter 1:15
Capital
4 Cut
Machine Dressed
Sub.B.
Roughpoint
98 3½
El. 111.307

FIG. 2.

The points of largest disturbance are at the quarters of the main span, and at the middle of the end spans. At those points the maximum bending moments from live load are greatest. The height of the stiffening frame at those points determines the rigidity of the arch, similarly as does in an ordinary truss its height at the middle.

Therefore, a form of stiffening frame having its greatest height at the quarters in the main span, and at the center of the side spans, will offer greater rigidity and also greater economy than any other; and, when attached directly to the arch, there will be the further economy of the saving of an entire chord.

Let us now bear in mind the familiar fact, that when the height of the bracing at points of greatest flexure attached to the cable (or arch) is equal to or greater than one-quarter of the sag of the cable, no additional material will be required for the cable sections for the bending strains from a one-sided live load.

We have then a combination of frame and arch or cable, resulting (similarly as in a truss) at once in great stiffness and great economy, which is an evidence of good design.

The suspension structures shown in Fig. 1 belong to that class, embodying the above-named features. The bottom chord is farthest from the cable at the quarters of the middle span and at the center of the side spans, and it coincides with the line of the floor for the middle half of the main span and for one-half of the side spans, where it can be utilized also for the wind trusses.

In the end spans we have to meet the important fact that, if they be half the length of the main span, the sections of the stiffening frame for the same height will be twice as heavy as that for the middle span. Therefore, it will be advisable to make the height at the middle of the end span greater, if possible, than at the quarter of the middle span, or to shorten the end spans.

With none of the known systems of arch or cable bracing is such great height readily obtainable between the chords at points of greatest flexure without incurring waste of material at other points.

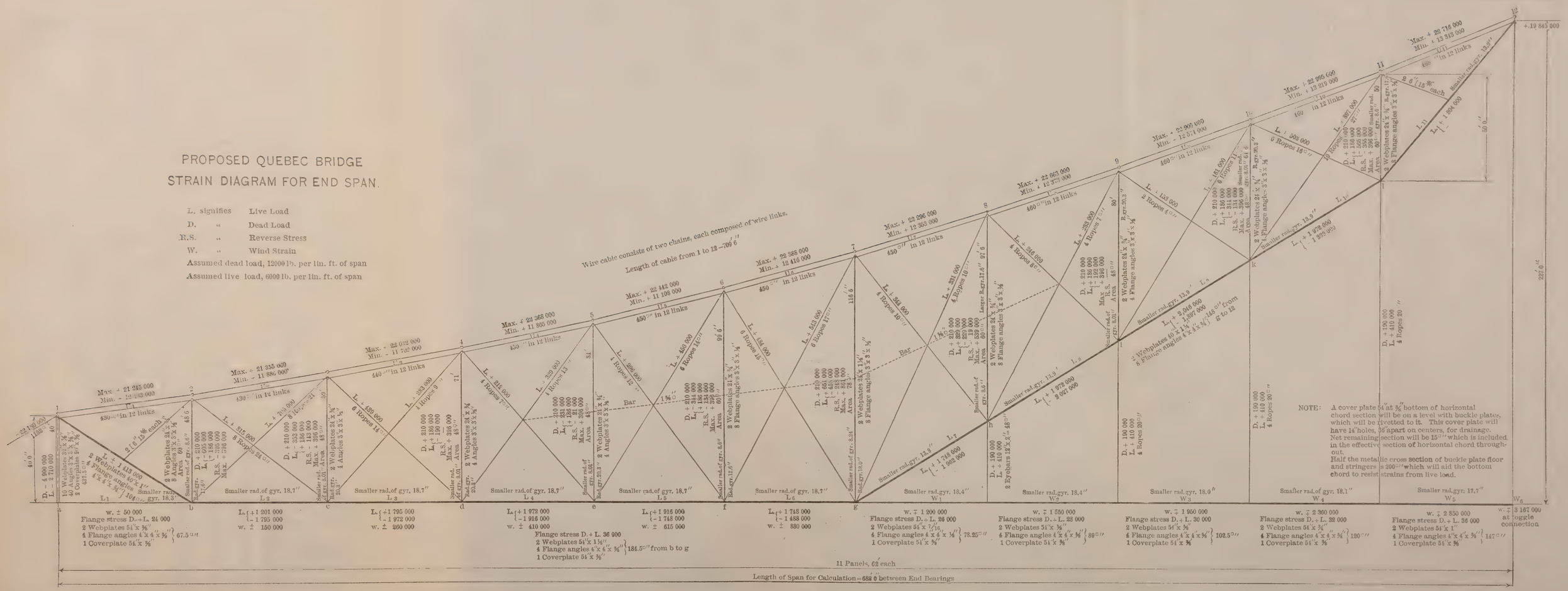
This system may be designed with or without a hinge at the center of the main span.

In the first case the stresses will be statically determinate. For this one advantage, however, there would be the great disadvantage of making the lower chord useless for a wind truss just at the point of largest bending strains from wind pressure.

PROPOSED QUEBEC BRIDGE
STRAIN DIAGRAM FOR END SPAN.

L. signifies Live Load
D. " Dead Load
R.S. " Reverse Stress
W. " Wind Strain

Assumed dead load, 12000 lb. per lin. ft. of span
Assumed live load, 6000 lb. per lin. ft. of span


Wire cable consists of two chains, each composed of wire links.
Length of cable from 1 to 12 = 709' 6"
Max. + 21 245 000
Min. − 12 283 000
Max. + 21 355 000
Min. − 11 886 000
Max. + 22 032 000
Min. − 11 702 000
Max. + 22 142 000
Min. + 11 108 000
Max. + 22 368 000
Min. + 11 895 000
Max. + 22 388 000
Min. + 12 410 000
Max. + 22 296 000
Min. + 12 363 000
Max. + 22 063 000
Min. + 12 573 000
Max. + 22 005 000
Min. + 12 571 000
Max. + 22 995 000
Min. + 13 219 000
Max. + 22 716 000
Min. + 13 343 000
+ 19 845 000
D. − 4 900 000
L. − 2 710 000
− 32 190 000
1 162 000
40'0"
430 in 12 links
440 in 12 links
450 in 12 links
460 in 12 links
480 in 12 links
2 @ 15' 0" each
2 @ 16' 0" each
Bar
Smaller rad. of gyr. 18,7"
Smaller rad.gyr. 18,4"
Smaller rad.gyr. 18,1"
Smaller rad.gyr. 17,7"
Smaller rad.gyr. 13,9"
Smaller rad.gyr. 13,0"
Smaller rad.gyr. 18,0"
Larger R.gyr. 17,6"
Smaller rad. of gyr. 8,01"
Smaller rad. of gyr. 8,66"
Smaller rad. of gyr. 8,64"
Smaller rad. of gyr. 8,6"
Smaller rad. of gyr. 18,0"
Smaller rad. of gyr. 18,1"
Smaller rad. of gyr. 8,8"
Smaller rad. of gyr. 8,5"
Smaller rad. of gyr. 8,0"
Smaller rad. of gyr. 8,1"
Smaller rad. of gyr. 8,2"
Smaller rad. of gyr. 8,3"
Smaller rad. of gyr. 8,4"
Smaller rad. of gyr. 8,7"
Smaller rad. of gyr. 8,9"
Smaller rad. of gyr. 9,0"
Smaller rad. of gyr. 9,1"
Smaller rad. of gyr. 9,2"
Smaller rad. of gyr. 9,3"
Smaller rad. of gyr. 9,4"
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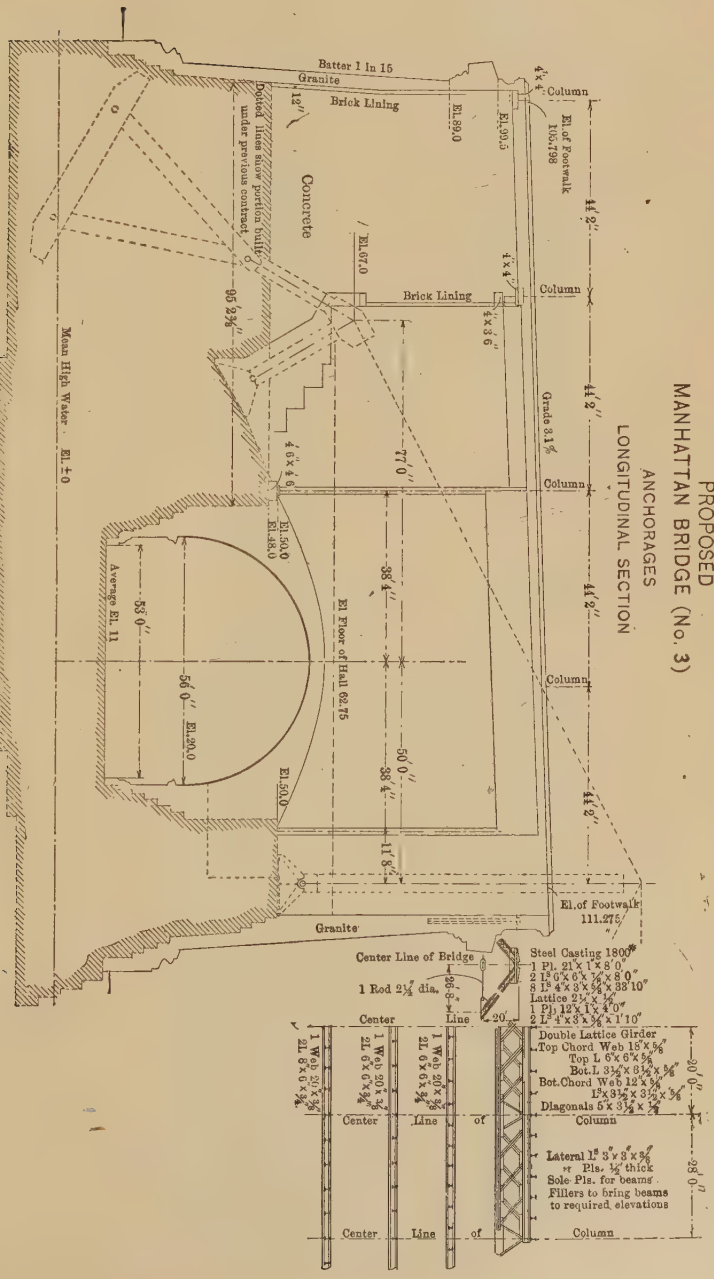
PROPOSED
MANHATTAN BRIDGE (No. 3)
ANCHORAGES
LONGITUDINAL SECTION
Batter 1 in 15
Granite
Brick Lining
Column
El. of Footwalk 105.798
El. 89.0
El. 99.5
Dotted lines show portion built under previous contract
Concrete
El. 67.0
Brick Lining
Column
Grade 3.1%
Mean High Water El. ±0
Column
El. 50.0
El. 48.0
Average El. 11
El Floor of Hall 62.75
Column
El. 20.0
El. 50.0
El. of Footwalk 111.275
Granite
Center Line of Bridge
1 Rod 2¼" dia.
Steel Casting 1800#
1 Pl. 21"x 1"x 8'0"
2 Ls 6"x 6"x ⅞"x 8'0"
8 Ls 4"x 3"x ⅝"x 33'10"
Lattice 2¼"x ½"
1 Pl. 12"x 1"x 4'0"
2 Ls 4"x 3"x ⅝"x 1'10"
Center Line
Double Lattice Girder
Top Chord Web 18"x ⅝"
Top L 6"x 6"x ⅝"
Bot. L 3½"x 3½"x ⅝"
Bot. Chord Web 12"x ⅝"
Ls 3½"x 3½"x ⅝"
Diagonals 5"x 3½"x ½"
Center Line of Column
Lateral Ls 3"x 3"x ⅜"
Pls. ½" thick
Sole Pls. for beams
Fillers to bring beams to required elevations
Center Line of Column

FIG. 3.

The middle hinge has no decided advantage as regards temperature stresses. The writer has shown before,* that in stiffening trusses the middle hinge does not eliminate temperature effects, but that they are merely shifted.

The total amount of extra material for temperature stresses, in stiffening frames with parallel chords, is nearly the same in both cases. It is merely distributed differently. A similar rule holds good with other forms of stiffening, modified, however, in each case by the form of the frame. In order to keep the temperature stresses down, when there is no middle hinge, the frame should be made as low as possible along the middle third of the center span.

In Fig. 1, the upper figure represents the form proposed by the writer for the bridge over the St. Lawrence River at Quebec.

At the center of the middle span, the stiffening is 40 ft. high, or one-forty-fifth of the span (1 800 ft.). At the quarters, the stiffening frame is 110 ft. high, which is nearly one-sixteenth of the span, or six-tenths of the sag or deflection of the cable. A stiffening truss with parallel chords of equivalent rigidity under partial loads would require to be more than 100 ft. high, and its upper chord is shown by a dotted line.

That economy of material is not synonymous with economy of cost, found a new illustration in the competition for the Quebec Bridge. On the basis of the same specifications and unit stresses, the writer's suspension bridge was lighter in weight of metal, and in the amount of masonry no greater, than any of the competitive designs. But, nevertheless, the high prices for wire work made it more expensive than the cantilever design, which was adopted and is now in process of construction.

At the writer's request, Professor Melan afterward made an independent calculation of the stresses, sections and deflections, with the result that no changes were required in the sections, as shown on the strain sheet, Plate II.

It should be remarked that in this system it is preferable that the cable assume its proper equilibrium curve under full dead load, and that the diagonals, which are without strain at a middle temperature, be made adjustable in length. The diagonals are strained only from live load and from temperature changes.

*_Engineering News_, March 10th, 1888; Appendix D of the Report of Engineer Officers, published in _Engineering News_, November 22d, 1894.

DESIGN PROPOSED FOR
MANHATTAN BRIDGE
OVER THE EAST RIVER AT NEW YORK CITY

PROPOSED MANHATTAN BRIDGE No. 3—ANCHORAGES—LONGITUDINAL SECTION THROUGH STAIR WELLS.

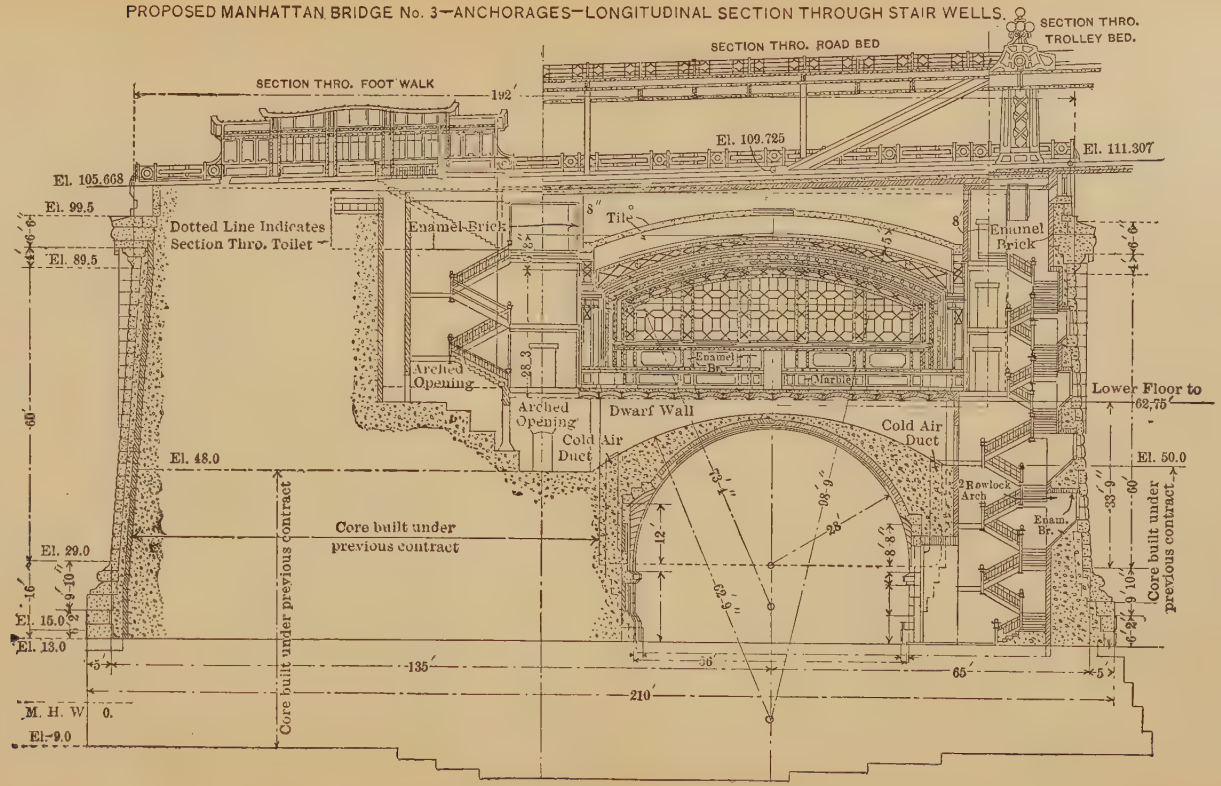

FIG. 4.

If a settlement should take place in the foundation of the towers, any irregularity resulting therefrom, in the stresses of the stiffening frames, is readily corrected by a readjustment of the diagonals.

The pivotal form of tower simplifies greatly the details of cable bearings, avoids eccentric pressure, and facilitates the erection of cables. These great advantages have been recognized also in the recent designs for suspension bridges abroad; thus, the towers of the new Buda Pest Chain Bridge and of the proposed suspension structure over the Rhine at Cologne are both pivoted at the bottom.

The lower design on Fig. 1 represents the form of the proposed Manhattan Bridge, of New York, on the same scale as that of the Quebec Bridge. At the center of the middle span, the stiffening frame is 22.15 ft. high, or one-sixty-sixth of the span (1 470 ft.). At the quarters, the height is 58 ft., which is about one-twenty-fifth of the span, and more than one-third of the sag or deflection of the chains in the main span.

A stiffening truss with parallel chords of equivalent rigidity under partial loads would be more than 55 ft. high, as shown by the dotted line. The trusses in the Williamsburg Bridge, 1 600 ft. span, are only 40 ft. high, equal to one-fortieth of the span.

The unsightliness of high stiffening trusses is not the only objection. Their greater weight and greater cost, and the greater temperature stresses in them, require no special proof. Strain sheets based on the same conditions are more reliable than general formulas.

In this paper the writer proposes to give merely the salient features of a type in the evolution of the rigid suspension bridge. That it is readily adapted to very satisfactory architectural treatment may be observed from the perspective view of the proposed Manhattan Bridge. The two tower piers are completed.

For a better understanding of the plans of this bridge, it may suffice to state, briefly, that the main span and the two symmetrical side spans are 1 470 and 725 ft. long, respectively, and are supported from two steel towers resting on comparatively low masonry piers and rising to a height of 400 ft. above high water.

The superstructure of the main spans is suspended from four lines of eye bar chains with fixed connections at the tops of the towers and at the shore anchorages. The eye-bars are of nickel-steel, which is 50% stronger than ordinary structural steel.

The advantages of pin connections with the bracing and suspend-

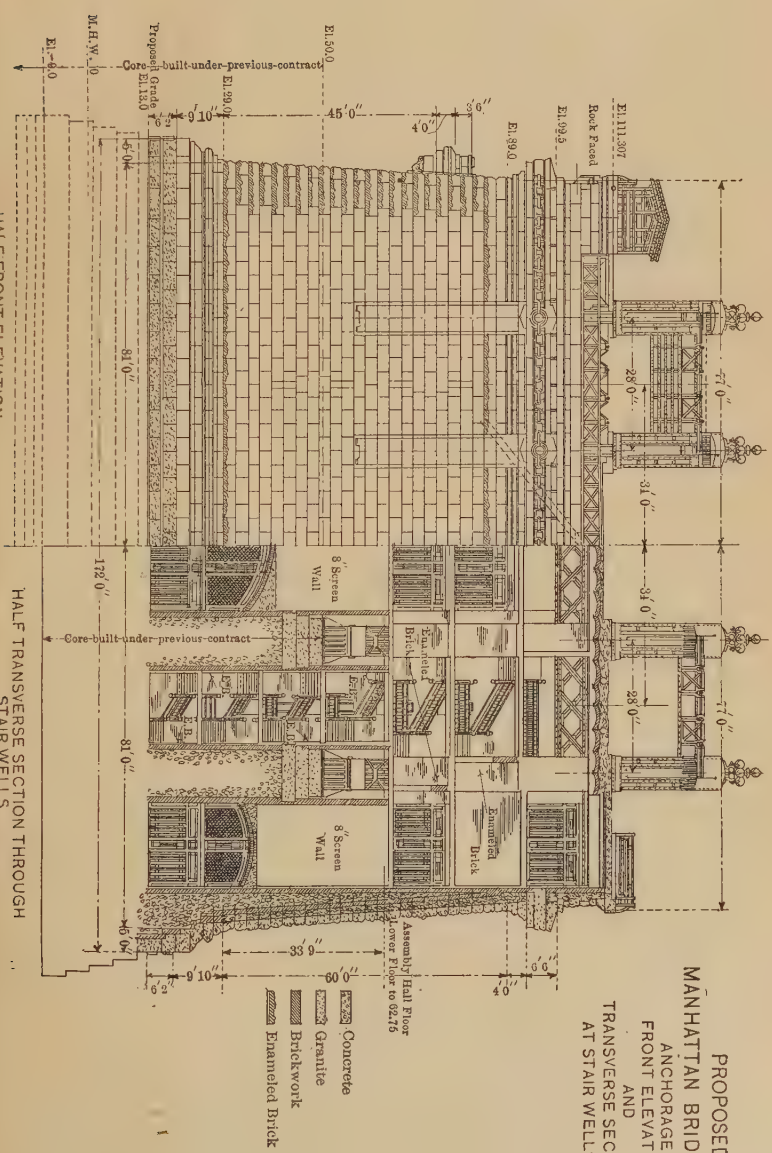
PROPOSED
MANHATTAN BRIDGE (No. 3)
ANCHORAGES
FRONT ELEVATION
AND
TRANSVERSE SECTION
AT STAIR WELLS.
HALF FRONT ELEVATION
HALF TRANSVERSE SECTION THROUGH
STAIR WELLS
Core-built-under-previous-contract
Proposed Grade
El. 13.0
El. 50.0
El. 89.0
El. 99.5
Rock Faced
El. 111.307
M.H.W. 0
El. -9.0
45′0″
9′10″
4′0″
3′6″
6′2″
5′0″
81′0″
172′0″
28′0″
77′0″
34′0″
8″ Screen Wall
Enameled Brick
Assembly Hall Floor
Lower Floor to 62.75
33′9″
60′0″
6′6″
Concrete
Granite
Brickwork
Enameled Brick

FIG. 5.

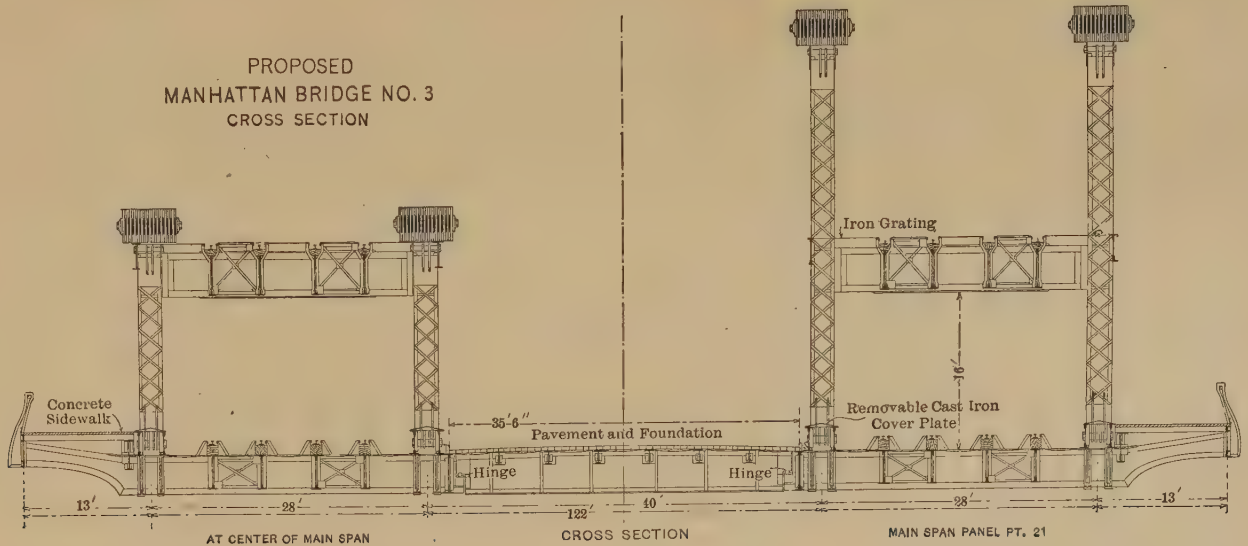
PROPOSED
MANHATTAN BRIDGE NO. 3
CROSS SECTION
Iron Grating
16′
Concrete Sidewalk
35′6″
Pavement and Foundation
Removable Cast Iron Cover Plate
Hinge
Hinge
13′
28′
122′
40′
28′
13′
AT CENTER OF MAIN SPAN
CROSS SECTION
MAIN SPAN PANEL PT. 21

FIG. 6.

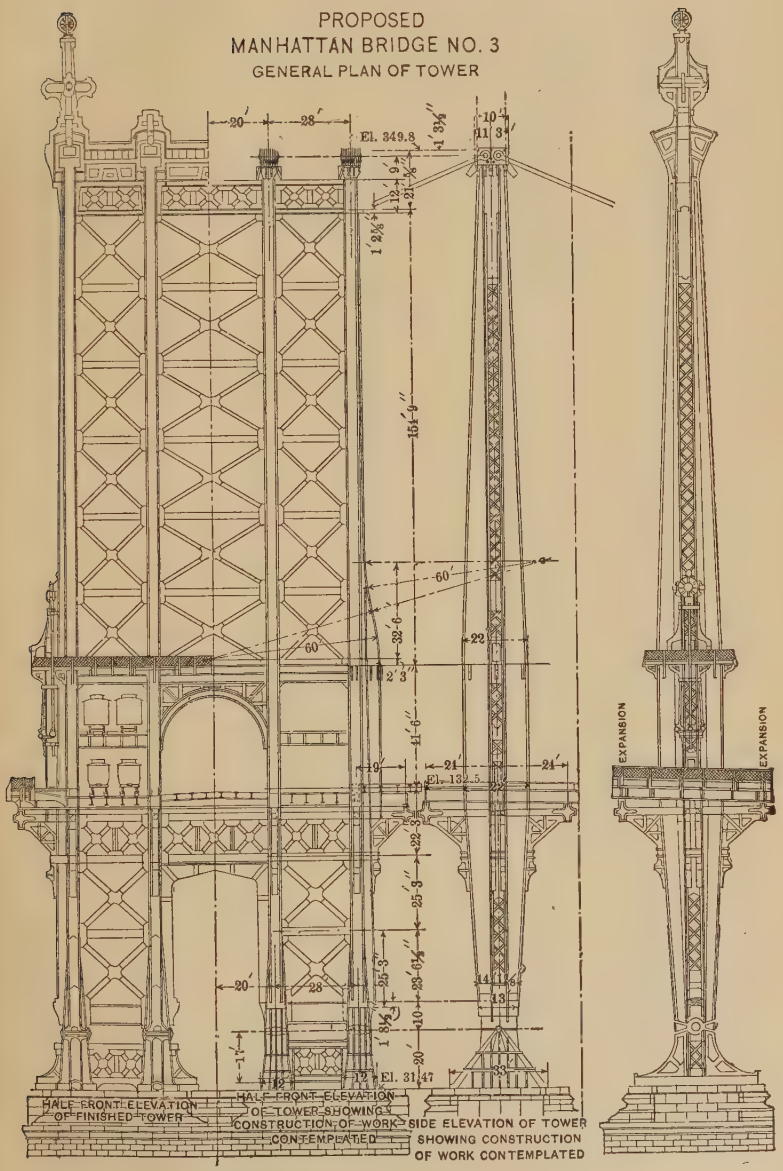
PROPOSED
MANHATTAN BRIDGE NO. 3
GENERAL PLAN OF TOWER
El. 349.8
El. 132.5
El. 31.47
EXPANSION
EXPANSION
HALF FRONT ELEVATION OF FINISHED TOWER
HALF FRONT ELEVATION OF TOWER SHOWING CONSTRUCTION OF WORK CONTEMPLATED
SIDE ELEVATION OF TOWER SHOWING CONSTRUCTION OF WORK CONTEMPLATED

FIG. 7.

ers, and of manufacturing and erecting the superstructure as a connected whole, are too obvious for extended description.

The main chains lie in vertical planes, 20 and 48 ft. from the axis of the bridge, and, at the shore ends, deflect from vertical steel posts into the heavy anchorage masonry.

The platform for the lower deck rests on plate-girder floor beams supported from the suspension chains. These floor beams are hinged at the interior chains, so that the loading of one side of the bridge will not affect the chains on the other side, and all ambiguity in the distribution of load is thereby prevented.

Each tower consists of a single transverse bent of four massive columns on the center lines of the chains. The four columns are vertical, and are thoroughly united by heavy transverse girders and by rigid cross-bracing.

The posts taper up and down from the roadway level in the plane of the chains. The lower end of each post has a pin bearing on a cast-steel shoe, which widens out to a broad base to distribute the pressure on the masonry.

The tower columns can be erected without staging, in the same manner as high chimneys are built.

During erection, wedges are inserted at the bottom of the tower columns, on a line with the pin bearings, as shown on the drawing. By means of these wedges, the verticality of the posts during erection can be readily secured. The wedge details are proportioned to resist a high wind pressure upon the tower columns during erection; after erection the wedges are removed.

Each anchorage is provided with a large transverse arch to allow street traffic through it. The interior cavities in the upper part of each anchorage are utilized for large auditorium halls, comfort stations and shelter rooms, to be accessible by elevators and stairways.

Like the tower piers, the anchorages are to be of concrete with ashlar facing, and will have heavy cornices and mouldings.

The aim has been to combine in the design of both superstructure and substructure simplicity of construction with architectural beauty of outline and details. The structure will be fire-proof throughout. The roadways will have solid buckle-plate floors. The design was not treated as for a mere utility structure, to be handed over to some decorator for adding the usual senseless ornaments, but as a truly artistic

work of the engineer, working together with the architect in the study of form and expression of purpose.

The rigidity of the structure obtainable with this simple method of stiffening is most remarkable, as may be judged from the computed deflection of 22 in. (one-eight hundredth), at the center of the middle span, when this is fully loaded with 4 000 lb. emergency live load per lin. ft. of each chain, and both side spans unloaded.

The plans for this proposed bridge have been passed upon by a board of bridge engineers appointed at the time by Mayor Low, and, from an æsthetic point of view, they have also been approved by the Municipal Art Commission of New York City.

DISCUSSION.

Mr. Hildenbrand.

W. HILDENBRAND, M. AM. SOC. C. E.—Looking at Mr. Lindenthal's paper in the light of a "study," similar to the paper read by the late George S. Morison, Past-President, Am. Soc. C. E., some eight years ago,* the speaker would place it in the same line with the latter, as a work of merit and interest. The design described possesses features which are worth examining and which may find practical application in the construction of suspension bridges.

The foremost feature, as indicated in the title of the paper, is the form of stiffening construction, in the shape of an inverted braced arch. This system has often been applied to smaller bridges, but, the speaker agrees with Mr. Lindenthal, it may also be adapted with advantage to large bridges, though he does not agree with him that it is the only rational stiffening construction, as the paper seems to indicate. Mr. Lindenthal is over-estimating the merits of his design, by attributing to it too many good qualities.

As a special feature of the design, it is pointed out by the author that the height of the stiffening construction is greatest at the points of the greatest moments. The same feature is contained in the Point Bridge, at Pittsburg, built in 1875, which, altogether, represents precisely the same theoretical principles as the present design.

It is certainly good construction, to give a stiffening truss the greatest height at the right place, but it is doubtful whether it is economical in this case, because the bottom chord (or, in the Point Bridge, the top chord) runs to the top of the towers. As it would be very injudicious to transfer the wind pressure to the top of a slender column, special wind chords become necessary, which, as seen from the strain diagram, are almost as heavy as the bottom chords themselves.

In regard to the artistic effect, the drawings and pictures do not convey a complete idea. They give merely the outlines, which are pleasing, and show the appearance of the bridge from a great distance, but the speaker is sure that the large, latticed, box-shaped, bottom chords and posts running high up in the air, will be an unpleasant disturbance to the eye of an observer who is on the bridge itself. The beauty connected with an open suspension bridge will be lost, and this bridge will not differ in appearance from that of a truss or cantilever bridge.

Evidently, there are reasons why all engineers do not believe in the superiority of the braced-arch system. This is demonstrated by examples of suspension bridges recently built. The speaker will not refer to the Williamsburg Bridge, because it has wire cables, but the

* *Transactions*, Am. Soc. C. E., Vol. XXXVI, p. 359.

new Elizabeth Bridge, at Buda Pest, may be mentioned, as the author himself has often quoted it as a model of modern engineering. This bridge has eye-bar chains and other features in common with the Lindenthal design, but it is made rigid with stiffening trusses, and not by the braced-arch system. Mr. Hildenbrand.

Of American engineers, the speaker will quote the opinion of Mansfield Merriman, M. Am. Soc. C E., who was a member of the board of engineers called upon to examine the possibility and feasibility of executing Mr. Lindenthal's plans for the Manhattan Bridge. In his book, "Higher Structures" published in 1898, Professor Merriman concludes a paragraph on stiffening systems of suspension bridges with the following words:

"The suspension truss with center hinge is more rational and more effective than one with unbroken chords. If these laws of development hold good for very long spans, the system of bracing the cables, and thus introducing complexity, does not seem to be in the right direction."

Another American engineer, Joseph Mayer, M. Am. Soc. C. E., whose reputation as a mathematician is well established, states that he made a comparison* between a stiffening truss and an inverted braced arch for a railroad bridge of 3 000 ft. span, and found, by calculating separate strain sheets, that the braced cable would weigh over 5 000 lb. per lin. ft. more, than if stiffened by trusses. He says:

"In view of the practical impossibility of calculating accurately the braced-chain design, and of adjusting it as assumed in the calculations, the unit stresses ought to be taken at least 5% less in this design than in the design with three-hinged suspended trusses, if the same degree of safety is aimed at.

"This would make the difference in economy between the two designs still larger."

These examples show that distinguished engineers in America and Europe prefer other stiffening methods than the braced-cable system.

The strongest argument, if correct, against the usefulness of this system, is furnished by Mr. Lindenthal in the following words of his paper:

"The only recommendation of stiffening trusses is that of a convenient construction independent of the cable, which can be made more or less efficient or costly as may suit the fancy of the designer, or the financial resources of the owners, frequently on the principle that 'half a loaf is better than no bread.'

"From a scientific point of view, the stiffening truss is a crude device, as compared with other modes of stiffening, which do not permit of the same laxity in dimensioning."

In other words, Mr. Lindenthal states that stiffening trusses can be constructed with any desired degree of rigidity or flexibility and cost, but that the braced arch has but one rigidity, one dimension,

* "The Stiffening System of Long-Span Suspension Bridges for Railway Trains," *Transactions*, Am. Soc. C. E., Vol. XLVIII, p. 371.

Mr. Hilden-brand.

and one cost. The speaker does not share this view, that the braced arch could not be proportioned for various flexibilities; but, if this were true, the braced cable would be the most impractical of all stiffening methods, because, rationally, it could only be used in rare and isolated cases for railway bridges, and would be prohibitory for the construction of highway bridges.

A hungry man (using the author's parallel), who is wise, takes half a loaf if he cannot get a whole one, and if the inverted braced arch represents the whole loaf, it cannot readily be sold, as there are too few people who are able to pay for it.

What, for instance, would have become of the intercourse between Brooklyn and New York if the people of those cities had not accepted the Brooklyn Bridge, which, in Mr. Lindenthal's estimation, seems to possess only the value of a quarter loaf? On the other hand, have the people fared badly because they did accept it? The answer to the first question is that there would be no bridge to-day, but plenty of annoyance; and the answer to the second question is that the people have fared well, and are the owners of a large bridge which, every year, carries 100 000 000 passengers safely and comfortably without a single person, excepting Mr. Lindenthal, ever knowing or noticing, that the bridge is not "scientifically stiff."

This leads to the question of rigidity of suspension bridges.

It appears from Mr. Lindenthal's paper that the chief aim of his design is extreme rigidity, which he considers the paramount quality of a suspension bridge. Even strength and economy are subordinate to stiffness. Bridges with less stiffness than is attributed to his design are styled: limber, undulating, improperly or unscientifically designed, inferior, "makeshifts."

Some of these epithets may be applicable to some old bridges, where the speed of travel used to be restricted, but Mr. Lindenthal extends them also to bridges where such restriction does not exist, simply because their rigidity does not reach the degree which he considers standard.

The question is, what is the standard? There is none; but the speaker would like to ask, which of the following two suggestions seems to be the more practical and sensible: One specifying that a bridge should conform to the theory and formulas of a certain professor, with strict limits as to the deflection under enormous loads, regardless of cost, or one specifying simply that the bridge must be capable of carrying its intended traffic with perfect safety and comfort to travelers, regardless of the amount of deflection, but not to exceed a certain cost? It is not absolutely necessary that these two specifications must be separated; but, if they are, the speaker would select the second as a scientifically practical design, which would be chosen by capitalists, while the former would have only mathematical merit, and might be chosen by theorists.

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Referring again to the Brooklyn Bridge, the speaker would ask, is there anyone who was ever prevented from crossing, or who was inconvenienced while crossing that bridge on account of insufficient rigidity of the floor? Has anyone ever heard or read of any complaint that the flexibility of the bridge had restricted or retarded traffic? The speaker has never heard of such complaint, and claims that the Brooklyn Bridge is practically and scientifically stiff, notwithstanding occasional statements to the contrary by people who have never taken the trouble to obtain the proper information.

Mr. Lindenthal states that if the tracks of the middle span were loaded with cars and the side spans unloaded, that the stiffening trusses would be wrecked, if not the whole structure. The speaker would like to see this statement proved by exact calculations before answering it. He knows that it is not true, if the load does not much exceed 2 000 lb. per lin. ft., for which the bridge was calculated. Many unproved assertions are hastily made and carelessly repeated which do injustice to a structure and its builder or builders. It is but just to the latter that such statements are either proved or retracted.

Both Mr. Lindenthal and Mr. Moisseiff describe correctly the difference between an upright arch and a catenary or inverted arch, *viz.*, that the former is in unstable and the latter in stable equilibrium. Mr. Lindenthal continues:

"To that condition many badly designed suspension bridges owe their life; it covers a multitude of sins against good engineering."

Mr. Moisseiff thinks that it is "inducive to the sound sleep of the engineer." The speaker draws exactly the opposite conclusions. If a suspension bridge owes its life to the stable equilibrium of the cable, the engineer who built it committed no sin if he took advantage of that good quality of a catenary and relied upon it; and his engineering is better and he is more wide-awake than the engineer who throws away the free gifts of Nature, and who, after having seen an upright arch, does not look farther, but, by artificial means, forces the inverted arch into a condition which it already possesses naturally. To do so seems to be as injudicious as constructing a hanger, from which a weight is suspended, like a column which supports a weight on top.

The author further says: "Every suspension bridge is theoretically an inverted arch bridge." This may be true, but, practically, there is a great difference between the two, because an arch bridge must be almost absolutely stiff or it collapses, while a suspension bridge can be absolutely limber without collapsing.

The speaker does not wish to be misunderstood as arguing or advocating that a suspension bridge should be without any rigidity; this is not at all his view, as he has shown practically in the bridges he has built. To be serviceable, a bridge should, and must, have

Mr. Hildenbrand.

sufficient rigidity, but the speaker believes in the judiciousness of varying the rigidity according to the demands and conditions of each case. Mr. Lindenthal, on the other side, urges the greatest possible rigidity in all cases, and a bridge without it, though it may be sufficiently stiff, is, in his view, only a "makeshift." The extreme to which Mr. Lindenthal goes is best shown in his own words:

"The merit * * * of a bridge structure, * * * can be gauged by no other criterion so reliably as by the degree of rigidity."

"A suspension structure, to be comparable with other bridge systems, * * * must be dimensioned on the same conditions of strength and stability. When that is done its vaunted economy vanishes."

The last sentence is perfectly true, but, if the principle expressed in the preceding sentence is carried out, not only the economy will vanish, but the suspension bridge itself will go out of existence. Mr. Lindenthal has gone as far as to publish* pictures showing how the Manhattan Bridge, if built on his plan, would look if placed upside down, compared with the Williamsburg Bridge if reversed. He actually used these pictures as an argument for the superiority of his design, because the inverted Manhattan Bridge would stand while the inverted Williamsburg Bridge would collapse!

What is the advantage of such excessive rigidity? There is absolutely none. Is anything or anybody benefited by it? No. Bridges are not built to be put upside down or to satisfy certain abstract analytical formulas. They are for practical use, and a bridge which is safe and answers fully its intended purpose, and, also, is built at the least expense, is more of a scientific structure than one which can boast of nothing but excessive rigidity.

To sum up: The speaker will repeat that Mr. Lindenthal's paper, theoretically and constructively, is a meritorious contribution to suspension bridge literature, for which much credit is due him. If the paper had been confined to a decription of the design in which his theories and detailed constructions were exemplified, the speaker's discussion, if presented at all, would have been short. Mr. Lindenthal, however, has connected with his paper statements referring to other engineering structures and to special engineering principles, which are the cause of this more lengthy discussion.

First, it must be inferred from the paper that the inverted braced arch is the best, if not the only rational, stiffening system. The speaker has shown that distinguished engineers of America and Europe hold different opinions.

Second, the speaker has argued that bridges built on different systems and different principles are not "makeshifts," as they are styled by Mr. Lindenthal, and that the Brooklyn Bridge, which is men-

* *Engineering News*, October 1st, 1903.

tioned specially in the paper, fulfils its purpose and all the conditions for which it was designed. Mr. Hildenbrand.

Third, Mr. Lindenthal has attempted to establish a standard for the rigidity of suspension bridges, and the speaker has shown that this standard is based on arbitrary and unscientific assumptions; that it is uselessly severe and extravagant, and that it leads to absurdities if carried to the extreme.

Probably the essence of the whole of Mr. Lindenthal's paper may be comprised in this: That a suspension bridge, in order to be properly designed should have the highest attainable rigidity; while the essence of the speaker's whole discussion may be expressed in the words that a suspension bridge, if properly designed, should have the greatest possible flexibility which is compatible with safety and the practical purpose of the bridge.

Joseph Mayer, M. Am. Soc. C. E.—The title of this paper suggests that previous forms of stiffened suspension bridges are irrational, and the body of the paper contains various assertions condemning other forms which have been developed by experience and which are justified mainly by two fundamental facts which have controlled the growth of suspension-bridge design in the past and will do so in the future. These two facts are the superior strength of steel wire and the difficulty of making satisfactory connections between a wire cable and the web members of a stiffening truss. The independent stiffening truss has been a necessary consequence of these two facts. The expense of wirework limits the economic usefulness of suspension bridges to long spans. Mr. Mayer.

Highway bridges of more than 800 ft. span, and railway bridges of more than 1 300 ft. span, can, under favorable conditions, when rock anchorages are available, be built economically of the suspension type. The speaker designed a cantilever and a suspension bridge across Sydney Harbor with a span of about 1 300 ft., for a double-track railway and a 60-ft. highway. The bids submitted for both designs were nearly the same. Natural rock anchorages were available. Subsequently, he designed another suspension bridge for the same site. The design was found to be by far the lightest of many cantilever and suspension-bridge designs submitted. The specifications for both kinds of bridges were substantially identical. For much less than 1 300 ft. span, a railway suspension bridge is uneconomical and cannot compete with a cantilever bridge. At Quebec the chief engineer in charge preferred the cantilever type, and stated that a suspension bridge would only be considered if there was a difference in cost of at least $500 000 in its favor. Wire cables of more than 200 000 lb. ultimate strength and 180 000 lb. elastic limit per square inch cost erected at most two and one-half times as much per pound as eye-bar cables made of nickel-steel of one-quarter the elastic limit. The bar heads

Mr. Mayer. and pins of the latter add about 25% to the weight of continuous bars of the same strength. This shows that wire cables cost only one-half and weigh about one-fifth as much as eye-bar cables of the same strength. The difference in weight becomes of extreme importance for very long spans.

A suspension bridge of three spans made of eye-bar cables is inferior in economy to a cantilever bridge. The type of bridge recommended in the paper, therefore, could only claim superiority on account of its appearance. The pictures published of the design proposed are deceptive because they do not show the cross-bracing between the stiffening trusses, which will detract greatly from the graceful appearance secured by the use of cables without bracing.

A cantilever bridge can be built on substantially the same outlines as the bridge proposed by the author; it is much stiffer, which, in the opinion of the author, gives it superiority, and it is far cheaper and looks equally well.

To abandon wire cables means practically to abandon all the advantages of a suspension bridge. Any suspension bridge without wire cables, therefore, can hardly claim to be called pre-eminently rational. One of the great advantages of the suspension-bridge type of structure is the possibility of building a perfectly safe bridge which will show very little deformation under the usual highway-bridge loads, and which will avoid excessive stresses under partial loads extending over either one-half of the main span or one or two of three spans. This advantage can be secured by the use of shallow stiffening trusses which are independent of the cables.

Such partial loads may never occur during the whole lifetime of the bridge. A deflection of several feet at the center or the quarter of one of the spans, produced by such imaginary distribution of the live load is no disadvantage whatever. The high degree of stiffness insisted upon by the author is absolutely useless for a highway bridge or for a bridge with so many independent units of load as will use any East River bridge not intended for heavy steam-railway trains.

The serviceableness of a bridge is not proportional to its rigidity. A certain degree of rigidity is needed; anything beyond is useless. An 80-ft. plate girder has a much greater deflection than an 80-ft. truss bridge of equal strength; the former is preferred by most, in spite of it.

The degree of rigidity required is very different for a bridge used by heavy railway trains of half the length of the main span and for a highway bridge of many roadways subject mostly to approximately uniform loads. For the former, a three-span suspension bridge, with the side spans of half the length of the main span, is very unfavorable, unless the end spans are held in the center.

In most situations the three-span suspension bridge, for railway

purposes, is an uneconomical type. The shallow stiffening truss, which is the most suitable for highway suspension bridges, gives too large deflections in a railway bridge. The stiffening truss which reduces the deflections just sufficiently for the needs of a railway bridge allows so little deformation of the cables that it alone has to carry the larger part of the moments and shears produced by partial loads. A much deeper stiffening truss than required for stiffness then becomes economical, for securing the required strength. The depths of the stiffening truss given by the author are nearly those suitable for a railway bridge. Mr. Mayer.

The author's assertions in regard to the relative size of the temperature stresses in hinged and continuous stiffening trusses are totally in error, as the writer has shown in his discussion with him.* A deep stiffening truss must be hinged in the center and at the ends of the main span. In this case the temperature stresses become small. For the writer's design of 2 800 ft. span, of the Hudson River Bridge, they amounted to less than 4% of the moving-load stresses. Without hinges, a deep stiffening truss is uneconomical on account of the excessive temperature stresses.

The most economical depth for a railway suspension bridge stiffening truss, hinged at the center and ends, is about one-eighteenth of the span. This depth is needed at the center of the half span. The economic general dimensions of each half span are about those of an independent span of the total length of the half span of the suspension bridge.

It is easy to design a center hinge allowing vertical motion and yet preserving the continuity of the bottom chords, which serve as chords of the lateral truss; the drawback of a center hinge mentioned by the author, therefore, does not exist.

When the span is very long and heavy the center hinge is best designed so that there is a hinge in the horizontal wind truss as well as in the vertical trusses. The lateral truss of each half span is then separate. The cables take, in this case, without excessive lateral deflection, the wind pressures transferred by the wind trusses to the center of the main span. The wind stresses of the top lateral system, which come to the center of the span, must be taken down to the bottom chords by means of overhead cross-bracing with brackets. The weight of the lateral system is greatly reduced by this arrangement, since each lateral truss is only half as long as the main span. This arrangement was adopted in the writer's design of the Hudson River Bridge, and was approved by a commission of eminent engineers.

The author's design is an attempt to revive an almost defunct type of bridge which is much inferior in economy to a properly de-

* *Engineering News*, November and December, 1901.

Mr. Mayer. signed wire-cable suspension or a cantilever bridge, and much inferior in looks to a suspension bridge with a shallow stiffening truss offering abundant stiffness for highway traffic.

One feature deserving high praise in the design of the author is the hinged tower. Such a tower, however, harmonizes much better with light unbraced cables than with heavy braced trusses. It has been adopted in the revised design of the cable suspension bridge for the same site, and is a great improvement on all earlier designs.

Mr. Buck. R. S. Buck, M. Am. Soc. C. E.—The spandrel-braced suspension bridge is a most interesting study, with claims to merit chiefly due to the degree of rigidity furnished by the depth of trussing at the quarters, which, for certain service and conditions, is inadequately furnished by the familiar type of suspension bridge, with dip of cable and depth of truss of economical proportions.

Much interest is added to the subject by the exhaustive mathematical treatment to which it has been subjected in the two supplementary papers by Mr. Moisseiff* and Mr. Cilley.†

It may be unfortunate that bridge design and construction are not more exclusively a matter of mathematical calculation; but it is a condition which cannot yet be escaped that the controlling considerations in gauging the rationality of a bridge design are based upon actual experience, rather than upon the attenuating processes of the higher mathematics. While mathematics forms an essential part of the foundation of bridge designing, it has serious limitations, most of which can only be supplied by a rational study of accomplished works.

While there will probably be no disposition to question the correctness of the two mathematical demonstrations, especially as they are mutually confirmatory and not antagonistic, adequate knowledge of the controlling physical and economic conditions, as well as theories of suspension bridge design, construction and maintenance, precludes acceptance of the author's implied conclusion, that the familiar type of wire-cable bridge is hopelessly wrong in principle, as well as thus far imperfect in its applications, and that the spandrel-braced eye-bar chain type is the "rational" solution of all the difficulties and objections found in the other.

The present high development of the art of bridge construction is due rather to the gradual improvement in the works of the many along familiar lines than to the bold conceptions of the few on lines radically new. No type of bridge can attain a high degree of excellence in one or two applications. In such complex problems several applications are necessary to develop all the difficulties and prove their correct solutions.

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* *Transactions*, Am. Soc. C. E., Vol. LV, p. 94.

† *Transactions*, Am. Soc. C. E., Vol. LIII, p. 413.

Mr. Buck. While the spandrel-braced chain bridge may in time become as superior as the author now considers it, its superiority can hardly be considered as proved in the light of accomplished facts; no more than in the same light it can be proved that the controlling principles of the wire-cable suspension bridge are unsound and inadequate.

The chief claims made for the "rational form of stiffened suspension bridge" are

1.—Greater rigidity and consequent general superiority;
2.—Superiority of details, due chiefly to pin connections;
3.—Greater ease and speed of manufacture and erection;
4.—Greater economy, chiefly in the eye-bar chains, as compared with wire cables.

1.—The greater rigidity of the spandrel-braced chain bridge, as compared with the wire-cable bridge with a stiffening truss of practicable depth, is, as a matter of theoretical mechanics, clearly apparent, but the universal value and certainty of proper action of this rigidity are not yet demonstrated either theoretically or practically.

It should be remembered that the stresses in a stiffened suspension bridge of any form are more or less indeterminate under varying conditions of loads and temperature. This indetermination cannot be entirely removed by mathematical treatment, nor by the degree of accuracy of adjustment practicable in bridge construction.

When a high degree of rigidity is imposed upon an indeterminate structure, the risk of serious consequence, because of departure from assumed and theoretical conditions, becomes materially greater than in a structure where there is enough flexibility to permit any overloaded member to pass part of the overload along to others and obtain relief.

In the deep spandrel bracing with long panels, light sections with constant reversals in the stiffening members and with adjustable diagonals, there cannot be the same freedom from serious indeterminate stresses that there is in the familiar form of stiffening truss of moderate depth and minimum number of adjustable members. In the rigid attachment of comparatively light bracing to a long, broad and massive chain, supported independently of the bracing, maintenance of assumed conditions within proper limits is not assured.

The author states (p. 1) that:

"All the older suspension bridges are limber and undulating structures, without adequate provision against deformation under moving loads."

While the stiffening systems of the older suspension bridges have not proved adequate to meet the excessive and unanticipated demands on their strength and endurance, there are very few, if any, cases where failure has occurred, except when the loading was far beyond that provided for. Wherever inadequacy has developed or failure occurred

Mr. Buck. it has been due to imperfect structural details or to imperfect knowledge, in designing, of the actual conditions of stress created by the loading, not to any inherent or incurable weakness in the principle of design.

The author's illustrative comparison between the Forth Bridge and the Brooklyn Bridge (p. 2) is not quite clear as to point, especially as the Forth Bridge was duly designed to carry the load which he states it will carry, while the Brooklyn Bridge was not designed to carry the load which he states would wreck it.

The failures of the stiffening system of the Brooklyn Bridge were not due to the lack of rigidity, but primarily to the improper loading to which it was subjected, and secondarily to the interference with its uniform deflection by the overfloor stays, coupled with the inadequate form of section of the bottom chord. These are conditions and details of construction readily remedied, and not at all inseparable from the general form of design.

In suspension-bridge design, as in all engineering work, the most valuable lessons are to be learned from a rational study of failures and their causes. There is serious insecurity in speculation that indiscriminately consigns to the junk heap accomplished works, which, though imperfect, are valuable in that they have served often far beyond the original purpose of the design, and have marked by actual performance sound and unsound features of design.

The author states (p. 4):

"The merit and serviceableness of a bridge structure, other things being equal, can be gauged by no other criterion so reliably as by the degree of its rigidity."

Again (same page):

"Absence of sufficient rigidity marks the inferior bridge."

"Rigidity" is a term often misapplied to metal structures and confused with "Stability" and "Strength."

"Sufficient rigidity" is the condition in a bridge where there is no overstraining of parts, where the motion of parts under deflection does not occasion wear, and where the deflection is not so great as to impede traffic. These conditions can be fully met in the familiar type of suspension bridge, and at the same time permit of much greater deflection than is possible with safety in a simple truss, arch, cantilever, or spandrel-braced chain bridge.

The matter of deflections in suspension bridges was fully covered in the investigations and discussions of the North River Bridge project, whereby, as the author suggests, "Much of the fog that surrounds the theory of rigidity was cleared away."

Major Raymond, in referring to the conclusions of the Board of Army Engineers appointed to investigate this project, wrote as follows: *

* *Transactions*, Am. Soc. C. E., Vol. XXXVI, p. 459.

" The Army Board devoted considerable attention to this question. Mr. Buck.
It remarked that ' the great distinction between the stable equilibrium of a suspension bridge, which cannot break down from the failure of any stiffening member, and the unstable equilibrium of a truss, arch or cantilever bridge, in which the failure of a member may involve the collapse of the entire bridge, ought to receive full recognition in the adoption of unit stresses and safety factors.' Again, the Board remarked that ' rigidity is in this case of much less importance than it is in most other kinds of bridges; indeed, it may be shown that a certain small flexibility is a positive advantage in suspension bridges;' and still again, 'the Board does not doubt that within narrow limits a certain degree of flexibility is an advantage to the bridge. Deflections in a system of stable equilibrium do not impair the safety of the structure, as they do in an unstable system like the upright arch, and they may exert a very beneficial influence in modifying the dynamic effects of a rapidly varying live load.'"

George S. Morison, Past-President, Am. Soc. C. E., wrote as follows: *

"A long span suspension bridge necessarily changes its shape with every change of load, and changes, too, in such manner as to relieve local strains, every unstiffened suspension bridge having some shape of perfect equilibrium for every possible loading. These changes of shape play an important part in proportioning a suspension bridge, and so long as they are kept within limits which do not disturb convenience of operation, they are a source of strength instead of weakness. A suspension bridge must be permitted to change its shape within proper elastic limits, and this change of shape must be made the basis of calculations in proportioning the structure."

The foregoing conclusions have long remained unquestioned, and are sound principles of design, established by practice as well as by theory.

"Sufficient rigidity" is not a fixed function in suspension bridges for all classes of service. This must be much greater for the 1 800-ft. railroad bridge at Quebec than for the 1 470-ft. city highway bridge required in the case of the Manhattan Bridge. In the former case, the stresses in the stiffening system, due to the heavy trains it must carry, will be great and frequent, and the tendency to excessive deflection will be present with every passing train. In the latter, the loadings will rarely depart so greatly from uniformity as to cause considerable stiffening-truss stresses or material deflections. No loading that could possibly occur in the legitimate use of the bridge could cause serious stresses or objectionable deflections.

In this case, with ample strength in cables and suspenders, high unit stresses can be allowed in the stiffening truss with perfect safety, and the maximum deflections under assumed possible conditions of loading can be regarded as of only theoretical interest.

The speaker fully agrees with the author in his conclusion (p. 2) that:

* *Transactions*, Am. Soc. C. E., Vol. XXXVI, p. 367.

Mr. Buck. "A suspension structure, to be comparable with other bridge systems, therefore, must be dimensioned on the same conditions of strength and stability."

In fact, all suspension bridges are stronger and more stable than other forms of construction, not only on account of being in stable equilibrium, but because they are generally built with a greater factor of safety in the most vital parts—cables and suspenders.

There is far more strength and greater security, in the thoroughly tried material and high safety factors of the wire cables and suspenders, than can be supplied by the increased rigidity due to deep spandrel bracing.

The minimum elastic limit of the steel wire to be used in the cables of the Manhattan Bridge is 180 000 lb. per sq. in. The allowed stress under maximum working load is 60 000 lb. per sq. in. The factor of safety is 3.

The minimum elastic limit of the nickel-steel in eye-bars specified for the same bridge is 48 000 lb. per sq. in. The allowed stress under maximum working load is 30 000 lb. per sq. in. The factor of safety is 1.6. Therefore, the strength of the cables, based on elastic limits, is almost twice as great as that of the chains.

While the uncertainty, which now surrounds the successful and economical manufacture of 18 by 2-in. nickel-steel eye-bars, may in time be removed, it has not yet been removed, and the likelihood of the existence of a defective component unit, and the injury therefrom, are many times greater in a chain than in a cable. This cannot be compensated by rigidity.

The author states (p. 4) that:

"From a scientific point of view, the stiffening truss is a crude device, as compared with other modes of stiffening, which do not permit of the same laxity in dimensioning."

He then proceeds to demonstrate this from the evolution of the arch, on the hypothesis that "every suspension bridge is theoretically an inverted arch bridge." The analogy between the arch and the suspension bridge is extremely narrow, and is confined to the fact that the cable of the suspension bridge and the rib of the arch produce horizontal reactions and are both treated in calculations as force polygons. In possibilities of span, in construction, in instability of equilibrium and consequent necessity for greater rigidity, the arch is wholly removed from the suspension bridge, and is an extremely inadequate basis for rational conclusions regarding the latter.

The author's "principal difference"—stable equilibrium in the one case and unstable equilibrium in the other—completely destroys the practical value of the analogy.

2.—The author states (p. 10) that:

"The advantages of pin connections with the bracing and suspenders, and of manufacturing and erecting the superstructure as a connected whole, are too obvious for extended description."

Mr. Buck.

It is hoped that the author will ultimately furnish the "extended description" withheld, as the merits of pin connections are not recognized as universally as he infers.

Apparently, there has been some change in the author's views since he stated,* of the study of a North River bridge design, by Mr. George S. Morison, that: "* * * * Another good feature of his stiffening truss is the riveted connections, with web members riveted up also at their intersections. It greatly helps to diffuse the bending strains throughout the frame, and the so-called secondary strains at all the connections and intersections are a decided gain on the side, not only of greater stiffness, but greater safety, contrary to the received theoretical considerations on this subject."

"The received theoretical considerations on the subject" are presumably the prejudices in favor of pin connections.

In bridge practice, pin connections have been greatly superseded by riveted connections.

It has been found that pins wear seriously when members assembled on them are subject to frequent reversals. Especially is this the case in suspension bridges where the diagonal members of the stiffening system are adjustable rods. This feature is the most serious element of weakness and deterioration in the Brooklyn Bridge, not its lack of rigidity. It was a serious element of weakness in the Niagara Railway Suspension Bridge. In both cases, the constant shifting of the rods on the pins caused wear and loss of adjustment. On the Brooklyn Bridge there has been quite extensive renewal of pins and diagonal bars.

In the designs covered by the author's paper, there must inevitably be the same shifting of the diagonals on the pins with every passing load. While the wearing action in this case, of course, would not be as great as in the Brooklyn and Niagara Bridges, it must exist. The narrow limits of deflection of a spandrel-braced chain bridge would not prevent this wear, as the amount of distortion necessary to cause the shifting is very small. Should it be attempted to put such initial stress on the diagonals that they could not shift on the pins, except under unusual loads, there would be likelihood of serious overstressing due to temperature changes, as well as to serious indetermination in the stresses.

This is evidently recognized by the author, as he states (p. 8):

"It should be remarked that in this system it is preferable that the cable assume its proper equilibrium curve under full dead load, and that the diagonals, which are without strain at a middle temperature, be made adjustable in length. The diagonals are strained only from live load and temperature changes."

In the later designs of wire-cable suspension bridges, the use of adjustable members and pins is reduced to a minimum, the trusses

* *Transactions*, Am. Soc. C. E., Vol. XXXVI, p. 444.

Mr. Buck. being riveted throughout. In such trusses, flexure can take place without wear, and, on account of their shallowness, deflection can be much greater, without injurious effect, than can occur under the heaviest possible traffic.

The advantage of the so-called "positive attachment" of the suspenders to the chains by pins, over the attachment to the cables by bands, is not apparent. The older forms of cable bands, despite their crudeness and the weak grip they have on the cables, have proved remarkably efficient, and have given very little trouble. They sometimes slip when first put on, but it is a simple matter to check this and restore adjustment.

A cable band can be provided with almost any degree of grip on the cable, which will be as positive as can possibly be required, even by the demands of suspender loads greater than have yet been provided for.

Fig. 8 shows the cable bands of the Brooklyn Bridge, the Williamsburg Bridge and the proposed Manhattan Bridge.

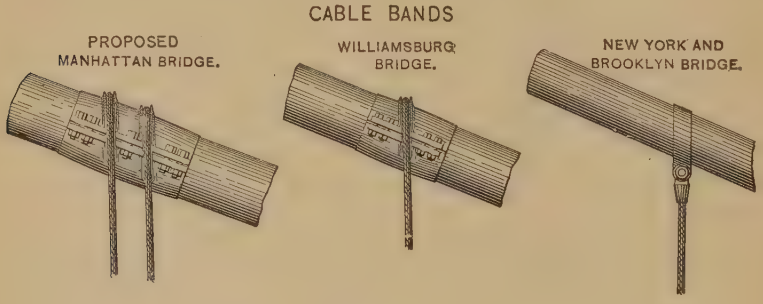


Fig. 8.

It can hardly be questioned that, as the first has proved reasonably efficient and satisfactory, the others are amply so.

In the cable design, the floor system is connected with the stiffening system in such a manner that the latter distributes the moving panel loads and their impact effects over a number of suspenders. In the chain design, the suspenders are interposed between the floor beams and the stiffening trusses, and, without any relief from the trusses, must receive the full dynamic effect of the moving loads. In other words, the suspenders are merely more or less lengthened floor-beam hangers—a form of construction now avoided when possible.

In the cable design, it is an added element of security, as compared with the chain design, to have more points of attachment between the suspended structure and the cables. In the case of the proposed Manhattan Bridge, these are about 43 ft. apart in the chain design

and 18 ft. apart in the cable design. The more numerous the suspenders, the less serious the effect of a defective unit, and the better the distribution of the load into the cables. Mr. Buck.

3.—It is difficult to make a positive comparison between the eye-bar chain and wire cable, in the matter of time and cost of construction, as what can be accomplished in the former case is wholly a matter of conjecture.

Previous to the construction of the cables of the Williamsburg Bridge, it was quite generally supposed that wire-cable construction must necessarily consume a serious length of time.

In this case, the time actually consumed in stringing the wires of the cables was seven months, including time lost on account of weather, the work being carried on during the winter.

There were, of course, many serious delays, but causes for the greater part of these were wholly foreign to the method of construction; while those for which it was responsible are now, in the light of the experience acquired, capable of being reduced materially or avoided altogether.

Causes for delay invariably attend novel methods of construction in greater numbers, where difficulties must first be developed by actual performance and then the means of surmounting them be worked out.

In cable construction, all the essential problems—both economical and structural—pertaining to the manufacture of material and erection have been solved.

In the construction of eye-bar chains of the size proposed by the author, most of the essential problems pertaining to commercial manufacture of material and erection must yet be solved, whether the chains be made of wire links or nickel-steel eye-bars.

In the case of the chains, the temporary bridge required for erection must be many times stronger and heavier than would be required in the construction of the cables, and, while threading 18-in. eye-bars weighing $3\frac{1}{2}$ or 4 tons on 18-in. pins, under distinctly difficult conditions, may be economical and expeditious, in the absence of explicit demonstration to that effect, skepticism is at least pardonable.

4.—The author, in the course of his paper, makes several statements regarding the economy of features of the spandrel-braced chain design, and lack of economy of the wire-cable design, from which it is natural to infer that he wishes to convey the impression that the former as a whole is distinctly more economical than the latter.

While this point is not capable of conclusive and specific settlement at the present time, on account of the lack of adequate information on which to base the cost of a spandrel braced chain bridge of the proportions of either the Manhattan or the Quebec Bridge, there are certain established facts which tend strongly to prevent the formation of this impression.

Mr. Buck. The relative cost of wire cables and so-called equivalent eye-bar chains may be, from known conditions and contract prices, so far deduced as to show that the chains must cost very much more than the cables.

The contract price of the cables of the Williamsburg Bridge was a little less than 14½ cents per lb., erected.

The contract price of the nickel-steel eye-bars for the Blackwell's Island Bridge was 8.03 cents per lb., and of the nickel-steel pins 10.03 cents per lb., erected. These bars were to be 16 by 2-in., with 38-in. heads.

On the one hand, the price of the Williamsburg Bridge cables was doubtless higher than could be obtained now, because many difficulties originally anticipated in procuring the specified grade of wire, and in attaining economical and reasonably rapid construction in building the cables, have now been satisfactorily solved. Further, prices of all such work were very much higher at the time this contract was let than have prevailed for the past year.

On the other hand, the cost of manufacture of the 18 by 2-in. eye-bars with 44-in. heads proposed is wholly unknown, as no such eye-bars have yet been made, and there are doubts as to the practicability of making them, except at a prohibitory cost.

Further, the difficulties and cost of erection are equally unknown. However, in view of the fact that elaborate special plant must be provided for the erection of the chains, which will be chargeable to these alone, while in the Blackwell's Island Bridge the same erection plant will handle and support the eye-bars which will handle and support the remainder of the work, the conclusion is fully warranted that the erection cost in the former case will be materially greater than in the latter.

In view of the known contract prices and the other conditions named above, it appears almost impossible that, at the present time or in the near future, the unit prices of wire cables and nickel-steel chains would be farther apart than 14 to 10, respectively, which, because of the chains being two and one-quarter times as heavy as the cables, would make the cost of the former, on this basis, 60% greater than the latter. There is warrant for the belief that the difference would be still greater.

The chains and cables are but single items of cost in the respective designs, but they are the items wherein material advantage in point of economy has been claimed for the chains.

A complete and conclusive comparison of cost of the two designs is not possible, in the absence of complete detailed plans of the chain design and fuller development of the actual conditions of manufacture and construction involved. However, there are no grounds whereon to base the claim that, as a whole, the chain design is more economical than the cable design.

To summarize: It does not appear that in either of the cases cited by the author—the Manhattan Bridge or the Quebec Bridge—is there any substantial evidence that the spandrel-braced chain design possesses adequate grounds for preference over the other designs proposed for the same cases. Mr. Buck.

If the bridge is for varied city traffic, where, whatever the weight of the aggregate loadings, material departure from uniformity is rare, and where appearance must count for so much, the rational design for long spans still appears to be, in the light of the best available evidence, the wire-cable suspension bridge.

If the bridge is for heavy railroad traffic, where departure from uniformity of loading is great and frequent, and where, therefore, a high degree of rigidity is essential, the rational design for long spans still appears in the same light as the cantilever, until the length of span approaches 2 000 ft., when the wire-cable suspension bridge again appears as a strong competitor, even for the heaviest class of railroad service.

W. W. Crehore, M. Am. Soc. C. E.—One point that the speaker would like to emphasize is that this question seems to turn on the degree of rigidity it is sought to obtain. Most engineers have had experience with smaller structures than these under consideration, and it will be pretty hard to convince any engineer who has had experience with railroad and similar bridges that rigidity is not a good thing. Most engineers believe that rigidity is a good thing; that rigidity is something which prevents abnormal wear and tear in a bridge; that is, of two bridges of the same span and for the same load, that which is the more rigid will last the longer, and, therefore, is more economical from that point of view, if not from any other. Mr. Crehore.

If rigidity is a good thing in a truss bridge, why should not rigidity be a good thing in a suspension bridge? The same laws of mechanics govern in a suspension bridge that govern in a truss bridge. Will not the distortion of a structure always cause some wear and tear which should be avoided as much as possible?

The point to be brought out is this: A suspension bridge takes a fixed position from its dead load. The amount of distortion caused by bringing on a live load depends upon two things, (1) the effectiveness of the stiffening system, and (2) the ratio of the live load to the dead load per linear foot. In other words, the inertia of the structure's dead weight prevents some distortion from taking place, and the stiffening system prevents some—the remainder takes place.

Of two bridges constructed alike, if a heavier live load is put on one than is put upon the other, it is expected that the former, in passing across the bridge, will produce the greater distortion. Now, the longer the span, the greater must be the dead load to carry the same live load; and, of two bridges constructed of the same span, that

Mr. Crehore. which must carry a heavier live load must have the greater dead weight. The amount of vibration or distortion, therefore, varies with the ratio of the live load per linear foot to the dead load per linear foot, other things being equal.

A suspension span across the North River would be about 3 000 ft. long, and one across the East River would be considerably less than 2 000 ft. long. Probably a sufficient live load for modern railway traffic over a North River span would be that assumed in Mr. Morison's design, *viz.*, about one-fourth of the dead load; whereas the emergency load which ought to be provided for any East River span being built to-day is about one-half of the dead load. The traffic for which the old Brooklyn Bridge was designed was one-fourth of the dead load; and, with such a traffic that bridge is comparable to a bridge carrying the modern railway traffic over the North River. In the speaker's opinion, the gentlemen who have have discussed this paper have made the mistake of comparing a bridge for modern traffic over the East River with a bridge of the same span to carry the traffic of twenty years ago. The two are not comparable, except when the live load (the only part of the load which causes distortion) is the same part of the total load in each case.

The modern East River traffic must be understood to include some provision for the future, such as heavily loaded automobile trucks massed close together, and an increase in weight of the railroad rolling stock. An emergency loading of this sort concentrated in one short stretch of roadway would amount to about one-half of the dead load of the bridge per linear foot; and, while this would have no injurious effect upon the cables of a suspended cable structure, it would cause excessive distortion, injurious to the stiffening system, unless the stiffening trusses were unusually deep. Here, again, the fact must be met that, when the stiffening trusses are of too great a depth, excessive loading will react injuriously on the cables, preventing their assuming an easy or natural position (as it were), and causing undue local strains. It works around in a circle, showing that when the limit of deflection is assumed to be fixed and must not be exceeded, the feasibility of preventing further distortion by means of stiffening trusses is limited, and the process cannot be continued indefinitely by adding to the depth or to the material of the stiffening trusses. To be really effective, the dead weight of the cables themselves would need to be increased so as to provide sufficient inertia to absorb a considerable part of the excessive loading before distortion should begin. Such a method of increasing the dead weight, however, would be very costly and is prohibitory on the ground of wasteful expenditure as long as any other method is known whereby sufficient rigidity may be obtained more cheaply.

Now, Mr. Lindenthal's paper illustrates this truth. It brings out

the fact that the braced suspension design will furnish the required degree of rigidity under the excessive loading referred to, that is, where the live load is about one-half the dead load. This result is accomplished by putting the principal dead weight of the bridge into the cables, and practically dispensing with the stiffening trusses entirely; or, better, incorporating the stiffening and the cables together in one system. If it were attempted to obtain the same amount of rigidity in a suspended-cable design as can be obtained in the braced-cable design—the live load per linear foot being one-half the dead load in both cases—it would be necessary to use stiffening trusses of such great depth that the design would be at once condemned on account of its disproportionate and ungainly appearance; and, on the other hand, the people of New York City do not want any more bridges on which the police are required to regulate the traffic to prevent possible congestion injurious to the structure. Mr. Crehore.

Regarding those passages quoted by Mr. R. S. Buck from the *Transactions* of this Society, in which Mr. Morison and Major Raymond express opinions on different designs for a 3 000-ft. span across the North River, the speaker understands them to refer to cases where the traffic load per linear foot would not exceed one-fourth of the dead load per linear foot. When it is proposed to bridge spans of from 500 to 1 000 ft., where the live load is as much as or more than one-half the dead load, a suspension bridge design is usually discarded as inappropriate and of insufficient rigidity, or as requiring too great an expenditure to make it rigid. Why should not this ratio of live to dead load receive the same consideration on longer spans? In his admirable paper, just referred to, Mr. Morison states that the first thing to do in designing a suspension bridge is to establish the limit within which deflection may be permitted, and then to design to it. This deflection limit, however, depends somewhat upon the ratio of the live to the dead load, as well as upon the material and the shape of the structure; and all of Mr. Morison's subsequent discussion of the subject leads to the conclusion that there is a practical limit to the use of a suspended-cable bridge, and that this limit is reached when stiffening trusses cannot be designed sufficient to restrict the amount of deflection permissible by the nature of the traffic.

Granting, then, that such a limit exists for the suspended-cable bridge, and that the author's braced-cable design furnishes a method of supplying the necessary rigidity far beyond that limit, it seems to the speaker that this discussion should be upon the question of where the economy of the suspended-cable design leaves off and that of the author's design begins, rather than upon the simple question of whether or not rigidity is a desirable quality in bridges of any kind.

Theodore Cooper, M. Am. Soc. C. E. (by letter).—The writer, in his report to the Quebec Bridge Company, on June 23d, 1899, stated Mr. Cooper.

Mr. Cooper. as follows, in reference to the plan described in Mr. Lindenthal's paper:

"The lines of the structure are very pleasing, giving a combined effect of grace and strength. The catenary curves of the cables are not crossed or broken by the stiffening trusses.

"The design appears, from an ordinary examination, to be in accordance with the requirements of the specifications. A stiffening truss of this kind could not be used successfully for bridges formed with continuous wire cables, as the connections of the various members of the truss would have to be made through the frictional grip of cable bands, which would not be trustworthy. The success of such a truss depends therefore upon the use of wire links for the cables and a positive connection of all the members by means of pins. That such links can be made is undoubted, but their successful and economic manufacture has yet to be developed."

At that time the suggestion was made that there might be future cases where eye-bars could be used for this form of bridge.

Engineers, through the daily and technical press, have conveyed the idea that "eye-bar cables" are new and untried forms.

Without detailing the earlier suspension bridges, which were all link-cable bridges, the fact that the great majority of bridges in North America, over 100-ft. spans, both railroad and highway, are dependent upon "eye-bar cables" for their lower chords, seems to have been overlooked even by some eminent engineers.

The writer would consider it a low estimate to state that there are more than 200 miles of "eye-bar cables" in successful use in the United States, many of which have done excellent duty for a full generation.

The first important structure with which the writer was identified, the St. Louis Arch Bridge, was erected by means of eye-bar cables more than 500 ft. in length, and these same cables, more than 2 600 ft. in total length, are now in use in the Fairmount Bridge, Philadelphia, and in bridges elsewhere.

The "eye-bar cable," forming the top chord of the Quebec Bridge, is now under construction and is formed of eye-bars 15 in. deep.

The eye-bar and the eye-bar cable are the peculiar and especial characteristic of American bridges, and it is rather late in the day for any engineer to discover that they are new and untried forms.

All the large suspension bridges built by the Messrs. Roebling at Niagara, Cincinnati and Brooklyn have eye-bar cables in their anchorages.

In the eye-bar cable, engineers know by actual test the strength of the component parts in large units, they know and can allow for the inclinations of the several members forming the cable, and, when erected and under work, they can inspect and see whether or not it is working according to the assumptions.

In the wire cable, engineers are asked to have "faith in things

unseen" and unproven. They must presume that the several thousand wires forming a large cable are all absolutely parallel after they have been banded together and wrapped, and that every wire is doing equal duty, except within a few inches of the splices. Mr. Cooper.

Engineers must accept the strength of a single wire multiplied by the total number of wires as the strength of the full cable. They must believe that, although all past methods of preserving the inclosed wires have failed to stand the test of time, the modern methods can be relied upon without waiting for a long experience.

For spans up to 2 000 ft., at least, engineers must allow the wire cable a large degree of flexibility, in order that it may enter into competition with the rigid truss under similar conditions of loading, even after accepting the above presumptions.

Until the limit of 2 000 ft. is passed, or the point where the wire-cable bridge may enter into fair competition with the rigid truss, it would seem unnecessary, if not undesirable, to accept a form of bridge with these questionable features, when the same graceful forms can be obtained in a rigid structure formed of large units the strength of which is known or can be known by actual tests, and which has all its parts open for inspection and preservation.

In the form of stiffened suspension bridge under discussion the strains on each and every member can be determined readily and positively.

The writer does not think the same can be said of the flexible suspension bridge, as heretofore treated. To consider a bridge, where several thousand tons of material are assumed as rising and falling several feet in a few seconds as a structure acted on by statical forces only, appears to the writer to be fallacious.

As to the competitive plans for the Quebec Bridge, the several plans were submitted to the writer for examination, and on his report the contract was let. There were submitted three plans for wire bridges. Two of these plans were by companies also presenting cantilever structures, and they stated that they could get no prices for the wirework which would justify tenders. The third plan, though accompanied by a tender, gave no strain sheets, sizes of parts, or plans of foundations. The schedule of material accompanying the tender gave but little more than half the wire and half the masonry contained in the other plans.

C. C. Schneider, M. Am. Soc. C. E. (by letter).—Suspension bridges with eye-bar chains in which the stiffening truss is attached to the chain have been built before, though on a smaller scale; in fact, the eye-bar chain was used before the wire cable in suspension bridges. Mr. Schneider.

The distinguishing features in the author's design are:

The shape of the stiffening truss and the tower;

Mr. Schneider.

The shape of the stiffening truss varying in height so as to conform with the theoretical requirements.

The latter is desirable for the greater stiffness it produces, as well as its economy.

The tower, consisting of vertical posts with the chain fastened to the top and free to rock on a pin-bearing below, the writer considers an improvement on the braced tower with the chain or cable fastened to a saddle moving on rollers.

The author's design of a stiffened suspension bridge compares favorably, for rigidity, with other types of bridges, and is destined to be a rival of the cantilever for long-span railroad bridges.

Rigidity is a desirable quality in any permanent structure, and more particularly in a structure of such magnitude, which should be built to last for generations.

The late George S. Morison, Past-President, Am. Soc. C. E., expressed himself in 1901 on the question of stiffness as follows:*

"Bridge specifications, many of which are drawn with great care, generally have the defect of specifying strength and not providing for stiffness. Under such specifications two structures can be accepted as equally good, though, under a load, one of them will have twice the distortion that the other will have. For immediate safety, strength is all that is needed; for long life of structure, stiffness is at least equally important. This objection may be raised to some special forms of trusses, favored because of their economy of material, in which much greater variations occur in the strains of individual members than in differently designed trusses containing a little more metal."

Some of the discussors have endeavored to make a comparison of cost between the wire-cable supension bridge with an auxiliary stiffening truss, and the author's design. A fair comparison is only possible if the designs of the two types of bridges are worked out under the same specifications and requirements, with the same unit strains for the floor system and stiffening trusses in each case.

The relative proportion of permissible unit strains on wire cable and eye-bar chain has been established, in the Brooklyn and Williamsburg Bridges, between cable and anchorage chain, which has never been questioned as being good practice.

The anchorage chain in the Brooklyn Bridge, with iron eye-bars, has approximately $3\frac{1}{2}$ times the section of the wire cable; and the anchorage chain of Williamsburg Bridge, with steel eye-bars, 3 times the section of the wire cable. Considering the difference in strength between the iron and steel eye-bars, the proportion in both cases is nearly the same.

In accordance with this established practice, the relative unit strains for ordinary steel eye-bars and wire cable should be as 1 to 3, and, for nickel-steel eye-bars, with 50% greater strength than carbon-

* *Transactions*, Am. Soc. C. E., Vol. XLVI, pp. 39 and 40.

steel eye-bars, this proportion becomes 1 to 2. If, therefore, a working strain of 60 000 lb. per sq. in. is allowed on the wire cable, 30 000 lb. per sq. in. should be allowed on the chain of nickel-steel eye-bars. Mr. Schneider.

Bids on the two designs should be asked at the same time, in order to have the same conditions as to prices of material, labor, etc. This, in the writer's opinion, is the only way to satisfy the public as to which of the systems is the more economical. If one design produces a stiffer bridge than the other at approximately the same cost, the stiffer bridge will be the cheaper in the end.

The writer, in examining the feasibility of the author's design, from a manufacturer's point of view, fails to find any difficult or unusual features, as far as the details of the structural steelwork are concerned. The details and connections can be designed so as to reduce the same to ordinary simple bridgework, such as any modern bridge shop equipped for heavy work can manufacture.

The members of the stiffening truss are of moderate sizes, and the maximum sectional area of the eye-bar chain is only slightly in excess of the maximum area of the tension chord of the Blackwell's Island Bridge.

From a manufacturer's standpoint, the work involved in the author's design of a stiffened suspension bridge is more desirable for a bridge shop than that of the Blackwell's Island Bridge, as it is more uniform and presents fewer complications in its details. Heavier and more complicated work has been manufactured, and more difficult problems of erection have been solved heretofore.

Some of the discussors have been questioning the propriety of the use of eye-bars for the cables. The eye-bar has been the principal distinguishing feature of American bridge building from the beginning of iron bridge construction. To condemn eye-bars as tension members in bridges would be a condemnation of the entire American practice of bridge building. Eye-bars as tension members have stood the test of time and have proved so satisfactory that they are now used exclusively in all railroad bridges of long span. To the writer's knowledge, no case is on record of a bridge failing on account of eye-bars being used as tension members.

Eye-bars, in connection with pin-connected trusses, have only proved unsatisfactory where they have been used improperly. In the early days of bridge building, when economy in weight of material was the first and almost only consideration of bridge designers, pin-connected trusses with eye-bars were used for very short spans, and consequently lacked the rigidity required in good railroad bridges. The writer has for years been an advocate of substantial riveted work for railroad bridges for small spans, and has been very persistent in his condemnation of the use of small eye-bars. Even at the present

Mr. Schneider.

time eye-bars are abused. The writer, only recently, has seen pin-connected highway bridges of 30 to 40 ft. span built with $1\frac{1}{2}$ by $\frac{1}{2}$-in. eye-bars. Any eye-bar less than 3 by $\frac{3}{4}$ in. should be condemned, even in the lightest kind of highway bridges. If the required section in the tension member is too small for 3-in. eye-bars, riveted members should be used, irrespective of the length of the span. Pin-connected trusses with eye-bar tension members for single-track railroad bridges of more than 200 ft. span and for double-track bridges of more than 150 ft. span are at the present time considered good practice by all American bridge engineers. Eye-bars have been used successfully in all the large cantilever bridges built recently, and will be used in the Quebec Bridge, which, when completed, will be the longest span in the world. It is well known to those who have had experience in the manufacture of eye-bars that they are generally the cheapest members in a bridge. The cost of manufacture per pound decreases as the size and length of the bar increases. The larger and longer the bar, the cheaper the cost per pound. Eye-bars of 14 and 15 in. in width have been manufactured in large quantities, and a number of experimental bars, 16 by 2 in., have also been manufactured successfully. Either of the above-mentioned sizes could have been used in the author's design. Eye-bars are used in the anchorages of most of the large suspension bridges, such as the Brooklyn and Williamsburg Bridge and, as the anchorage is a continuation of the cable, there is no good reason why eye-bar chains should be condemned for one portion of the cable and used for the other.

The eye-bar chain has the advantage that the sectional area can be varied to accommodate the variations of the strains in the same, while it is necessary to give the wire cable the required maximum sectional, area throughout its entire length.

Mr. Erlinghagen.

OSWALD ERLINGHAGEN, ESQ.* (by letter.)—A comparison of Mr. Lindenthal's design for a stiffened suspension bridge with the design for a suspension bridge proposed by the Harkort Company for the second bridge over the Rhine in Cologne, is interesting, for the reason that the Harkort Company, for which the writer was Designing Engineer, made two plans for this bridge, one with cables of eye-bars, and another, of precisely the same form, with cables of wire strands of a special form, manufactured and patented by the well-known firm of Felten and Guilleaume, in Mülheim on the Rhine.

The plans were made for a competition of designs in 1898-99. No decision on the plans has yet been reached, because considerable changes in the existing steam railroad tracks on the right shore of the river, opposite Cologne, will have to be made before the bridge can be built.

* Designing Engineer for the Harkort Company in Duisburg, Germany.

Mr. Erlinghagen.

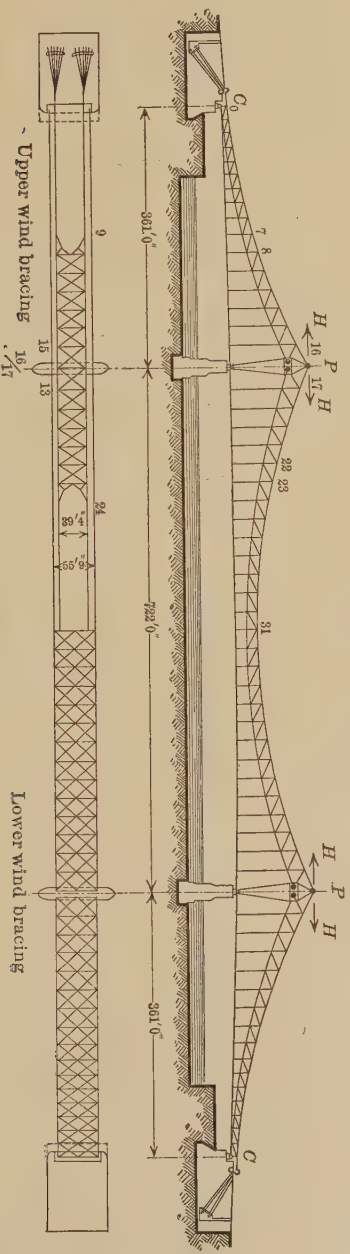


Fig. 9.

TABLE 1.—(*Continued.*)

STEEL EYE-BARS

361 FT. SIDE SPAN 361 FT.

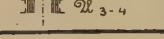
MEMBER	STRESS	AREA
ANCHOR	t	26 à 400·25
0-0		24 " " ·22+2 à 200·22
0-1	3283	26 " " " +2 " " " "
1-2	3661	28 " " ·25
2-3	3975	28 " " ·26+2 " " ·26
3-4	4177	30 " " "
4-5	4297	30 " " " +2 " " "
5-6	4341	32 " " "
6-7	4373	32 " " ·25+2 " " ·25
7-8	4341	34 " " ·24,5
8-9	4273	32 " " ·25+2 " " ·25
9-10	4161	32 " " ·25
10-11	4025	30 " " ·25+2 " " 25
11-12	3875	30 " " ·25
12-13	3780	28 " " ·24,5+2 " " 24,5
13-14	3560	28 " " ·24,5
14-15	3404	26 " " " +2 " " "
15-16	3250	26 " " ·25

722 FT. CENTER SPAN 722 FT.

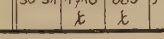
MEMBER	STRESS	AREA
17-18	3144	24 à 400·24,5+2 à 200·24,5
18-19	3149	24 " " ·24 +2 " " ·24
19-20	3139	24 " " ·24,5+2 " " ·24,5
20-21	3243	24 " " ·25 +2 " " ·25
21-22	3296	24 " " " +2 " " ·25
22-23	3347	26 " " "
23-24	3389	26 " " "
24-25	3420	26 " " "
25-26	3431	26 " " "
26-27	3425	26 " " "
27-28	3393	26 " " "
28-29	3342	26 " " "
29-30	3272	24 " " " +2 " " 25
30-31	3199	24 " " " +2 " " "
	t	

BOTTOM CHORD OF STIFFENING TRUSS

361 FT. SIDE SPAN 361 FT.

MEMBER	STRESS +	STRESS −	AREA
0-1	448	394	4 L 150·150·18
1-2	432	379	2 580·18+4 L 150 150·18
2-3	858	752	4 620·18+4 L 150·18-2 300·18
3-4	1202	1057	8 620·18+4 L 150 150·18
4-5	1449	1245	8 " " +8 L " " "
5-6	1541	1354	
6-7	1595	1401	8 620·18+8 L 150·150·18+
7-8	1594	1402	+4·160 18
8-9	1542	1352	
9-10	1435	1262	8 620 18+8 L 150 150 18
10-11	1281	1126	8 620·18+4 L 150·18+4 160·18
11-12	1095	964	6 " " +4 L " " +4 " "
12-13	896	786	4 " " " +4 L " " +4 " "
13-14	682	601	4 " " " +4 L 150·150 18
14-15	466	411	2 " " +4 L " " "
15-16	251	223	2 " " +4 L 100·80·10
	t	t	

722 FT. CENTER SPAN 722 FT.

MEMBER	STRESS +	STRESS −	AREA
17-18	203	144	2 620·18+4 L 100·80·10
18-19	380	265	2 " " " +4 L 150·100·18
19-20	557	379	2 " " " +2 300·18+
			+4 L 150 150·18
20-21	734	488	4 620·18+4 L 150·150 18
21-22	906	590	6 " " " +4 L " " "
22-23	1071	681	6 " " " +2 300·18+
			+4 L 150·150·18
23-24	1224	758	8 620 18+4 L 150·150 18
24-25	1358	815	8 " " " +4·160·18+
			+4 L 150 150 18
25-26	1473	855	8 620·18+4 160·18
26-27	1561	873	+8 L 150 150 18
27-28	1623	878	
28-29	1665	872	8 620·18+8·160·18
29-30	1697	876	+8 L 150 150 18
30-31	1716	885	
	t	t	

STRESSES ARE GIVEN IN METRIC TONS (2205 LBS)
AREAS ARE GIVEN IN MILLIMETERS (1" = 25,4 MM.)

Mr. Erlinghagen.

Mr. Erlingbagen.

TABLE 1.—COLOGNE BRIDGE: GREATEST STRESSES FOR EACH MEMBER, AND RESULTING CROSS-SECTION.

DIAGONALS

361 FT. SIDE SPAN 361 FT.				722 FT. CENTER SPAN 722 FT.			
MEMBER	STRESS +	STRESS −	AREA	MEMBER	STRESS +	STRESS −	AREA
15-16	218	245	$2\,\square\,\frac{300\cdot100}{10\cdot16}+2\ 300\cdot15$	30-31	253	264	
14-15	194	221		29-30	243	274	
13-14	193	219	$+4\ 115\cdot10$	28-29	233	284	$2\,\square\,\frac{300\cdot100}{10\cdot16}+2\ 300\ 15$
12-13	191	217	$\mathfrak{D}_{12-16}$	27-28	213	276	$+4\ 115\ 10$
11-12	184	210	$2\,\square\,\frac{300\cdot400}{10\cdot16}+2\ 300\cdot10$	26-27	191	265	
10-11	174	197		25-26	170	248	$\mathfrak{D}_{25-31}$
9-10	162	182	$2\,\square\,\frac{300\ 105}{15\cdot16}$	24-25	146	228	
8-9	138	152	$2\,\square\,\frac{300\cdot100}{10\cdot16}$	23-24	139	219	$2\,\square\,\frac{300\cdot100}{10\cdot16}+2\cdot300\ 15$
7-8	114	129		22-23	130	209	
6-7	86	87	$2\,\square\,\frac{300\cdot75}{10\cdot10}$	21-22	129	204	$\mathfrak{D}_{18-25}$
5-6	118	112	$2\,\square\,\frac{300\ 100}{10\ 16}$	20-21	128	199	$2\,\square\,\frac{300\cdot100}{10\cdot16}+2\ 300\ 15$
4-5	183	167	$2\,\square\,\frac{300\cdot105}{15\cdot16}$	19-20	129	196	
3-4	248	222	$2\,\square\,\frac{300\ 105}{15\ 16}$	18-19	130	192	$2\,\square\,\frac{300\cdot100}{10\cdot16}+2\ 300\ 15$
2-3	344	304	$2\,\square\,\frac{300\ 100}{10\cdot16}+2\ 490\ 18$	17-18	149	209	$2\ 300\cdot15+2\,\square\,\frac{300\cdot100}{10\cdot16}$
1-2	436	383	$4\cdot520\cdot18+2\,\square\,\frac{300\cdot75}{10\cdot10}$		ℓ	ℓ	$+4\ 115\ 10$
0-1	ℓ	ℓ	$2\,\square\,\frac{300\cdot75}{10\cdot10}+4\cdot530\cdot18$				$\mathfrak{D}_{17-18}$
			$\mathfrak{D}_{0-1}$				

VERTICALS

361 FT SIDE SPAN 361 FT.				722 FT. CENTER SPAN 722 FT.			
MEMBER	STRESS +	STRESS −	AREA	MEMBER	STRESS +	STRESS −	AREA
16	169	151	Ideeller Stab	17	142	101	Ideeller Stab
15	222	95	$2\,\square\,\frac{300\cdot100}{10\cdot16}+4\,\text{L}\ 100\ 100\ 10$	18	200	46	$2\,\square\,\frac{220\cdot80}{9\cdot12,5}+4\,\text{L}\ 100\ 100\ 10$
14	206	80		19	190	38	$\mathfrak{V}_{18-21}$
13	187	64	$4\,\text{L}\ 100\ 100\ 10+2\,\square\,\frac{260\cdot90}{10\cdot14}$	20	179	30	
12	169	48		21	171	23	$2\,\square\,\frac{200\ 75}{8,5\ 11,5}+4\,\text{L}\ 100\cdot100\ 10$
11	149	32	$2\,\square\,\frac{220\ 80}{9\ 12,5}+4\,\text{L}\ 100\ 100\ 10$	22	162	15	$4\,\text{L}\ 100\cdot100\cdot10+2\ 250\cdot10$
10	130	16	$4\,\text{L}\ 100\ 100\cdot10+2\,\square\,\frac{200\ 75}{8,5\ 11,5}$	23	160	16	$4\,\text{L}\ 100\cdot100\ 10+2\ 220\cdot10$
9	115	10		24	158	16	
8	98	3	$\mathfrak{V}_{10-15}$	25	157	21	$4\,\text{L}\ 100\ 100\cdot13+2\ 220\ 13$
7	103	18	$4\,\text{L}\ 100\ 100\ 10+2\ 200\ 10$	26	153	26	$4\,\text{L}\ 100\ 100\ 12+2\ 220\ 12$
6	107	32		27	149	29	
5	122	50	$\mathfrak{V}_{6-9}$	28	147	33	$4\,\text{L}\ 100\cdot100\cdot11+2\cdot220\ 11$
4	137	67	$4\,\text{L}\ 100\cdot100\ 10+2\cdot250\ 10$	29	147	43	
3	161	96	$4\,\text{L}\ 100\cdot10+2\cdot250\cdot10+4\cdot600\ 10$	30	151	55	$4\,\text{L}\ 100\ 100\ 10+2\cdot220\cdot10$
2	185	125	$4\,\text{L}\ 100\cdot10+2\cdot440\ 18+1\cdot600\cdot10$	31	64	•	
1	203	145	$4\,\text{L}\ 100\cdot100\cdot10+2\cdot420\ 18+$		ℓ	ℓ	$\mathfrak{V}_{22-31}$
	ℓ	ℓ	$+4\cdot100\cdot18+1\cdot600\ 10$				
0			$4\,\text{L}\ 100\cdot10+2\cdot420\cdot18+1\cdot600\cdot10$				

STRESSES ARE GIVEN IN METRIC TONS (2205 LBS.)
AREAS ARE GIVEN IN MILLIMETERS (1" = 25,4 MM.)

Mr. Erlinghagen.

Several designs of suspension structures for this bridge had been made in former years, and, in these designs, stiffening trusses of various forms, suspended from the cables, were proposed, but it was desired that the bridge should be above the commonplace in design, and that its architecture should be suited to the historical importance of this old and picturesque city.

Fig. 9 shows the general form of the bridge and the arrangement of the wind bracing.

One of the main conditions was that the free view from the bridge, over the surrounding landscape and over the city, should not be obstructed by the bridge trusses. For that reason the supporting cables and bracing were raised high above the roadway. The upper arch line represents the cable, which supports the entire length of the bridge. The lower arch line is the bottom chord of the stiffening frame. That form of stiffening was chosen for æsthetical reasons, the designer well knowing that it would not result in as rigid a bridge as would otherwise appear desirable.

The form of stiffening in Mr. Lindenthal's design for the Manhattan Bridge, and also for the Quebec Bridge, is more rational, because more economical, and more rigid under one-sided loads.

The Cologne Bridge is intended to have only two electric railway tracks on the roadway, on which the principal traffic will be by wagons and pedestrians. Crowding on such a bridge can easily be prevented by police regulations, and therefore the question of avoiding too great deflections did not receive special consideration.

On a bridge over the East River, with eight tracks, however, as in the proposed Manhattan Bridge, subject to frequently congested traffic, the subject of rigidity is of the first importance. Police regulations against crowding and the bunching of the heavy electric cars and other vehicles on eight tracks cannot as readily be enforced as on only two tracks. It is advisable, therefore, to have the most rigid system attainable. The same is true, also, for a railroad bridge, as in the case of the bridge over the St. Lawrence at Quebec.

The form of stiffening selected by Mr. Lindenthal has the advantage that it meets that condition very economically, because it provides for the greatest depth of the stiffening trusses at the points where it is most needed, and as is explained in the paper.

The form of stiffening chosen for the design of the Cologne Bridge has also the advantage, like the Lindenthal system, of saving one chord, because the cable itself forms the upper chord. But the form of the Cologne design requires heavier cable sections at the quarters of the middle span and at the middle of the side spans, while the Lindenthal system does not require them. The amount of metal is less, proportionately, and yet the author's system is more rigid. Therein consists its rationality and economy. The writer's

opinion, therefore, is that his system is particularly suitable for a suspension bridge of many tracks, which should be as rigid as it can possibly be made. Mr. Erlinghagen.

A lengthy description of the Cologne Bridge does not appear to be necessary. The system is statically indeterminate in the first degree. All the stresses were determined graphically with the aid of influence lines.

The dead load for the chain bridge was assumed at 8 tons per lin. m. (5 400 lb. per lin. ft.), and for the wire-cable bridge at 6.29 tons per lin. m. (4 230 lb. per lin. ft.).

The live load was assumed at 3.3 tons per lin. m. of each truss (2 220 lb. per lin. ft.).

Table 1 gives the greatest stresses for each member, and the resulting cross-sections.

The material for the chains was assumed to be open-hearth Siemens-Martin steel, having an ultimate strength of from 5 000 to 6 000 kg. per sq. cm. (71 100 to 85 300 lb. per sq. in.), with a stretch of 18 per cent.

Nickel-steel, with a strength 40% greater than Siemens-Martin steel, had also been under consideration, but offered no saving at the prices which then prevailed. These prices for nickel-steel were nearly three times as great as for structural steel. Since that time there has been a very large reduction in the price of nickel-steel, and also a great improvement in its quality and strength.

The alternative design for the wire-cable bridge provided that each cable should be composed of nineteen ropes, made of crucible wire of a special form. Each rope had an estimated ultimate strength of 13 200 kg. per sq. cm. (187 700 lb. per sq. in.), with a stretch of from 2 to 3 per cent. The largest tensional stress in each wire cable was 3 912 tons. A wire cable of this form offered greater certainty in the distribution of the stresses than a cable made up of parallel wires.

Steel-wire cables have the advantage over eye-bar cables of smaller weight for the same strength. But with that advantage there are some serious defects: the impossibility of inspection and careful maintenance of the interior of the wire cable; the much more difficult erection, and the higher cost of the material. The erection of the nineteen wire ropes of the form proposed by Felten and Guilleaume would, it is true, have decreased the difficulty of erection somewhat; but, on the other hand, the cable composed of eye-bars offers undisputed advantages, thoroughly tested by the experience of time. These are: The easy inspection and maintenance of the chain; the smaller cost of manufacture and erection, and the great solidity and certainty with which all parts of the bridge frame can be jointed. The greater weight of the chains has

Mr. Erlinghagen.

its effect upon the towers, piers and anchorages, but great weight, if it does not increase the cost, is rather an advantage in a bridge.

It will be noticed that the towers in the Cologne design have pin bearings, which were chosen because of the greater certainty with which the great pressures would be distributed upon the masonry piers.

It is gratifying to observe that the same thought occurred to the designer of the Buda Pest Suspension Bridge and to Mr. Lindenthal for the Quebec and Manhattan Bridges.

The comparison of the weights of the two designs is herewith given, and it remains only to state that, on the basis of these two carefully worked out estimates, there was no difference in cost between the chain bridge and the cable bridge.

	Weight of Superstructure:	
	With eye-bars.	With cable.
Floor and bracings	1 770 Net tons.	1 770 Net tons.
Towers	985 " "	935 " "
Chain or cable	3 345 " "	1 610 " "
Stiffening truss	1 205 " "	1 305 " "
Totals	7 305 Net tons.	5 620 Net tons.

Should a third design be prepared for the Cologne Bridge, of the same form, but with a chain of nickel-steel, there is no doubt that, at present prices for nickel-steel, the chain bridge would show a large saving over the wire-cable bridge.

From the æsthetical point of view, chain cables, because of their more solid appearance, are considered preferable. The fine and impressive lines of the two suspension bridges at Buda Pest, with double chains, bear testimony to that effect.

Among European bridge engineers, the opinion certainly prevails, and is fully justified by experience, that chain bridges are more durable than wire-cable bridges. That opinion was assigned at the time as one of the principal reasons for the choice of chain cables for the latest and largest suspension bridge in Europe, that which was built recently over the Danube at Buda Pest.

Mr. Hodge.

HENRY W. HODGE, M. AM. SOC. C. E. (by letter).—Mr. Lindenthal's paper brings to the attention of the Profession many features of suspension-bridge design which differ from previous practice. Some of these features have already been adopted, and doubtless many others will be adopted, in general practice, as, in the writer's opinion, this design is a great advance in many ways over any bridge of this type which has yet been constructed.

Mr. Hodge.

The discussions on this paper, thus far published, as well as the many articles on this type of suspension bridge which have appeared in the technical papers, seem to have centered almost entirely about the relative values of wire and bars for the main cables, and the question as to whether the stiffening trusses should be attached to and form part of the cables, or be made independent of the cables and placed at the floor line; and, while these are important features, they certainly do not by any means cover all the changes in practice used in the proposed design.

In the writer's opinion, the form of anchorage suggested is far better than any design thus far built, as the directions and amounts of the forces acting are clearly defined and localized, so that the masonry can be accurately proportioned and placed where it will be most effective, thereby obtaining a saving over the previous indeterminate designs of anchorages.

The hinged tower is also a great improvement over a tower intended to be rigid, as a rigid tower never fully realizes its intention, and although this hinged tower was severely criticized in the engineering publications when it first appeared, it has been adopted for the largest suspension bridge now in course of construction.

The use of a stiffening truss with maximum depth at the point of maximum bending is another important feature of this design, and, while it is not an entirely new idea, as it has been used to a certain extent in other suspension bridges, it certainly is more rational than the usual practice of making a stiffening truss of uniform depth throughout. Modern bridge designers are united in their opinion that in simple trusses of long span, the economic depth varies with the bending moment; and this principle certainly applies with equal force to stiffening trusses of suspension bridges.

The above-mentioned three features of this design are in themselves sufficient to place it in advance of any previous design, without taking up the question of relative values of bars or wire for main cables, and the consequent placing of the stiffening truss on the cable or on the floor. These last-named features seem to have brought out a most exhaustive discussion on both sides in the technical papers, and probably will never be settled to the full satisfaction of either side, though, in the writer's opinion, the final net difference, in dollars, between the two designs, for equal rigidity, would not be a very appreciable percentage of the total cost of a large suspension bridge, where there are so many other items which remain constant with either design of cable or stiffening truss, and the writer believes that an eye-bar chain can be erected more rapidly than a wire cable and will give a more rigid bridge for the same cost.

The writer does not agree with some who have discussed this paper, and who appear to consider a flexible bridge to be practically

Mr. Hodge. as good as a rigid structure. Rigidity is a most essential requirement for all framed structures, and (within reasonable limits) money expended to obtain rigidity is well expended. Therefore, no fair comparison of cost of two structures can be made unless both structures are of equal rigidity, as well as of equal strength.

Mr. Schüle. F. SCHÜLE,* ESQ. (by letter).—The design of the Manhattan Bridge has awakened the justifiable interest of bridge engineers, as has been shown already by the discussion of this paper.

The objections raised to Mr. Lindenthal's project refer less to questions of detail than to the principles underlying the factors of safety for wire cables and chains, and to the comparison of parallel stiffening trusses for wire cables with stiffening trusses attached directly to the chains. These remarks are presented because of the great and general significance of these questions.

Up to the present time, the factor of safety of a bridge has always been determined with reference to the ultimate strength of the material used, and not with reference to the yield point of the material. The reason for this is that the ultimate strength for the same material varies only within narrow limits, whereas the position of the yield point may vary quite considerably, due to mechanical treatment, and because stresses, exceeding the lowest elastic limit of steel or iron, reduce the subsequent elongations, up to the point of fracture.

This is at once clear from the theoretical treatment of compression members, for which, indeed, the elastic limit is the danger point. For such members, the ultimate load producing fracture must never exceed the elastic limit—the yield point. The safe working load, however, is not based upon the elastic limit in tension, but upon the ultimate compressive strength of the member; that is to say, the safe working load is obtained by dividing the ultimate or breaking load by from 3 to 5, according to circumstances. That the practice of basing the factor of safety upon the breaking strength, and not upon the elastic limit, of the material prevails in the dimensioning of large suspension bridges, is clear from the proof given by Mr. Lindenthal† in the comparative check calculations of the sections for the cables and anchor bars of the Brooklyn Bridge and the Williamsburg Bridge.

A method of calculation, of course, is conceivable, which assumes the yield point of the material to be determinative for the dimensioning. But, in that case, the allowable stresses cannot be determined by means of the same safety factor for tension and compression members. On the contrary, a distinction would have to be made between parts exposed to compression, in which, as men-

* Zürich, Switzerland.

† *Engineering News*, December 10th, 1903.

tioned before, buckling may lead to collapse when the yield point is reached, or in long compression members even before it is reached; and between tension members, for which even the passing of the yield point, as a rule, involves no danger, although the yield point and the ultimate strength may have been raised by artificial treatment of the material, as in wire cables. It is difficult, however, to conceive what advantage for the practice could be gained with a changed basis for the factor of safety. Mr. Schüle.

A prudent constructor would require, under all circumstances, a greater factor of safety, based upon the yield point for such material, when that yield point had been raised in an artificial manner, as in wire, than for material the natural and lower yield point of which had been maintained by careful treatment.

To declare one-third of the elastic limit of the material necessary in the dimensioning of eye-bars, as in wire cables, would be equivalent to saying that all American pin-bridges built in the last decade are too weak, an assertion which nobody would advance seriously.

The proof that the usual coefficient of safety for tension members in bridges, corresponding to an admissible stress between 1:1.5 and 1:2 of the elastic limit of the material, is all sufficient, is furnished by tests, made in Switzerland, with discarded bridges, up to fracture. Among these tests the following may be mentioned:

The railroad bridge across the Emme, in Wolhusen (Switzerland), with a span of 48 m;* the railroad bridge near Mumpf (Switzerland), with a span of 28 m.; and the Erlenbach Railroad Bridge, near Bilberach-Zell (Baden), with a span of 19 m.

The first two bridges failed solely in consequence of the bending of a web compression member; the third bridge failed in consequence of the bending of web compression members and of the subsequent shear of rivets in the tension members of the end panels. Such tests show that the danger is not so much with tension members, insufficiently proportioned, as with compression members; therefore, a greater factor of safety for tension members does not appear to be justified.

The purport of stiffening the cable or the chain by suitable trusses is to transmit to a greater length of the cable, or of the chain, the deformations from local or concentrated loads. These deformations are partly due to the change in the equilibrium of the cable or chain from such loads, and partly due to the elastic alterations in the length of the cable, or of the chain, from the inner forces.

The first cause of the deformation is approximately the same for chain and cable, but the second cause is essentially greater for the

*"Rapport sur les Épreuves de Charge jusqu'à Rupture de l'ancien Pont sur l'Emme à Wolhusen." Berne, 1895.

Mr. Schüle. wire cable. Consequently, the heavier chain can be stiffened more easily than a wire cable of the same span and deflection. This holds true for statically determinate stiffening trusses with a center hinge, and still more so for a stiffening truss without a center hinge.

In the former case, the sections of the stiffening truss are determined by its depth, and any arbitrary reduction and variation from this rule requires the regulation and restriction of traffic.

In the second case, the dimensioning of the stiffening truss without a center hinge can be done with somewhat greater freedom, but only under the condition that non-uniform distributed loads shall be limited in intensity, so that the deflection and unit stresses caused by them shall be within the allowable stresses of the material.

The direct use of the cable or chain as a chord of the stiffening truss means, under all circumstances, a saving, (1) on account of the omission of a chord of the truss, (2) on account of the great height (in Mr. Lindenthal's design) at the quarters of the center span, and (3) on account of the small depth of the truss in the middle of the center span and the consequently smaller stresses from temperature changes.

For the purpose of comparing the different stiffening systems for the same bridge, minute calculations of the deformation are, before all things, necessary, and the publication of this paper shows that the scientific treatment of such problems, which was not possible some years ago, is now achieved, although requiring much labor.

The direct stiffening of the chain, as in Mr. Lindenthal's design, met, until a few years ago, with the theoretical difficulty that an analysis of the forces in the truss, which would be free from objections, could not be made. This difficulty is now removed, not by the application of mathematical hypotheses, but by the proper application of methods, which, for various structures of modern times, have proved to be true, and without which hardly any statically indeterminate construction of importance will be dimensioned and executed in the future.

Prof. Melan. J. Melan,* Esq. (by letter).—The most effective and most suitable stiffening of suspension bridges—a question brought up by Mr. Lindenthal in his paper—is not only interesting in regard to the Manhattan Bridge, but is of much importance in the future development of suspension bridges. For that reason a non-American expert is glad to avail himself of the opportunity to take part in the discussion.

The question concerns two different systems of suspension bridges. One system is the stiffening truss with parallel chords,

* Prof. Ingenieur, Lehrkanzel für Brückenbau, an der k. k. deutschen techn. Hochschule, Prag.

the elastic deformation of which limits the alteration in the equilibrium curve of the cable or chain under moving load; in the other system, already used by Barlow in the Lambeth Bridge, in London, but with imperfect development of details, the chain or cable itself represents the chord of a stiffening truss. Prof Melan.

The possibility of producing with both forms of construction every required degree of stiffness, admits of no doubt, but it is equally doubtless that, assuming the same stiffness and the same unit stresses, the stiffening truss, which is separated from the chain or cable, requires more material than the other truss, particularly more than the one proposed by Mr. Lindenthal, in which the chords intersect at the supports, thereby avoiding excessively long web members.

The degree of stiffness can be judged by the deflection of a bridge under the moving load. Assuming first one half, and then the other half, of a stiffened suspension bridge, loaded with p per unit of length, then the total deflection at the quarter of the span is given by the equation:

$$y = \frac{5}{384}\,\frac{p}{E\,I}\left(\frac{l}{2}\right)^4 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In this equation l denotes the length of the center span, E the modulus of elasticity, and I the moment of inertia. While this formula cannot be used for a very small moment of inertia, it is theoretically exact for a constant, I, and holds also approximately true if I is variable, provided, however, that an average value of I, in each half of the span, or its value at the quarters, is substituted.

Let h = height of truss at the quarter point,

F = sectional area of one chord,

$\frac{1}{60}\,p\,l$ = bending moment (approximate),

s = maximum unit stress due to the live load, p, per unit of length.

Then we have:

$$F\,h\,s = \frac{1}{60}\,p\,l^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 2)$$

Substituting in Equation 1,

$$I = \frac{F\,h^2}{2}, \text{ and } F\,h = \frac{p\,l^2}{60\,s},$$

then

$$\frac{y}{l} = \frac{600}{6144}\,\frac{s}{E}\,\frac{l}{h} = \frac{1}{10}\,\frac{s}{E}\,\frac{l}{h} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The proper unit live-load stress for the chords of the stiffening truss is probably 15 000 lb. per sq. in., so that $\frac{s}{E} = \frac{1}{2000}$, and, con-

Prof. Melan. sequently, the vertical deflection is determined approximately by the equation:

$$\frac{y}{l} = \frac{1}{20000}\,\frac{l}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

For a stiffening truss, the top chord of which contains the large sectional area of a chain or cable, the above unit stress, or $\frac{s}{E}$, is rather too high than too low.

For instance, should it be required to restrict the deflections to the value of $\frac{1}{1000}\,l$ it would be necessary to make the height of the truss at the quarter point equal to one-twentieth of the span.

While it is easy to realize such a height in the Lindenthal suspension truss, it would be quite objectionable in a stiffening truss which is separated from the chain or cable, and having, as a rule, parallel chords, on account of the inadmissibly great temperature stresses in the central section, and quite apart from the general unsuitability of such high trusses.

These stresses, due to a change in temperature of 60° Fahr., are approximately:

$$s_t = 0.00085\,E\,\frac{h_o}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where h_o signifies the height of the truss in the middle of the span and f the sag of the cable.

For $E = 30\,000\,000$,

$$s_t = 25\,500\,\frac{h_o}{f} \text{ lb. per sq. in} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Without decreasing the stiffness of the Lindenthal truss, its depth in the central part may be assumed as small, thereby restricting the temperature stresses to sufficiently low limits. The same object can also be accomplished with a cable and stiffening truss by making the sag of the former and the depth of the latter great enough, but this would be an unsuitable arrangement on account of the increase in its cost.

In order to render these objections of no force, the advocates of the stiffening truss with cable advance two points, *viz.:*

1.—When great stiffness is a requisite, the temperature stresses in high trusses may be eliminated by a center hinge.

2.—That generally there is no necessity for great stiffness in highway suspension bridges, hence no need for high stiffening trusses.

Regarding the first assertion, it should be mentioned that Lindenthal has pointed out that the same does not hold true, and the more exact investigations* made by the writer upon this subject

* *Österreich. Wochenschrift f. d. öffentlichen Baudienst*, 1903. Heft 28.

brought out the fact that, although the temperature stresses are in general lessened by a center hinge, they cannot be eliminated. If H = the greatest horizontal pull in the cable, ΔL its alteration in length due to the effects of temperature and live load, and if, further, F = the sectional area of the chords in the stiffening truss, and h = its depth, then the approximate temperature stress in the chords is Prof. Melan.

$$s_t = \frac{1}{16}\frac{H\,l}{F\,h\,f}\,\Delta L \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Calling g the dead load, p the live load per linear foot, L the cable length, and s_k the maximum unit stress in the cable, then

$$H = \frac{1}{8}(g+p)\frac{l^2}{f},$$

and, for a variation in temperature of 60° Fahr., we have,

$$\text{for } \Delta L = \left\{\frac{1}{2500} + \frac{s_k}{E}\,\frac{p}{p+g}\right\}$$

By making $\frac{s_k}{E} = \frac{1}{625}$, Equation 7 gives, after some transformation:

$$s_k = \frac{1}{320\,000}\left(g + 5\,p\right)\frac{l^3\,L}{F\,h\,f^2} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

This last approximate equation is quite suitable for stiff trusses, but it cannot be applied to those stiffening trusses which are weak and flexible, in which case the stresses, s_k, become materially greater and nearly as great as for the truss without a center hinge.

Granted, that by means of a center hinge a material reduction in the temperature stresses could be obtained, the writer believes that every designer will take it earnestly into consideration before he decides to build a stiffening truss with a center hinge for a larger suspension bridge. Quite apart from the fact that its up-and-down deflections are larger, the hinge itself is a weak place in the truss; it is a constructive detail subjected to forces acting unfavorably in a high degree.

It is necessary to bear in mind that the vertical shearing forces at the hinge constantly change their intensity, and that they also suddenly change in direction when the rolling load passes the hinge. This condition also applies to the wind forces, which are very considerable in long spans, and which, too, must go through the hinge and change direction. Under such circumstances, and in view of the great up-and-down deflections, a pin-connected center joint, resisting both vertical and horizontal shearing forces, presents practical difficulties which make it objectionable.

A possible substitute for such a hinge would be a flexible joint, built of plates, and partaking of the nature of a spring, a construction used by Köpcke in the stiffened suspension bridge across the

Prof. Melan. River Elbe near Loschwitz (having a central span of 147 m.), but the practicability of this arrangement for great spans and forces must first be proved. It deserves to be mentioned that, disregarding the Point Bridge, in Pittsburg, the condition of which is said not to be the best, the above-mentioned Elbe Bridge is the only existing truly stiffened larger suspension bridge with a center hinge, and that even here it was found necessary to mitigate the motions of the hinge by a construction detail, gripping both halves at the meeting point, in order to obtain a kind of braking arrangement for impact.

If a center hinge in a deep stiffening truss of a great suspension bridge cannot be considered practicable, then the only recourse is a shallow, flexible truss, and therewith we give up the attainment of greater stiffness. It has been asserted that this is admissible, at least for highway bridges, and that the realization of great stiffness in such bridges is not even economical on account of the higher cost.

It will hardly be denied that even highway suspension bridges must be stiffened, although they may have the advantage of cables with fixed supports on the towers, contributing to a more stable equilibrium; it is only in regard to the amount or degree of stiffness that opinions differ.

There can be no question that, of two bridges, designed with the same unit stresses and under the same specifications, that one is the more perfect which shows the greater stiffness. Want of stiffness goes hand in hand with a large up-and-down motion of the whole construction, and, consequently, with fatigue, deterioration of the material, and great secondary stresses at the joints.

The assertion that, by reducing the depth of the stiffening truss, its weight, and consequently its cost, can be reduced at our pleasure, needs to be put into the proper light. The value of the maximum bending moment from live load in the stiffening truss is nearly independent of the stiffness of the latter. Only in high stiffening trusses—which, however, must be left out of practical consideration on account of the high temperature stresses—would the increased bending moments in the trusses be of any consequence. The most favorable depth of stiffening truss is that which corresponds to the least amount of material in the chords. If the reduction in the depth of truss is carried too far, no saving in weight is possible without overstraining the chords.

Published investigations made by the writer upon the subject of a single span have shown that the most favorable depth of the stiffening truss is approximately obtained for

$$h = 0.8 \frac{(p+g)\,s}{p\,s_0 + (p+g)\,E\,\omega\,t}\,\frac{l}{L}\,f \qquad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

In this equation, g = the dead load and p = the live load per linear foot, s = the maximum unit stress in the chords, s_0 = the

maximum unit stress of the cable, l = the length of the span, L = the entire cable length, and f = its sag. Assuming $t = 60°$ Fahr., then we have $E \omega t = 12\,000$ lb. per sq. in., as the temperature unit stress in the chords. From these data, the most favorable depth of the stiffening truss for an East River Bridge would be about $h = \frac{1}{5} f$. Prof. Melan.

A further reduction in the depth would only result in an increase in weight. The same, of course, will occur for an increase in the depth. The economical depth, therefore, gives a shallow stiffening truss subject to very considerable up-and-down deflections.

The foregoing considerations refer only to the central span, the trusses of the side spans having been assumed as perfectly stiff or not suspended from the cable; but, owing to the flexibility of the side trusses, the bending moments and deflections in the stiffening truss of the center span will be considerably increased.

For a suspension bridge stiffened with parallel-chord trusses, the results of the foregoing considerations can be summarized as follows:

1.—Effective stiffening, by means of which the deflections up and down are kept within correspondingly narrow limits, requires high stiffening trusses, which are not rational on account of the high temperature stresses. These latter cannot be entirely eliminated by means of a center hinge, which is, moreover, an objectionable detail and a weak place in the truss.

2.—A shallow stiffening truss with an economical amount of material admits of only an imperfect stiffness, in consequence of which the great deflections up and down, from partial loading, cause local overstraining and deterioration of the material, which is certainly not favorable to the durability of the bridge.

3.—The suspended spandrel-braced arch or truss, the top chord of which is formed by the cable or chain itself, possesses great stiffness, and, as it is subjected to only moderate temperature stresses, on account of the small height in the center of the span, it must be considered, for great bridges, as the more advantageous form of construction.

It has been claimed as a principal point that a wire cable could be used for the first-mentioned system, which is alleged to be cheaper than a nickel-steel eye-bar chain. Granted, that such is the case, a proper comparison bringing out the true economic value between the two systems can only be made on the basis of complete estimates of cost.

The difference in cost, for instance, between chain and cable for the Elizabeth Suspension Bridge, in Budapest, was only of small account, and for this reason the chain was adopted. As for the remainder, it is entirely feasible to construct the tension chords or

Prof. Melan. cables of braced suspended arches, either as a wire chain, composed of wire links, as proposed by Mr. Lindenthal for a design of the Quebec Bridge (and North River Bridge), or as a continuous cable, as is shown in Kübler's prize design for a bridge across the Rhine at Bonn. The difficulties of the panel-point construction of such cable trusses, therefore, can be solved readily, in most cases, by proper studies and experiments.

Mr. Moisseiff. LEON S. MOISSEIFF, ASSOC. M. AM. SOC. C. E. (by letter).—In his discussion of Mr. Lindenthal's paper Mr. Hildenbrand says:

"Both Mr. Lindenthal and Mr. Moisseiff describe correctly the difference between an upright arch and a catenary or inverted arch, *viz.*, that the former is in unstable and the latter in stable equilibrium. Mr. Lindenthal continues:

"'To that condition many badly designed suspension bridges owe their life; it covers a multitude of sins against good engineering.'

"Mr. Moisseiff thinks that it is 'inducive to the sound sleep of the engineer.'"

Mr. Hildenbrand then proceeds to state that he "draws exactly the opposite conclusions."

Mr. Hildenbrand evidently misunderstood the writer's statement. Within the English meaning of the sentence "inducive to the sound sleep of the engineer" implies that the structure is safe and the engineer need not worry much about its safety. If Mr. Hildenbrand had taken the sentence in its correct meaning he would have found that he does not draw "exactly the opposite conclusions," but, on the contrary, is in perfect agreement with the writer's statement.

Dr. Rieppel. DR. A. RIEPPEL* (by letter).—A comparison of the Lindenthal system of stiffening with the usual system of wire cables and attached stiffening trusses suggests the following points for discussion:

1.—The form of the stiffening truss, and the stiffness of the bridge;

2.—The manufacture of the principal tension members of the bridge as a cable or chain;

3.—The æsthetic effect of the bridge.

1.—The maximum moments of a stiffening truss are greatest at about one-fourth of the span, and may be assumed to have an average value for the entire span of one-ninth of the maximum moment for a beam on free supports. Therefore we can write:

$$m_1 = \frac{1}{9}\frac{p\,l^2}{8} = \frac{p\,l^2}{72}.$$

Assuming the most favorable depth of a truss on free supports to be from one-sixth to one-eighth of its length, then the depth of a

*Director d. Verein. Masch. Ges. Augsburg und Ges. Nürnberg, Germany.

stiffening truss should be from $\frac{1}{54\,l}$ to $\frac{1}{72\,l}$, or, in round figures, $\frac{1}{60\,l}$. Dr. Rieppel.

Recently built trusses show a greater depth—for instance, the stiffening trusses of the Williamsburg Bridge, with $\frac{1}{40\,l}$. The Lindenthal trusses have, at the center of the span, a depth of $\frac{1}{60\,l}$, and, at the quarters, $\frac{1}{25\,l}$. The form of stiffening truss selected by the author, therefore, is rational, and is materially stiffer than the stiffening truss generally used with wire cables.

In the shore spans the stiffening trusses act, as regards live load, very nearly like trusses on free supports, and therefore require to be heavier than in the middle span. Rocker posts, supporting the stiffening trusses from below, reduce the deflections and offer a good solution where it can be had.

The placing of large iron masses high above the floor system, however, is very objectionable, because the enormous areas of four trusses, one behind the other, exposed to wind pressure, require an extraordinary amount of wind bracing. Owing to the shape of the trusses, wind chords must also be inserted in the plane of the floor system. In stiffening trusses with parallel chords this is not necessary, the bottom chords by themselves acting conveniently as the wind chords.

The following question is proper: How much stiffness should a bridge have for the kind of traffic over it? The answers vary. Some prefer rigid trusses, while others claim that a yielding bridge is good and useful. In general, great deflections are not considered admissible in railroad bridges, and for them the simple trusses or cantilever trusses appear to be preferable; but, in a highway bridge, there are two factors in its favor, *viz.*, the great dead weight and the almost uniform traffic. Taking the old Brooklyn Bridge as an example, it carries crowds of people and vehicles, an almost uninterrupted string of electric cars, and elevated trains following each other within a few seconds, the whole representing an almost uniformly distributed load. The loads are transmitted to the cables almost directly, and the stiffening trusses carry little moving load. Partial loadings are possible only in times of war. The dead load of the proposed Manhattan Bridge will be about double that of the maximum live load, for single concentrated loads produced by electric trains cannot cause large or dangerous vibrations. The deflections of the cable bridge take place slowly, depending upon the more or less dense traffic at different times of the day and upon the temperature changes.

A stiffening truss with parallel chords, therefore, is sufficient

Dr. Rieppel. for a highway bridge; but it is better to use riveted instead of pin-connected joints, to avoid injurious effects from local impact.

The practicability of center hinges, if such are desired, and of the tall rocker columns for the towers, may be admitted. Engineers are somewhat timid, and are prejudiced against such tower constructions, because they are accustomed to the sight of massive masonry or heavy iron towers for carrying the cables.

2.—Cable or Chain?—The following points deserve consideration: *a,* the construction of cables and chains; *b,* the quality of the materials; *c,* the time of erection; and *d,* the costs.

a.—All the older wire bridges have cables with parallel wires, and there is little doubt that the manufacture of these cables, composed of thousands of thin, single wires, was formerly very defective. It was not possible to draw all the wires of uniform diameter and weight, and this prevented them from hanging parallel during erection and sometimes necessitated cutting and splicing. It was difficult to keep track of individual wires and be sure of their condition in the cable.

In spite of this defective construction, no case of rupture of a cable is known. It has always been defective stiffening, or the requirement of greater carrying capacity, which has led to the reconstruction of cable bridges.

Wire cables built up of strands (according to a system introduced by Roebling), and fastened to the eyes of anchor bars, permit of accurate adjustment of length before being compacted and wrapped. In the Williamsburg Bridge the wire cables, moreover, are protected against the weather by a mantle of sheet iron. Cables made in this way form ideal tension members.

b.—It is claimed that the steel wire for the proposed Manhattan Bridge cables can be had with an ultimate strength of 140 kg. per sq. mm. (200 000 lb. per sq. in.) and an elastic limit of 128 kg. per sq. mm. (180 000 lb. per sq. in.). For the eye-bars of the chain bridge it is proposed to use nickel-steel having an ultimate strength of from 60 to 70 kg. (85 000 to 100 000 lb. per sq. in.) and an elastic limit of 34 kg. (48 000 lb. per sq. in.).

Assuming a maximum working stress of 42 kg. per sq. mm. (60 000 lb. per sq. in.) in the wire, its factor of safety against permanent alteration of length would be: $n = \frac{128}{42} = 3.$

The maximum stress for the contemplated eye-bars is given as 21 kg. per sq. mm. (30 000 lb. per sq. in.), which gives a factor of safety, based upon the elastic limit, $n_2 = \frac{34}{21} = 1.6$, consequently considerably smaller.

Regarding rupture, the factors for safety are:

Dr. Rieppel.

For wire, $\frac{140}{42} = 3\frac{1}{3}$.

For nickel-steel, $\frac{60}{21} = 2.9$.

On these assumptions, the wire cable, in respect to safety, is superior to the chain.

c.—The time required for the erection of the cables for the Williamsburg Bridge is claimed to have been seven months, including all delays on account of bad weather. In erecting the cables, a light temporary bridge was used, which had to carry its own weight, the erection gang, and several light apparatus.

No experiences whatever are available for the erection of a chain without fixed falseworks. Probably a heavy temporary bridge, capable of carrying the entire chain, will be necessary. It will hardly be possible, without such a bridge, to put in place the eye-bars, which weigh from 3 to 4 tons, in such a way that each will receive its share of the stresses. The erection of the chain for the Budapest Bridge took at least one year, and was done on falseworks. The time of erection for the chain would be at least as long as that for the cable, and the cost of the erecting plant for the chain would be greater than that for the cable.

d.—A comparison of costs between chain and cable, made for the Elizabeth Bridge in Budapest, shows that the cables were more expensive.*

The costs were doubtless arrived at from different points of view, and, even on a superficial check calculation, it appears that everything was done to make the cable appear expensive and the chain cheap. What a large sum of money the latter has cost is known.

Comparing the data for the Manhattan Bridge, the following weights are obtained:

Let the weight of the cable per unit of length be 1, then, theoretically, that of a chain of equal capacity will be:

$1 \times \frac{42}{21} = 2.0$	2.0
Adding about 20% for pins and eyes	0.4
	2.4

This figure must be increased, owing to the greater weight of the chain itself, but this increase is compensated for by the fact that the uniform section of a wire cable must be dimensioned for the maximum stress in the backstays.

The contract price for the cables of the Williamsburg Bridge is claimed to have been 1 340 marks ($290) per ton, a price which

* S. Z. d. J., 1900, Nos. 18 and 19.

Dr. Rieppel. would probably be lower for the proposed Manhattan Bridge on account of the experience obtained; it will be assumed at 1 240 marks ($265) per ton. The price paid for the eye-bars of the Blackwell's Island Bridge may serve as a guide to estimate the nickel-steel chain. This price is 740 marks ($158) per ton, and, for the pins, 930 marks ($200) per ton. The bridge is on the cantilever principle, and can be erected partly on falseworks and partly in cantilever fashion. By making the cost of the falseworks, plus the cost of the appliances necessary for the erection without falseworks, equal to the cost of the heavy suspended structure for the erection of the chains of the proposed Manhattan Bridge (per ton) it is found that the chain is $2.4\left(\frac{740 + 35 \text{ (for pins)}}{1\ 240}\right) = 1.5$ times more expensive than the cable. This computation rather favors the chain.

The weight of the cables in the Williamsburg Bridge is about one-fifth to one-sixth of the entire weight of the bridge. The weight ratio between chain and bridge would be about one-third. This result (always assuming large dimensions and an equal arrangement of the stiffening trusses) shows that a cable bridge will be at least 15% cheaper than a chain bridge. This does not take into consideration the cheapening of the anchorages by reason of the smaller tension in the wire cables.

3.—The Aesthetic Point of View.—The Lindenthal system is possessed of an unusual configuration. The broken lines of the bottom chords and the mass of ironwork placed high up in the air give a disturbing impression. An ordinary cable bridge with stiffening trusses having parallel chords is more beautiful and affords a completely open view.

From the foregoing, therefore, it may be concluded:

(1)—The form of stiffening truss selected by Mr. Lindenthal must be designated as suitable for the purpose, in particular for the central span. For the side spans, an increase in the depth of the trusses or a support from below would be advisable.

(2)—A stiffening truss with parallel chords would accomplish the same purpose, and with sufficient stiffness against vibration in the case of a highway bridge.

(3)—To judge from the progress made in the manufacture of cables with parallel wire in recent times, the cable must be considered as equivalent to the chain.

(4)—A cable constructed with the above-mentioned qualities, in reference to safety against permanent deformations, and also against rupture, is superior to a chain (particularly because experiences are not available as to whether it is possible to manufacture a homogeneous material for the nickel-steel eye-bars of such dimensions as required for the proposed Manhattan Bridge).

(5)—An essential saving of time during the erection of a chain compared with a cable can hardly be expected. Dr. Rieppel.

(6)—Based on present prices for cable and chain material, not only the cable, but the entire superstructure with cables, will be materially cheaper than the chain and the chain bridge.

(7)—The old cable bridges, open above the floor and with shallow trusses, are more satisfactory, from an æsthetic point of view, than Mr. Lindenthal's suspended arches with their masses of ironwork at high elevations.

G. Lindenthal, M. Am. Soc. C. E. (by letter).—The object of this paper is to point out a form of stiffening frame, for important suspension bridges, offering greater economy and efficiency than that obtainable by means of stiffening trusses. The extensive discussion indicates the great interest of bridge engineers in that subject, and the endorsement of the writer's views by engineers whose opinion is most highly regarded in professional circles is gratifying to him. Mr. Lindenthal.

There is great variety in suspension structures, as may be seen from the partial list of diagrams on Plate IV, to which reference is made later.

The only suspension bridges dealt with in the paper are those which are fit for permanent structures, meaning those for which no rules shall be necessary for the regulation of speed or for keeping the loads uniformly distributed, to prevent rupture or buckling of the stiffening frame. The Brooklyn Bridge, referred to by others in the discussion as an example, is not of that class.

Comparatively few permanent and heavy suspension bridges will ever be built. The system is economically suited only for very long spans, which do not occur frequently, or where exceptional local conditions favor it in competition with the cantilever system.

A distinction between stiffened, unstiffened and poorly stiffened suspension systems has always been made. Recently, there have arisen advocates of the so-called "flexible suspension bridge," which is alleged to be a kind having not too much rigidity and not too little, but just "reasonable," whatever that may mean. These engineers are under the delusion, that by relying on the stable equilibrium of the suspension structure (designated in one of the discussions a "free" gift of Nature), the problem of stiffening can be cheaply solved with light stiffening trusses, so as to preserve "the true principle of suspension bridge construction," which is to regard "the material required for stiffening as a luxury."

The question is asked: What is rigidity in a suspension bridge? The obvious answer is that it is the same thing as in all other kinds of metal bridges. There are existing suspension bridges of shorter

Mr. Lindenthal.

spans which do not differ in rigidity from truss bridges, like the Point Bridge, in Pittsburg, and the Grand Avenue Bridge, in St. Louis, which are chain bridges; the wire-cable bridge over the Ohio at East Liverpool, the side spans of the Tower Bridge, in London, and others. No question of reasonable or unreasonable rigidity has arisen with them.

All bridges are necessarily subject to more or less elastic deformation. The deflection under full load may vary from $\frac{1}{800}$ to $\frac{1}{3000}$ of the span. Low trusses are more flexible and yet more expensive than high trusses, so are shallow arch ribs and low cantilevers. It is not a necessary conclusion that rigidity is a measure of economy. But there is no fact better established in bridge practice than that economically designed bridges are rigid. The writer has endeavored to show that the same rule applies also to the suspension bridge, scientifically designed.

Long-span bridges are very expensive, and, therefore, should be made durable. Flexibility is not conducive to durability in any kind of bridge. There is no such thing as "excessive rigidity," any more than there is excessive durability. One after another of the poorly stiffened suspension bridges wear out, and are replaced by more rigid structures, which, as a rule, are not of the suspension type. For the claim that flexibility is economical with any one kind or all kinds of stiffening, no practical or theoretical proof has been produced in the discussion, nor can it be produced in the only way which is conclusive, that is, on the basis of strain sheets.

The writer has discussed on a former occasion* the influence of elastic deformation upon the strains in stiffening trusses. How much the weight and cost of stiffening trusses depend upon the sag or dip of the cable and the suspended dead load can be readily shown in the following manner:

Let l = the length of span;
g = the dead load per linear foot;
p = the live load per linear foot;
f = the sag of the cable;
y = the distortion of the cable or floor at the quarter;
w = the excess load per linear foot, which is that part of the live load, p (when covering one-half the span), causing the distortion of the cable, and which is not resisted by the stiffening frame, whatever be its form. The stiffening frame has to resist only the remainder of the live load $(p - w)$ per linear foot.

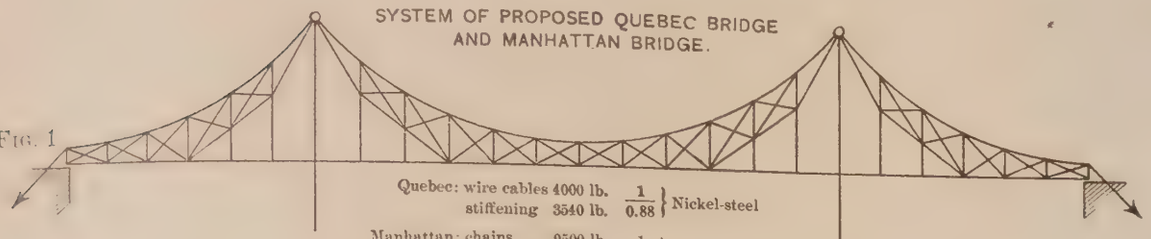
The largest distortion of the cable will occur when approximately one-half of the span is loaded and the other half unloaded.

* *Transactions*, Am. Soc. C. E., Vol. XXXVI, pp. 442, 443.

PROPORTIONS OF WEIGHT OF CABLES OR CHAINS TO STIFFENING IN VARIOUS SYSTEMS OF STIFFENED SUSPENSION BRIDGES.

SYSTEM OF PROPOSED QUEBEC BRIDGE AND MANHATTAN BRIDGE.

FIG. 1

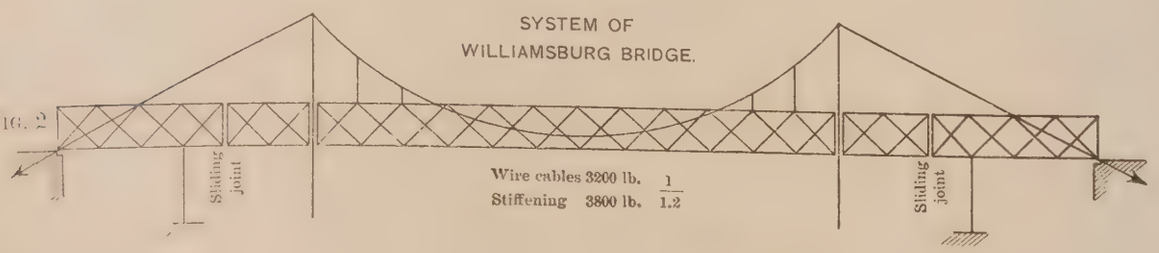


Quebec: wire cables 4000 lb. $\frac{1}{0.88}$ } Nickel-steel
stiffening 3540 lb.

Manhattan: chains 9500 lb. $\frac{1}{0.41}$ } Nickel-steel
stiffening 3910 lb.

SYSTEM OF WILLIAMSBURG BRIDGE.

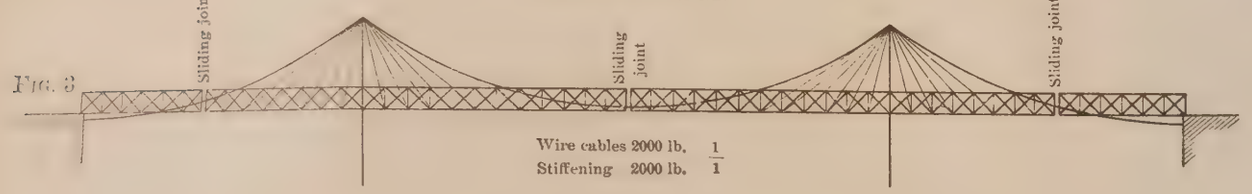
FIG. 2



Wire cables 3200 lb. $\frac{1}{1.2}$
Stiffening 3800 lb.

SYSTEM OF BROOKLYN BRIDGE.

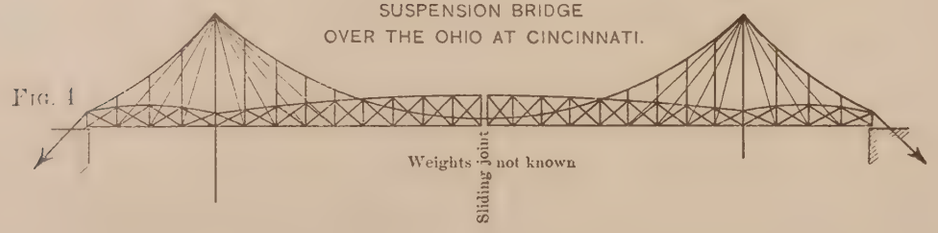
FIG. 3



Wire cables 2000 lb. $\frac{1}{1}$
Stiffening 2000 lb.

SYSTEM OF SUSPENSION BRIDGE OVER THE OHIO AT CINCINNATI.

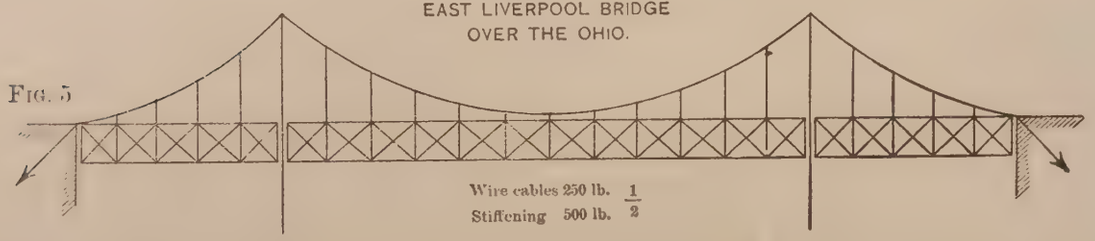
FIG. 4



Weights not known

SYSTEM OF EAST LIVERPOOL BRIDGE OVER THE OHIO.

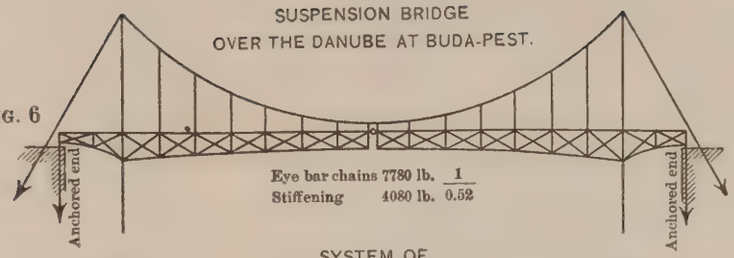
FIG. 5



Wire cables 250 lb. $\frac{1}{2}$
Stiffening 500 lb.

SYSTEM OF SUSPENSION BRIDGE OVER THE DANUBE AT BUDA-PEST.

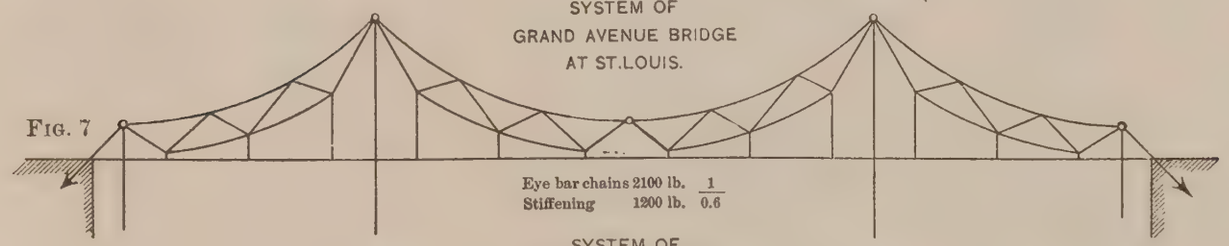
FIG. 6



Eye bar chains 7780 lb. $\frac{1}{}$
Stiffening 4080 lb. 0.52

SYSTEM OF GRAND AVENUE BRIDGE AT ST. LOUIS.

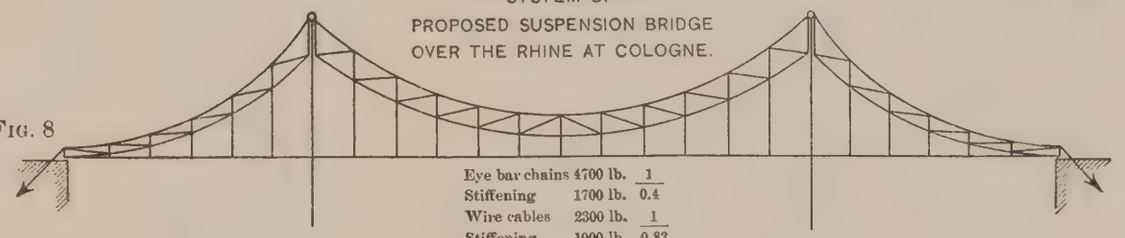
FIG. 7



Eye bar chains 2100 lb. $\frac{1}{}$
Stiffening 1200 lb. 0.6

SYSTEM OF PROPOSED SUSPENSION BRIDGE OVER THE RHINE AT COLOGNE.

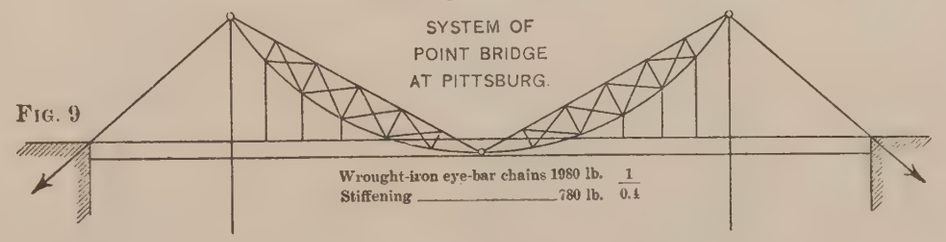
FIG. 8



Eye bar chains 4700 lb. $\frac{1}{}$
Stiffening 1700 lb. 0.4
Wire cables 2300 lb. $\frac{1}{}$
Stiffening 1900 lb. 0.83

SYSTEM OF POINT BRIDGE AT PITTSBURG.

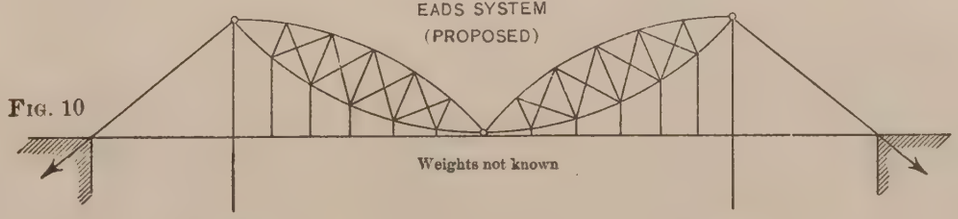
FIG. 9



Wrought-iron eye-bar chains 1980 lb. $\frac{1}{}$
Stiffening ——— 780 lb. 0.4

EADS SYSTEM (PROPOSED)

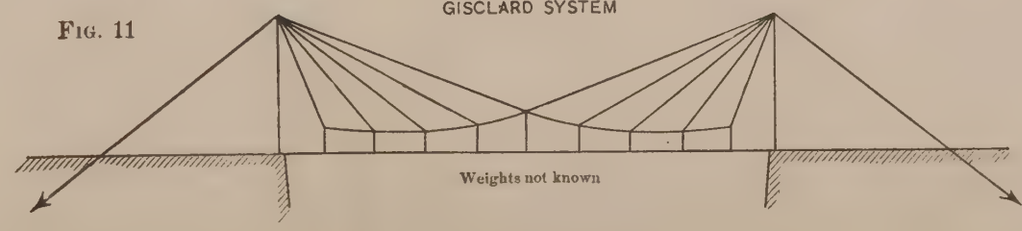
FIG. 10



Weights not known

GISCLARD SYSTEM

FIG. 11



Weights not known

Mr. Lindenthal.

The deflection of the loaded half is somewhat larger than the corresponding rise in the unloaded half, because the apex of the distorted cable moves toward the loaded part, and the curvature of the loaded half sharpens, while that of the unloaded half flattens. Each cable half takes up one-half of the bending effect of the one-sided load ($w \frac{l}{2}$). The bending moments (and the moment areas) in both halves are the same, but of opposite signs. y can be taken without material error as a mean value of the distortions in the two halves.

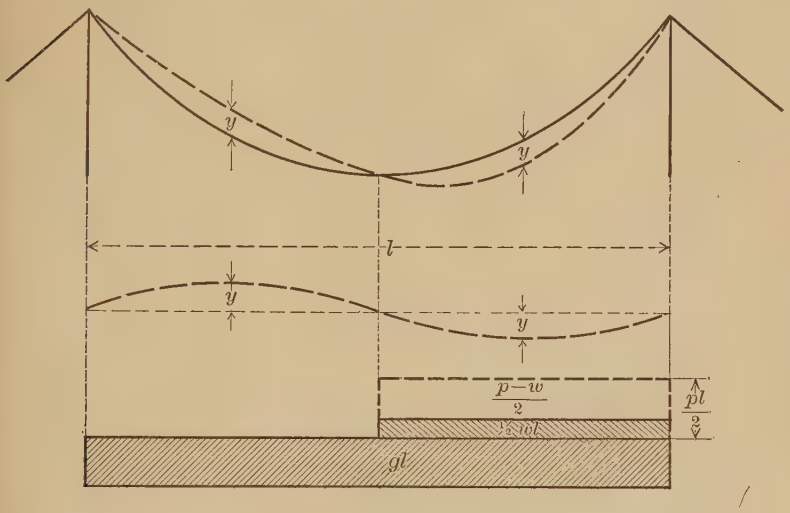


FIG. 10.

The bending moments at each quarter of the distorted cable will be $\frac{\frac{w}{2}\,\frac{l^2}{4}}{8} = \frac{w\,l^2}{64}$ and the horizontal component of tension in the cable will be $\frac{w\,l^2}{64\,y} = H$.

This horizontal component must also be equal to that from the dead load, g, plus the excess load, w, covering one-half the span.

For the values f, g and w, occurring in practice, the value of H will be, with close approximation, $H = \frac{g\,l^2}{8f} + \frac{w\,l^2}{16f}$.

We can therefore write

$$\frac{w\,l^2}{64\,y} = \frac{g\,l^2}{8f} + \frac{w\,l^2}{16f}.$$

Mr. Lindenthal.

From which is found:

$$w = \frac{8\,g\,y}{f - 4\,y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This simple equation is instructive, and very useful in the preliminary study of a suspension design. It shows:

1.—That the length of span has no influence upon the rate of the excess load.

2.—That the excess load increases directly with the dead load and with the rate of distortion. The proportion of live load to dead load, therefore, is of great importance.

3.—That the excess load increases as the sag of the cable decreases, in the proportion indicated by the divider $(f - 4\,y)$.

4.—As the stiffening frame is affected only by the difference between the live load and the excess load $(p - w)$, it follows that it must be made stronger and heavier as the sag increases, and as the dead load decreases, which is also confirmed by the analysis of Professor Melan (see his Equation 9), and by practical observation.

In a suspension bridge without stiffening, w is equal to p, and

$$p = \frac{8\,y\,g}{f - 4\,y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$g = \frac{(f - 4\,y)\,p}{8\,y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The distortion at the quarter

$$y = \frac{f\,p}{4\,(2\,g + p)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

If it is desired, for illustration, to apply the data to the structure discussed in Mr. Morison's paper,* with the object of finding the proportions of a suspension bridge without stiffening trusses, but, promising the same degree of rigidity, it may be done by assuming a flat catenary with $f = 200$ ft. or one-sixteenth of the span, which is nearly the same proportion of sag as in the late Niagara Railroad Suspension Bridge.

$f = 200$ ft.,
$l = 3\,200$ ft.,
$p = 12\,000$ lb.,
$y = 3.5$ ft.,

then g will be found from Formula 3 $= \dfrac{(200 - 4 \times 3.5)\,12\,000}{8 \times 3.5}$ $= 79\,718$ lb., or round 80 000 lb. For a sag of 400 ft. (one-eighth of the span) g would be 165 400 lb. or more than twice as large.

As the cables and floor construction would weigh much less than 80 000 lb. per lin. ft., the difference can be made up most cheaply

* *Transactions*, Am. Soc. C. E., Vol. XXXVI, p. 399.

with stone ballast. (About 30 000 lb. of stone ballast per linear foot would be required.) Mr. Lindenthal.

Therefore, it is entirely feasible to build a comparatively rigid suspension bridge without any stiffening system whatever. But it would not be cheap.

The total suspended load per linear foot in Mr. Morison's design is	39 000 lb.
Of which the cables and suspenders weigh (for a sag of one-eighth of the span)	11 600 "
The stiffening trusses weigh	14 400 "
Together	26 000 lb.

The cables alone (for a sag of one-sixteenth) in the bridge without a stiffening system would weigh 36 000 lb., and the weight of the towers, anchorages and floor construction would be more than twice as great as in Mr. Morison's design. The "luxury" of stiffening is saved, and, moreover, there are no temperature stresses, but the bridge, nevertheless, would be more than twice as costly.

This illustration is chosen because Mr. Morison's paper is on record, and the data therein are fully explained. The writer is also convinced (as he stated in his discussion at the time) that, for the system with stiffening trusses of nickel-steel, continuous through the towers, the design is conceived on economical lines. The excess load agrees closely with the writer's Equation 1, and the height of truss accords nearly with Equation 9 for economical height deduced by Professor Melan, when applied to length of span between points of contraflexure.

As a further practical illustration there may be mentioned the old Budapest Chain Bridge of 660 ft. span, weighing 10 000 lb. per lin. ft., the sag of the cables being one-fourteenth of the span. The bridge has no stiffening trusses worth considering (simply four hand-railings), but, nevertheless, is remarkably stiff in the central span. The writer's Equation 4 will confirm the statement that a load of 1 000 lb. per lin. ft. of bridge, representing a crowded condition on that structure, covering half its length, will produce a deflection of only 6 in. at the quarter, changing the grade less than 0.75 per cent. The function of the stiffening trusses, therefore, is merely nominal in the central span; but the trouble is in the side spans, where the deflections up and down are twice as large as in the central span. Further, if the bridge had wire cables instead of eye-bar chains, the undulations in all spans would be 50% greater. This flexible structure carries no street-car tracks.

Comparing now Mr. Morison's above-mentioned design with the

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system of spandrel bracing, it is readily seen that the weight of the top chord would be saved, say, 5 000 lb. per lin. ft. of bridge. In reality, the saving in weight would be greater, because the chord stresses would be smaller on account of greater height at the quarters. The total weight of web and bottom chord would remain about the same, the height at the quarters would be about double (135 ft.) and at the middle one-half (35 ft.). The rigidity of the bridge would necessarily be twice as great,, and the temperature stresses at the center of the span half as large.

With this system the distortion at the quarter is so small as to be negligible. The distortion of the stiffening frame, therefore, need not be taken into account in proportioning the members of the spandrel bracing. This fact was recognized, and is stated in the report of the Commission of Engineers on the proposed Manhattan Bridge, of which Mr. Morison was Chairman. (See Appendix.)

Where Mr. Morison's design for a 3 200-ft. span provides for a stiffening frame weighing 14 400 lb. per lin. ft., the writer's system of spandrel bracing would require only 9 400 lb., and be twice as rigid. The cables, back-stays and anchorages would be at least 10% lighter, because the total dead and live load of 50 000 lb. per lin. ft. of bridge would now be less than 45 000 lb., and the estimate of cost would be reduced about 10 per cent. This gives a theoretical illustration of the fact that rigidity and economy go together in a suspension bridge, as it is known that they do in trusses, arches or cantilevers.

It is not pertinent to this comparison that the writer's design for the North River Bridge, with the vertical bracing inserted between the chains consisting of wire links, was still more economical than with spandrel bracing. A consideration of that system is here out of place, and does not affect the following propositions, believed to be true beyond contravention:

1.—Given a certain quantity of steel for stiffening, it will produce greater rigidity with the spandrel bracing than with stiffening girders.

2.—Given a certain amount of rigidity (expressed by y), the spandrel bracing will require less steel than stiffening girders. If the spandrel bracing is desired to be shallow, then the form of the lower chord merges into that shown in Fig. 7, Plate IV, for the Grand Avenue Bridge, in St. Louis, or as in the design for the Cologne Bridge, Fig. 8, Plate IV.

3.—Of all stiffening systems, that with parallel-chord stiffening girders requires the largest quantity of steel, other things being equal.

In further confirmation of the economy of the spandrel-braced

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cable, a few data from the designs for the Quebec Railroad Bridge may be of interest. As already mentioned in the paper, and also in the discussion by Mr. Cooper, a number of competitive designs were made for this bridge, some for suspension and others for cantilever, on the same basis of loads and specifications. Mr. Mayer, in his discussion, states that a suspension structure was expected to be at least $500 000 cheaper than a cantilever design, before it would be considered. The writer had not heard of that condition before, although it was known that Mr. Mayer participated in that competition with a design which was not complete. The design adopted for the Quebec Bridge, and now in process of construction, is that of a cantilever structure 3 280 ft. long over all, with a middle span of 1 800 ft., containing about 62 600 000 lb. of steel, including the anchorages and side spans.

The writer's design, as described in his paper, was for a wire-link suspension structure, 3 200 ft. long over all, with spans of 682, 1 800 and 682 ft., and contained 52 700 000 lb. of steel, including anchorages, or nearly 10 000 000 lb. less than the cantilever design. The higher cost per pound of the wire cables than the steelwork for the cantilevers annulled the advantage of the lighter suspension bridge.

The masonry, as already stated, was about the same.

Table 2 furnishes a comparison of the quantity of stiffening required for spandrel and for stiffening trusses:

TABLE 2.

	Weight of steel per linear foot of bridge.		
	Pounds in chains or cable, including connections.	Pounds in stiffening frame.	Stiffening to chain or cable.
Quebec Bridge, 1 800-ft. span, wire links, spandrel-braced	4 000 unit stress, 50 000 lb. per sq. in.	3 540	11½ % lighter than cables.
Quebec Bridge, 2 000-ft. span, Pencoyd design, straight wire cables, stiffening trusses, 70 ft. high	5 000 60 000 lb. per sq. in.	7 800	56 % heavier than cables.
Morison's design,* 3 200-ft. span	11 600 50 000 lb. per sq. in.	14 400	24 % heavier than cables.
Same, if spandrel-braced	11 600 45 000 lb. per sq. in.	9 400	19 % lighter than cables.
Manhattan Bridge, spandrel-braced, nickel-steel for chain and stiffening. Span 1 470 ft	9 500 30 000 lb. per sq. in.	3 910	59 % lighter than chain.

* Of his paper, *Transactions*, Am. Soc. C. E., Vol. XXXVI, p. 359.

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In Table 2 only such suspension designs are compared as are proportioned for any condition of loading occurring on them. No police regulations, either for distributing the live load or for the regulation of speed, would be necessary.

It will be observed that in each instance the steel required for spandrel bracing is much less than that required for parallel-chord stiffening trusses.

That there are other systems of stiffening, where the same condition holds true, may be seen from Figs. 7, 8, 9, 10 and 11, of Plate IV, which show systems in all of which the stiffening is lighter than the cables or chains; but the spandrel bracing is specially suitable in cases where side spans occur.

Fig. 11 shows how much greater are the bending moments in the side spans than in the middle span, and how desirable, therefore, to have a greater height of stiffening frame at the middle of the side span. Such height in the end spans is obtainable more readily with the spandrel bracing than with the other systems.

Stiffening trusses in the side spans must either be made heavier or higher than in the middle span, or they require to be supported from underneath, as first practiced by the writer twenty years ago, in the Seventh Street Suspension Bridge, in Pittsburg, and afterward proposed in his design for the North River Bridge.

A sliding joint at the center of the side spans, as in the Brooklyn Bridge (Fig. 3, Plate IV), which is the point of greatest bending moment, is conceivable only on the theory that the live load will be distributed more or less over the entire bridge.

M. Gisclard's system, Fig. 11, Plate IV, will be recognized as a modification of the Ordish system, used for the Forty-third Street Bridge, in Philadelphia, also for the Francis Joseph Bridge, over the Moldawa, in Prague, which was found unsafe and was rebuilt a few years ago. It was considered by the inventor to require very little stiffening and therefore be economical. The same idea underlies the Gisclard system. But strain sheets for long spans, here, again, tell a different story, when compared with other systems, in which the cable or chain is a member of the stiffening system. For shorter spans, of a temporary character, wire-rope bridges of that kind have advantages of erection to offer, which make them very useful for slow speeds or for military operations.

For spans of 2 000 ft. or less, a wire suspension bridge with stiffening trusses, cannot, at present prices, prevail against a cantilever design, under the same specifications. That fact has been proven in the competition for the Quebec Bridge and for the Sydney Bridge (1 300 ft. span). And unless there is economy there is no other inducement for the use of wire. The greater extensibil-

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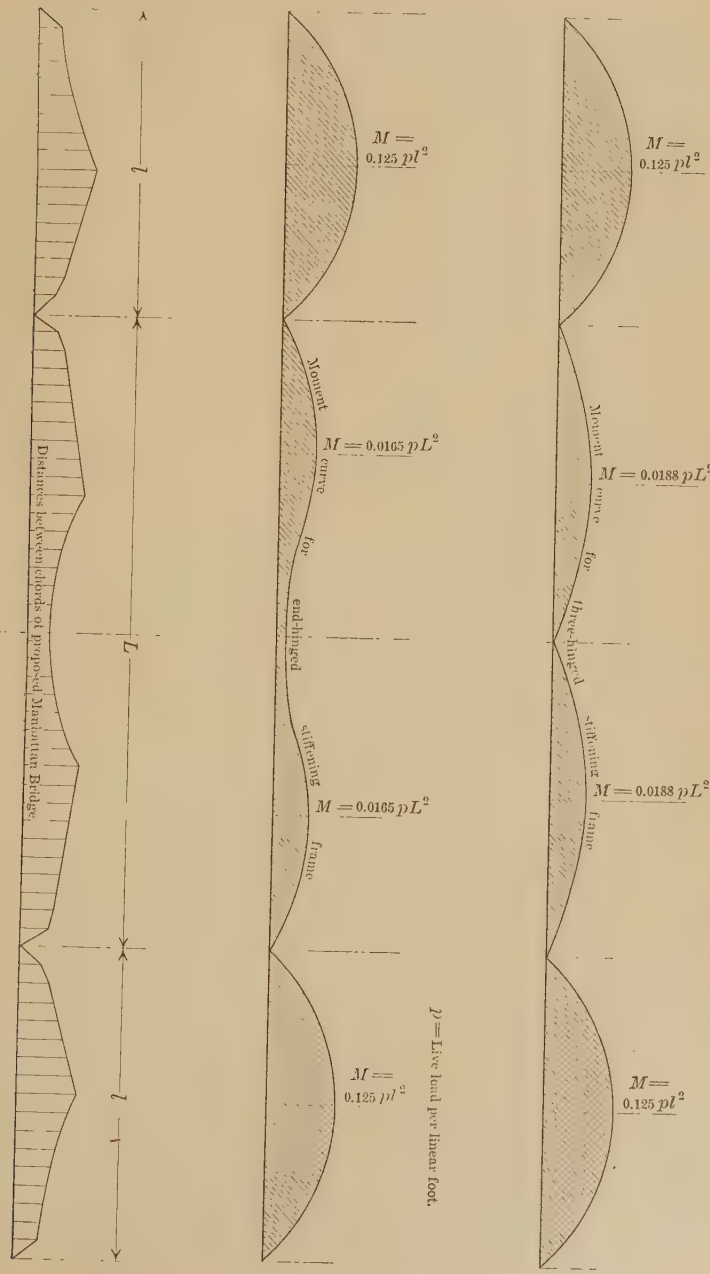

FIG. 11.

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ity of wire members, by reason of the larger unit stresses, is an important argument against their usefulness, as compared with solid steel members. There would certainly be a great saving in weight, in a long cantilever span, by making the heavy tension members of wire links, for instance, but the greater elastic motion introduced into the bridge frame with them must be considered an objection.

The writer is on record as being in favor of wire cables for long spans, and therefore cannot be accused of being prejudiced against them. But he agrees with Mr. Cooper that the durability of wire cables is still a matter of much concern, and that the claims made for the strength of wire cables on the basis of the strength of individual wires are exaggerated. The writer desires to call serious attention to the fallacious notions on the strength of large wire cables which Dr. Rieppel, among other engineers, in his discussion, regards as ideal tension members on quite superficial grounds. There are no tests in existence with wire cables of several hundred wires, let alone several thousand wires. Such tests as are on record are with wire ropes and small wire strands.

Appendix A to Mr. Morison's paper* gives tabulated results with tests of strands made up of straight, parallel wires, coiled wires, and of patent linked wire ropes. Only the first will be considered here. The average strength of five wires made of special steel is given as 172 588 lb., and the strength of straight wire strands, made up of 37 No. 8 wires, is given as 150 000 and 146 640 lb. per sq. in., respectively, or from 14 to 16% less in strength than the individual wires. The average strength of five wires made of plow-steel was 226 504 lb. per sq. in., and the strength of straight wire strands, made up of 37 No. 8 plow-steel wires, is given at 188 980 and 187 360 lb. per sq. in., respectively, which is again 16% less than the average strength of the individual wires.

The wires were fastened in sockets, and, although Mr. Morison implies that better results might have been obtained, if the inside of the sockets had been better finished, it is more than doubtful, in the absence of recorded tests, whether the usual method of making up cable strands by bending the wires around shoes is superior in accuracy to that of socket fastenings, although its greater convenience and simplicity has made the former the standard practice in America for the last fifty years. Mr. Morison, himself, preferred a cable, composed of ropes of twisted wire, which he considered superior to straight wires,† and he properly took the strength of the ropes (about 180 000 lb.), and not that of the individual wires (226 504 lb.) as the basis for estimating the strength of his cables.

There is no reason for believing that cables from 15 to 20 in. in

* *Transactions*, Am. Soc. C. E., Vol. XXXVI, pp. 414–416.

† *Transactions*, Am. Soc. C. E., Vol. XXXVI, p. 369.

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diameter, and composed of many thousand parallel wires, have an elastic limit or an ultimate strength equal to the sum of the strengths of the individual wires. It presupposes a mathematically uniform tension and the simultaneous arrival at the elastic and ultimate limit in all of them, as in a solid bar, a condition of which there exists not only no proof, but which is contrary to the observations in the tests quoted. These show that the rupture begins with individual wires and progresses successively to all of them; which explains why the strands and ropes break with a much smaller ultimate limit than the sum of the individual wires; and, if strands break in that way, large wire cables must break in the same way. A deduction, therefore, of at least 15% is dictated by prudence, to be on the safe side.

But further deductions are necessary, arising from the bending of the wire cables at the saddles on the towers. In practice, the wires are compacted into a cylindrical form, close to the saddles, in order, first, to preserve their parallel position, and next to hold tightly the paint, or other protective substance against corrosion, in the interstices between the wires, if possible to the exclusion of cavities in which moisture from condensation or otherwise could lodge. The compacting of the cable precedes in practice the attachment of the suspended structure.

The cable will deflect at the saddles through an angle, α, caused by:

1.—The weight of the suspended structure,

2.—A rise in temperature, and

3.—The live load.

The deflection is shown in Fig. 12 on an exaggerated scale. The wires on the upper side must extend, and, on the under side, may contract. A simple calculation shows that the extension, C, is approximately $\frac{1}{30}$ in. for each inch in the diameter of the cable, having a sag of one-eighth to one-tenth of the span. For a cable 20 in. in diameter, C will be, therefore, $\frac{2}{3}$ in. Assuming that the wires in the middle or heart of the cable are not affected by the bending, then the uppermost wires will lengthen by the amount, $\frac{C}{2}$, and the lowermost will shorten by the same amount, $\frac{C}{2} = \frac{1}{3}$ in.

The wires must be able to slide on each other, for that purpose. It is evident that they cannot do so in the saddles, because held there by the friction from the great pressure of the cable reaction. The only sliding which is possible is in that part of the cable between the saddle and the first clamp, which, in practice, is not more than 6 ft. from the saddle; but, assuming that the first clamp is put on with

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somewhat less pressure, to reduce the friction between the wires, so that these may slide on each other for a distance of 25 ft., to the next clamp, then, for $E = 25\,000\,000$, the uppermost and lowermost wires in a 20-in. cable will be strained about $\pm$ 25 000 lb. per sq. in. from bending alone.

The distance, 25 ft., is longer than in any large wire cable in existence, and is probably a more favorable assumption than could be obtained in practice without inviting other trouble from the uncompacted parallel wires. Nor is allowance made for strains set up by the friction between the wires, which, if ever so slight, produces shearing stresses and flexure moments, with increased strains in the outer wires.

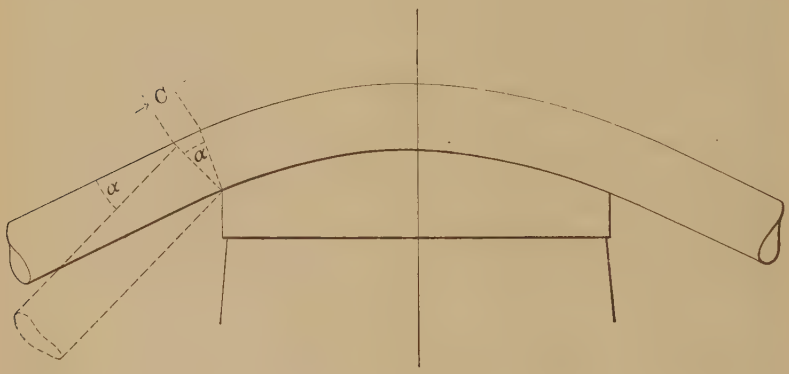

FIG. 12.

The only practicable way to reduce the bending stresses somewhat appears to be, not to put the clamps on the cables near the towers until the greater part of the superstructure is suspended. That may reduce the stresses in the extreme fiber about 50%, or to 12 500 lb. per sq. in.

The rule for eye-bar members in chords prescribes an increase of section, when the extreme fiber stress from bending, under its own weight or from other cause, exceeds 10% of the unit stress. Applying the same rule to wire cables, with a unit stress of 50 000 lb., the cable section requires to be increased, so that the extreme fiber stress shall not exceed 55 000 lb. Therefore the gross cable section should be proportioned for 55 000 — 12 500 = 42 500 lb. per sq. in. That means either an increase of about 17% in the cable section at the tower, or a further deduction of about 15% from the ultimate strength of the cable for bending.

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The writer has called attention before to these serious bending stresses in cables at the towers, in his discussion of Mr. Morison's paper, and Mr. Morison, in his closing discussion, acknowledged the "point well taken." It was easy enough in Mr. Morison's construction to limit the over-straining, but the same remedy cannot be used so readily with large cables of parallel wires, resting in saddles, without making the last condition worse than the first.

There can be no doubt that the bending stresses at the towers cannot be ignored, and that through their omission the strength of wire cables is greatly over-estimated. Until proof is furnished, from observation or tests, that the bending stresses are less than above computed, we are not justified in assuming them to be smaller, or in neglecting them altogether.

Assuming in the cables a wire having an average strength of 180 000 lb. per sq. in. (stronger and harder steel wire can be had at a higher price), 15% must first be deducted for unevenness of strain in the strands, as shown, and another 15% for bending at the tower, making necessary a total deduction of at least 30% to obtain the safe value of the wire cable at the weakest point. That makes (180 000 — 54 000) = 126 000 lb. per sq. in. A working stress of 50 000 lb. per sq. in. will give a factor of safety of 2.50 at the tower, and a working stress of 60 000 lb. will give a factor of only 2.1, instead of 3.0, as ordinarily assumed.

A further uncertainty in the cable stresses is produced in the case when the cable saddles become immovable on the towers, as in the Brooklyn Bridge, through corrosion of the rollers, which have not turned these many years, or in the case of saddles fastened to fixed towers.

The tension in the cables on each side of the towers may differ greatly, from uneven loading of the spans, subjecting the tower to bending stresses. That difference in the tension must be transmitted to the tower through the saddles in which the cables rest. Let that difference be only 10 000 lb. per sq. in. of the entire cable section, then the strain in the wires on top, in the saddle, from that cause will be zero, and at the bottom of the saddle will be 20 000 lb. per sq. in.

Analyzing the stresses in the cable at the bearings, as we do those of other bridge details, it is evident that under no circumstances can the tensional stresses be assumed as uniformly distributed over the entire cable section at the most dangerous point, that is, at the tower bearings; and that the before-mentioned deductions are the least which must be made for the sake of safety.

The foregoing considerations, among others, have induced the writer to prefer wire links in place of straight wire cables, if wire is to be used at all. The strength of wire links is easily determined

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in a testing machine. It is also easy to increase the section of wire-link cables at the towers to meet the bending stresses, as may be seen from an inspection of the strain sheets for the Quebec Bridge, Plates I and II. The first set of links at the towers has a cross-section of 468 sq. in., when 430 sq. in. would have been sufficient without the bending stresses, an increase of 8%, fully covering the bending of the shallow strands of the wire links.

The construction with the wire links has the further advantage that the cable section can be varied in accordance with theoretical values. In the Quebec Bridge, the variation is from 468 sq. in. at the towers to 400 sq. in. at the center of the middle span. An equivalent straight-wire cable, under the rules of safety above noted, would require a section at the towers of at least 500 sq. in., which would be continuous throughout its length, and, on the whole, would weigh 15% more than the wire-link cable, including pins, shoes, etc.

The same advantage appertains to a chain of eye-bars. An inspection of the strain sheet for the proposed Manhattan Bridge will show that the section at the towers is 597 sq. in. and in the next panel only 531 sq. in. The large increase of 12% includes the bending stresses in the eye-bars at the towers from the change of angle, α, caused by loads and by variations in temperature.

A comparison of the strength of a straight-wire cable with that of an eye-bar chain, therefore, is not admissible on the superficial basis of the strength of the individual wire.

Any one who has ever seen a large cable of parallel wires, after the compacting clamps had been applied, must have observed the unevenness in the tension of the wires. Some would be straight, while others would bulge out slightly. If it is difficult to adjust 37 wires into a strand with uniform tension, still greater is that difficulty when adjusting thousands of wires in a long cable.

Tests with as large and long strands as our largest testing machines will take are imperatively needed, and, when made, it will very probably be found that the deductions above noted are not large enough to arrive at correct conclusions as to factors of safety.

On the other hand, there can be no question as to the strength of a chain of eye-bars, with the high degree of workmanship in vogue. The usual specifications for eye-bars permit no greater variation in length than $\frac{1}{64}$ in. in 40 ft. Another clause provides that bars of one set shall be so nearly equal in length, that upon being piled on each other, the pins fitting with no more than $\frac{1}{32}$ in. play, shall slide down through the holes at both ends simultaneously without driving. This accuracy in length is readily obtained in American bridge practice. The difference of stress in eye-bars of the same set, therefore, cannot possibly exceed $\pm$ 1 000 lb. per sq. in. (for $E =$

29 000 000). For nickel-steel eye-bars, having an average ultimate strength of only, say, 80 000 lb., that variation would amount to only 1¼%, as against the 15% from the same cause observed in wire strands, as previously stated. Hence, the true proportion of the working or unit stress of nickel-steel eye-bars, having an average ultimate strength of 80 000 lb., and that of an equivalent cable made of straight wire with an average ultimate strength of 180 000 lb., is as Mr. Lindenthal.

$$\frac{80\,000 - 1\,000}{180\,000 - 54\,000} = \frac{1}{1.60}.$$

If the working stress in nickel-steel is taken at 30 000 lb., that for the full section of the wire cables at the towers, the weakest point, should be not more than 48 000 lb.

The cross-sections are in inverse ratio to the working stresses, hence $\frac{1.60}{1}$. Adding now to the cross-section of the eye-bars 25% for pins and heads, and adding to the cross-section of the wire cables 8% for details (bands, shoes, sheathing, etc.) gives the equivalent values of weight as

$$\frac{1.60 + 25\%}{1 + 8\%} = \frac{2.00}{1.08} = \frac{1.85}{1}.$$

No allowance is made for the increase of section and weight in the eye-bar chain for carrying its own greater weight. It is nearly balanced by the excess weight of the cable for uniform section, as remarked, also, in Dr. Rieppel's discussion, although he arrives at a proportion of weights which is clearly erroneous.

Comparing the wire cables with eye-bars of structural steel, an average ultimate limit of 57 000 lb. will be assumed for them. Hence the proportion of cross-section is

$$\frac{126\,000}{57\,000 - 1\,000} = \frac{2.25}{1}.$$

Allowing, again, 25% for heads and pins and 8% for cable details gives

$$\frac{2.25 + 25\%}{1 + 8\%} = \frac{2.82}{1.08} = \frac{2.61}{1}.$$

Therefore the equivalent cross-sections or areas for wire cables, nickel-steel eye-bars and structural-steel eye-bars are as 1 : 1.60 : 2.25; and the corresponding weights per linear foot are as 1 : 1.85 : 2.61. Higher ultimate as well as unit stresses can be assumed and obtained for all three kinds of tension members, but that would not materially affect the relative proportions of the three equivalents.

If the price per pound of wire cable in place be taken at 14 cents, that of an equivalent nickel-steel eye-bar chain would be 7.57 cents,

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and that for structural-steel eye-bars 5.36 cents. The opinion expressed in the report of the Commission of experts on the proposed Manhattan Bridge, that the cost of eye-bar chains and wire cables would be about the same, is therefore well founded (See Appendix). The prices of nickel-steel eye-bars for the Blackwell's Island Bridge at 8.03 cents and for ordinary steel bars at 6.52 cents per pound, also the prices for pins, will be recognized as being disproportionately high, and much higher than if the same work were done for a railroad company or other private corporation, notwithstanding that prices have increased very much, and in some cases doubled, during the last seven years.

The objections to large eye-bars would be worth serious consideration if they came from steel and bridge manufacturers; but such objections have never been made. On the contrary, inquiries have shown that steel and bridge manufacturers are ready to furnish the bars and erect all the necessary plant for their production, provided the contract be large enough.

An order for 14 000 tons of nickel-steel bars would surely justify them in doing so. The method of cutting out bars from plates, as was done for the new Buda Pest Chain Bridge, is particularly well adapted for large eye-bars from 16 to 22 in. wide, and from 2 to $2\frac{1}{2}$ in. thick, and is cheaper than forging. The steel is not affected by repeated heating, as in forging and annealing, and, therefore, full-size tests with such bars become unnecessary. The advantage of heavy eye-bar chains over wire cables for a permanent bridge structure is so manifest to unprejudiced minds as to be worth an extra price, if necessary. That was the decision in the case of the Buda Pest Bridge, and the writer considers it a wise decision. He is equally convinced that heavy eye-bars for such large work, in America, can be obtained at a much lower price than was paid for Blackwell's Island Bridge. But even with higher prices, the cost of the spandrel-braced chains, in a fair competition, will be below the cost of wire cables stiffened with independent trusses.

It is evident that, by reason of the smaller extensibility of the eye-bars, as compared with wire cables, there is a smaller rate of distortion under live load, in the proportion of the unit stresses in eye-bars and wire cables.

Because of this greater distortion, stiffening trusses in wire-cable structures must be made heavier than the same trusses with eye-bar chains; any saving in cost with the lighter wire cables is offset by the greater cost of stiffening. The eye-bar chain combined with spandrel bracing is not only stiffer but cheaper than the system of wire cables combined with stiffening trusses, as the sequel of the Manhattan Bridge will undoubtedly show, when the bids for the wire-cable structure are received.

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Interesting if not novel is the proposition to base factors of safety on the elastic limit instead of on the ultimate limit universally used, and to dimension the areas of chains and wire cables accordingly. Professor Schüle, of Zurich, whose experience and experiments entitle his views to special consideration, also points out in his discussion the incorrectness of basing the purely theoretical conception of the factor of safety upon the elastic limit. But one might go further in that proposition, and, on the same fictitious principle, claim special merits, for instance, for compression members of cast iron, in which the elastic limit is so near the crushing limit (about 80 000 lb.) as to be almost indistinguishable. If the elastic limit and not the breaking limit were the basis for factors of safety, then the old bridges with cast-iron posts and top chords were safer than the wrought-iron and steel structures which were substituted for them. If that rule were applied to tension members, and if it had been practised for the anchor chains in the Brooklyn Bridge and the Williamsburg Bridge, their cross-section and weight should have been twice as great as the chains actually in place.*

It was not practised for the anchor chains of the Cincinnati Suspension Bridge, rebuilt by Mr. Hildenbrand, nor in any other structure anywhere, but it is seriously brought forward, nevertheless, to make it appear that an eye-bar chain is inferior in strength to a wire cable, unless it is made 5¼ times as heavy as a wire cable.† The incorrectness of that contention is sufficiently indicated in the preceding comparisons of the equivalent areas and weights of wire cables, nickel-steel and ordinary steel, but can be further shown by an examination of the tests made with wire strands by Mr. Morison, and already referred to.

Hard steel wire has an elastic limit of about 85 to 90% of the ultimate limit, and an elongation of from 2 to 3% before rupture. About ½% of this elongation is used up when the elastic limit is reached, leaving, therefore, a margin of only 1½ to 2½% before rupture. The elongation after the elastic limit is passed occurs so rapidly, that, for all practical purposes, the strength of the wire is gone when the elastic limit is reached. The reserve strength of only 15 000 to 20 000 lb. beyond the elastic limit, and the reserve elongation of only 1½ to 2½% are too narrow limits for safety, and too small a margin for the wires in a strand to adjust themselves under strain to each other. That is clearly proven by the tests with the strands of straight wires. The elastic limit of the wires is not given, but it must have been not less than 150 000 lb. for the wire

* *Engineering News*, December 10th, 1903, page 523.

† *Engineering News*, March 12th, 1903, page 230.

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of average 172 588 lb. ultimate limit. Two strands broke with 150 000 and 146 640 lb. per sq. in., respectively, which is at or below the elastic limit of the wire. For the wire made of plow steel, having an average strength of 226 504 lb. per sq. in., the elastic limit could not have been less than 190 000 lb. The two wire strands again ruptured at or below the elastic limit of the single wire with 188 980 and 187 360 lb., respectively.

If such is the case with small strands of only 37 wires each, on what grounds can we expect a greater strength for strands made up of 200 wires each, or of a cable of from 6 000 to 7 000 wires?

It is probable that could the wire have the same elongation, of from 8 to 12%, as a steel bar, permitting greater length of adjustment between wires during elongation, the strands would show higher ultimate limits; but hard steel wire of that kind is not made. From all we know, we are not justified in assuming for the cable a higher breaking limit per square inch than the elastic limit of individual wires.

In a chain of large eye-bars with modern workmanship, there is, on the other hand, no possibility of one bar in a set being strained in excess of its neighbors more than 1 000 lb. per sq. in. Rolled steel has an elastic limit of a little more than one-half the ultimate strength. Between the two limits the difference may be from 30 000 to 45 000 lb. per sq. in., and the bar will elongate from 8 to 14%, that is, from 4 to 7 times as much as wire, before rupture. By the very nature of the construction, the entire cross-section of the chain must develop the sum of the eye-bars, as regards both the elastic limit and the ultimate limit. Such an effective reserve of strength beyond the elastic limit is absent in wire strands and it must be so in cables.

The claims, therefore, for the strength of straight-wire cables are built up from first to last on the inconclusive evidence of the individual wire. They do not agree with analytical reasoning, and the few tests made with strands contradict them absolutely. In the whole field of testing materials, there is not another subject so important, and yet investigated and experimented with so little, as the relative values of single wires, strands and cables of wire. No other branch of steel construction presents such an amount of exaggerated claims, in place of facts, on the question of safety.

But this is not the only serious question with wire cables. The other is the greater liability to corrosion. A section of 1 sq. in. of No. 6 wires (the size used in the Williamsburg Bridge) has 20 times the surface of a section of 1 sq. in. in a bar 18 by 2 in. The danger from corrosion, therefore, is 20 times as great in a wire cable as in a

chain of eye-bars. The necessity of preventing corrosion is in the same proportion. Mr. Lindenthal.

In a chain of eye-bars the surfaces are open to inspection and painting. The spaces between the heads can be puttied up and filled with molten paraffin or other protective substances, and, if necessary, cleaned out and filled again. It is a daylight operation. But, in a wire cable, such inspection is not possible. It is a matter of speculation whether the interstices between the wires are filled up moisture-tight, which is necessary to protect the wires against corrosion. The wires in the Williamsburg Bridge showed signs of corrosion before they were compacted, and a supplementary method of protecting them was applied, not included in the original specification and contract. The writer is not convinced that it is as efficient as the simpler method used for the wire cables of the Brooklyn Bridge, although this, also, seems to be far from perfect. The risk and danger of electrolytic corrosion has been added in recent years, making it doubly desirable that every part of the steel structure should be accessible to inspection in daylight.

Questions have been raised as to the cost and time of erection of eye-bar chains, as compared with wire cables.

In America bridge erection is a specialized branch of bridge construction. There are superintendents and contractors, with complete equipment and working organizations, who do nothing else but erect bridges. These are the responsible and experienced men consulted by bridge engineers and manufacturers on questions of erection. In the case of the Quebec Bridge, the cost of erection, estimated by such contractors, was $50 000 less for the wire-link suspension bridge than for the cantilever bridge. As regards erection, wire links do not differ from eye-bars. The stringing of eye-bars, suspended from a temporary wire-rope bridge, into a chain, and the pushing of additional eye-bars upon the pins with the aid of pilot nuts, when the eye-bars already in place are under tension, is considered a simple operation of bridge erection in America. Dr. Rieppel is mistaken in believing that it is necessary to have a temporary wire-rope bridge strong enough to carry the weight of the entire chain. It requires to be strong enough to carry only the first set of from 2 to 4 bars, alternatively, in each set, and the pins. This was the method used 30 years ago with the chains of the main span for the Point Bridge, weighing 1 600 tons, the erection of which required only six weeks, after the temporary wire-rope bridge had been completed. Erection contractors, with the improved appliances of the present day, could complete the same operation in four weeks. For the proposed Manhattan Bridge six months was regarded by them as ample time for the same operation. The ques-

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tion of having all the material, including the bars, stored near the bridge site, ready for uninterrupted erection, was of more concern to them than the question of erection itself.

If the objections as to cost and time of erection came from erection specialists or contractors they would have some force. It is entirely safe to say that none can be produced from that quarter, which is the only one worthy of consideration.

On the other hand, there can be no delusion as to the time required for the erection of wire cables, and the delay caused thereby to all the work of erection, which cannot proceed until they are completed.

The possibility of erecting large wire cables in shorter time than has been done heretofore is not questioned. In the case of the Williamsburg Bridge the contract time was 10 months from the day that the saddles on the towers were ready to receive the cables. The actual time consumed was 23 months, not including 12 months of preparation at the shops. It would have taken much longer than 23 months, but for the steps taken by the writer to urge the work along. The time of 7 months, for spinning the cables of the Williamsburg Bridge, as stated in the discussions, is misleading, as that is only a part, and it was by far the shorter part, of the erection work. The delay and disorganization of the work, which had to wait until the cables were completed, proved also to be a serious feature.

Cable contractors are not bridge manufacturers, and *vice versa.* The great advantage, in time and cost, of having the entire steelwork made in one shop by one responsible manufacturing firm, after competitive letting, is so manifest as to require no explanation.

In regard to pin connections, argument at this late date, and the needless repetition of elementary knowledge, which may easily be derived from American bridge specifications and practice, would seem to be unnecessary. Can serious attention be given to the "shifting of the diagonals on the pins with every passing load," or the view "that pins wear seriously when members assembled on them are subject to frequent reversals," when, on the other hand, the novel objection is made of "excessive rigidity?"

The Brooklyn Bridge, with its light trusses, is cited as an example. The poor quality of the truss details in that bridge was recognized even while it was building (1880).

Pin details are not now used with light structures, except in our discredited county bridges, which are not built by first-class bridge works; but, when the pin details in heavy bridges comply with the specifications in force for many years on our large railroads, there is no more danger of pin connections wearing than riveted connec-

tions, and they are cheaper and can be erected more quickly. Field riveting has become very expensive, although power riveting machines are used. The average cost of the field rivets in the stiffening trusses of the Williamsburg Bridge was 25 cents apiece. Mr. Lindenthal.

The writer finds himself quoted by Mr. R. S. Buck (page 28), on the subject of riveted connections. The quotation is misapplied, because it relates to the beneficial effect of "secondary strains at all the connections and intersections" in latticed trusses, and the diffusion of "bending strains throughout the frame," which, of course, has nothing to do with the heavy pin details in large trusses.

The steel arch bridge at St. Louis, completed 30 years ago, has pin-connected web members subject to severe reversal of stress many hundred times a day. The increasing loads have required the rebuilding of the floor construction, but the pin connections in the steel arches have not suffered. There are many hundreds of heavy pin-connected bridges in America. The St. Louis Bridge is mentioned because it is the earliest in which the strains of pin details were carefully analyzed, and engineering literature possesses a complete account of its construction; but there is nothing of the kind about the Brooklyn Bridge. Instead of Mr. Hildenbrand calling upon the writer for strain sheets, it is suggested that, having been connected with its construction, he should favor the Society with them, and also with those of the Cincinnati Bridge, which he rebuilt.

It seems to be assumed, in some of the discussions, that the system of spandrel-braced chains had not received careful consideration. To dispel this impression, there are given in the Appendix the two reports of the Commission of five experts appointed by Mayor Low to pass upon the plans. It will be recognized that upon this Commission were engineers of acknowledged great ability, who had designed and executed large structures, who knew how bridgework is done, how cost is estimated, and how structures are erected. That Commission approved the plans and recommended their execution. The plans were laid aside by a new city administration, and announcement was made that new plans would be prepared for a wire-cable structure of the same capacity and on the same specifications, but costing several million dollars less than the chain bridge.

One would think that the new project would also be submitted to the scrutiny of a board of bridge experts for a comparison with the plans already approved, so that the city should have the benefit of whatever plan be judged to be the better or cheaper, and in case of doubt bids be invited on both designs. But to anyone acquainted with the administration of our cities, it is not surprising that noth-

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ing of the kind was, or will be, done. At the time of this writing the details and strain sheet of the new plans have not been published, and, therefore, are not available for comparison with the chain design. Theories are advanced in the meantime that, being a city bridge and not a railroad bridge, it should be made more flexible.

Let it be first remembered that the completed structure will cost the city more than $20 000 000, including right of way and approaches; that it is built for the future, perhaps of centuries; that the traffic over it will be certainly larger than that over the Brooklyn Bridge; and that the structure must certainly be designed to carry safely the frequently congested traffic on eight rapid transit tracks, besides one wagonway and two promenades; that on such a structure police regulations for keeping cars and teams equally distributed (which are required to be enforced on the Brooklyn Bridge for the sake of its safety) are impracticable; then may be recognized the very great importance of building a bridge having at least the same

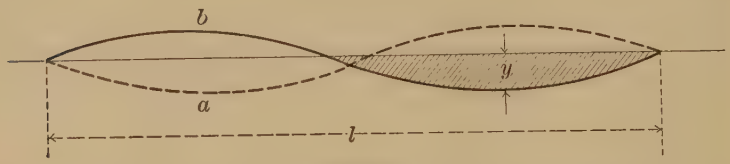

FIG. 13.

rigidity which a cantilever structure would naturally have in its place. And let it be remembered that a cantilever structure, to meet all these conditions, would be cheaper. When all this is considered, the question is justified: On what grounds is a flexible suspension bridge to be preferred?

The Manhattan Bridge will really consist of two bridges side by side, each carrying four tracks on two decks with the roadway between them.

If the tracks of one side be loaded for one-half the span, and the tracks of the other side be loaded on the opposite side of the span, which will not be an infrequent occurrence during rush hours, the deflections will be as shown in Fig. 13. Let these deflections up and down at the quarters be only 18 in., which the advocates of flexible suspension bridges probably do not regard as excessive, then the difference in the floor level of the roadway may reach 3 ft., as shown in Fig. 14.

This is not a fanciful condition. The ends of floor-beams on the Brooklyn Bridge are frequently 3 ft. out of level during rush hours.

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What would be thought of a cantilever or arch structure exhibiting that amount of see-saw motion of its floor? If the floor-beams were not hinged as shown, the bridge would become impossible of calculation, and the excessive bending stresses in floor-beams and stiffening frames could not be controlled.

In the chain design such great difference of levels cannot occur under any grouping of loads; but the question is asked: "What is the advantage of such excessive rigidity? Is * * * anybody benefited by it?" And another expert suggests as a guide: "the controlling considerations in gauging the rationality of a bridge design * * * based upon actual experience, rather than upon the attenuating processes of the higher mathematics."

The progressive breaking, in the Brooklyn Bridge, of a number of hanger rods (induced by the bending stresses from the never-ceasing motion of the trusses), in the summer of 1901, seems to have been entirely forgotten.* It is within the bounds of conservative state-

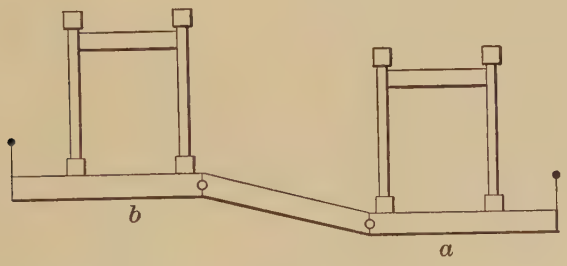

FIG. 14.

ment to say that, if the fractures had not fortunately been discovered in daylight (when rail traffic over the bridge was at once suspended), part of the superstructure would very probably have dropped into the river by next morning, at the commencement of the early rush hours, and an appalling catastrophe would have been almost inevitable.

And still the question is put seriously: "Is there anyone who was ever prevented from crossing, or who was inconvenienced while crossing that bridge on account of insufficient rigidity of the floor?"

The writer does not desire to be understood as not recognizing that the bridge carries a much larger traffic than it was intended for, with just enough safety as regards cables, towers and anchorages, all of excellent workmanship; but it is also true that the stiffening trusses and floor system are faulty, and require continuous costly

* *Engineering News*, August 1st, 1901; *Railroad Gazette*, August 2d, 1901.

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repairs. The rebuilding of the bridge is only a question of a short time, and plans for its reconstruction had already been taken under consideration by the writer while in office.

Admitting that the best was done that could be done, in the backward state of knowledge of suspension bridges 30 years ago, it is no reason for decrying radical improvement. The system of cables, trusses and stays, some fastened to the towers and others fastened to the cable saddles, defies computation, and a large amount of flexibility is almost a necessity for its safety. Improvement in design is always in the direction of simplification, of compactness, of self-contained frames having the fewest possible parts, and having single points of support on the towers and anchorages. In this light the system of cables and stiffening trusses, even without the stays, represents a composite that is an irrational structure. By this is meant a structure in which the bending and shearing stresses from live load are divided among two or more systems, according to their respective deflections, and are expected to act harmoniously in carrying the loads to more than one point of support at each end. The respective reactions of cable and truss cannot be determined accurately, because of the elastic elongation of the long suspenders near the ends.

The question as to how much of the live load is carried by the cable and how much by the trusses cannot be answered with precision. Any theoretical apportionment of load and work between them cannot be maintained. It changes not only with each position of the load, but with the temperature. The cross-section of the cables does not participate in resisting flexure moments. The resistance for that purpose is provided by a system entirely outside of the cables, and that increases the weight of metal. The composite and uneconomical character of the combination of cable and truss may be likened to a combination of two trusses, one of which is proportioned to carry the entire dead and live load equally distributed, and the other truss, joined to it, proportioned for only those positions of partial live load giving maximum bending and shear. The two trusses could be made to work together only with great difficulty, and would certainly require more metal than one truss doing the work of both. This principle of simplification is acknowledged to be self-evident in trusses, cantilevers and erect arches, and why not also in suspended arches?

Instead of arguing about a reasonable or unreasonable amount of rigidity in a suspension bridge, let it be ignored altogether for a while, and let attention be concentrated upon finding the best and simplest forms of frame for meeting the bending and shearing stresses under loads, as is done in other bridge types.

PLATE V.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 998.
LINDENTHAL ON
STIFFENED SUSPENSION BRIDGES.

PROPOSED MANHATTAN BRIDGE

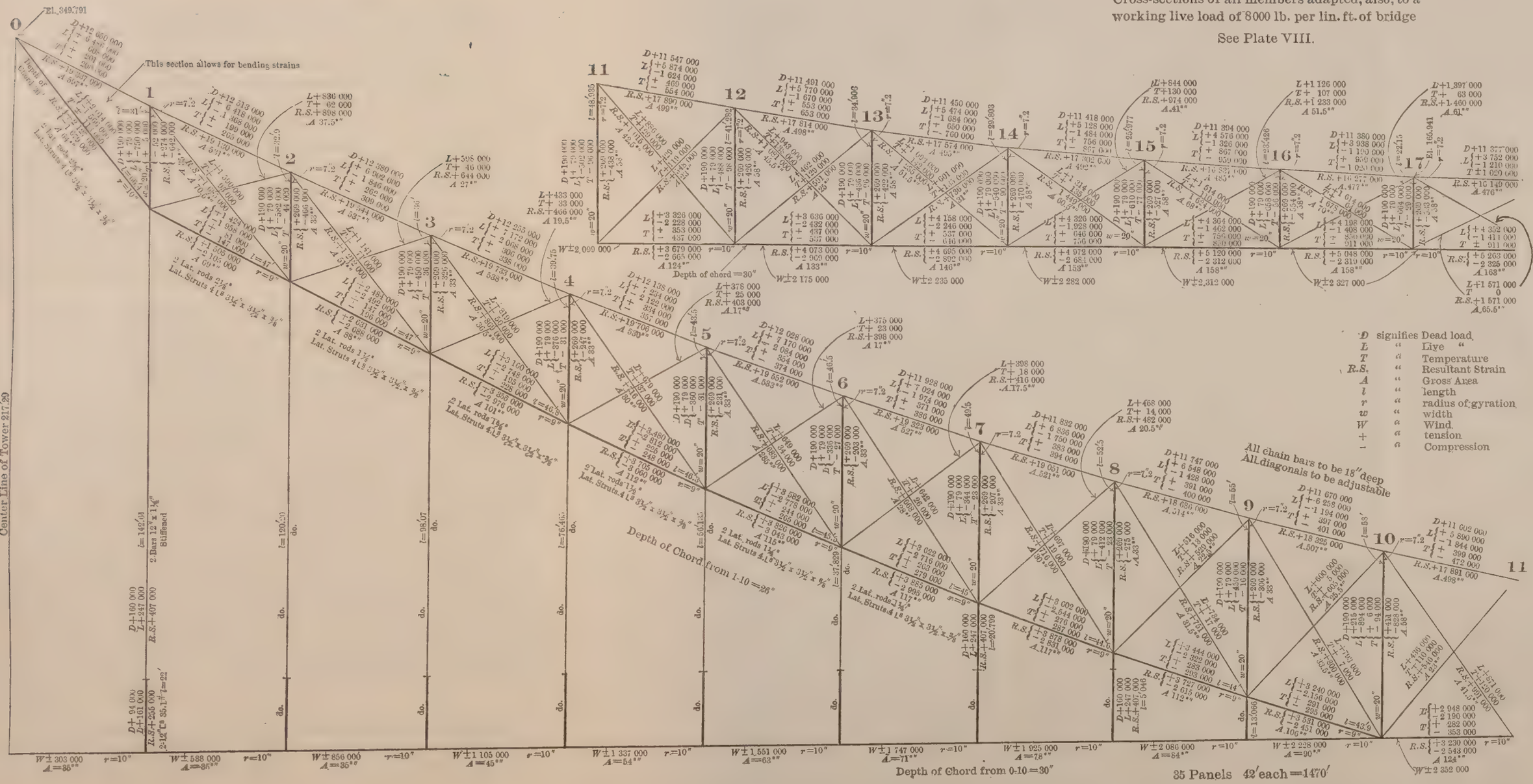
CONGESTED LOAD STRAINS {16 000 lb. per lin. ft. LIVE (for the 4 trusses) / 31 000 " " " " DEAD (" " " ")

MAIN SPAN STRAIN AND SECTION SHEET.

STRAINS ARE FOR ONE TRUSS

Cross-sections of all members adapted, also, to a working live load of 8000 lb. per lin. ft. of bridge

See Plate VIII.


Center Line of Tower 217.20
El. 349.791
This section allows for bending strains
Depth of Chord from 1-10 = 26"
Depth of Chord from 0-10 = 30"
Depth of chord = 30"
2 Bars 12" x 1¼" Stiffened
35 Panels 42' each = 1470'
All chain bars to be 18" deep
All diagonals to be adjustable
D signifies Dead load
L " Live "
T " Temperature
R.S. " Resultant Strain
A " Gross Area
l " length
r " radius of gyration
w " width
W " Wind
+ " tension
− " Compression

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Having done this, advantage can then be taken of the "gift of Nature," in the suspended arch of long span, by using higher unit stresses for all members, not essential for the stable equilibrium of the structure. This then obtains a large and legitimate economy over the erect arch, and the only economy justified by scientific reasoning.

The unit stresses in a stiffening system attached directly to the cable or chain can be taken, with perfect safety, 50% higher than that customary for the web members of the erect arch or of the cantilever. If in the latter that unit stress is based upon 16 000 lb. for structural steel (reduced for compression) then in the suspension stiffening it can be based upon 24 000 lb. This degree of economy, however, is not advisable for the composite system (of stiffening trusses), in which the strains cannot be determined so accurately, nor can economy obtained with the aid of police regulations for the distribution of the live load be commended as a good principle of suspension-bridge construction. Besides, no police regulations can be made for temperature stresses, which become very large if an attempt is made to economize on the bending from live load with the aid of high trusses, as clearly pointed out by Professor Melan.

Quotation is made from the Report of the Board of Army Engineers on Suspension Bridges in 1894, of which Major, now General, Charles W. Raymond was Chairman, to make it appear that the Board favored flexible suspension bridges in preference to more rigid forms. The context of the Report shows that it dealt with a span of 4 335 ft., the suspended weight of which was 56 503 lb. for a live load of 6 353 lb., the cables, of 17 917 lb. per lin. ft. of bridge, were combined with stiffening trusses weighing 25 202 lb. Of course, the Board properly argued for economical stresses, that is, high unit stresses, in the heavy stiffening girders, which are nearly 50% heavier than the cables, and which, otherwise, would be heavier still. And high unit stresses cause greater flexibility. No other form of suspension bridge was considered by the Board. Does anyone believe that the Board would have rejected a design in which the stiffening frame is lighter than the cables, simply because it is stiffer?

Their belief that other designs than those with stiffening trusses may be more economical is expressed in the concluding paragraph of the report, as follows: "If sufficient inducements were offered to competent engineers, to prepare competitive designs and estimates," etc.

The writer's opinion is that, should ever competitive designs be invited for a large suspension bridge, the specifications should prescribe the minimum degree of rigidity. It is more necessary in a

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suspension bridge than in any other kind of bridge. The specifications should also insist that the structure be designed for groupings of live load producing maximum bending stresses. The writer ventures to predict that, in a competition of this sort, the most economical design will be found to have very ample rigidity, but that it will not be found among the class relying upon stiffening trusses.

Professor Melan, in his admirable discussion, points out clearly that the economy of the stiffening truss has no relation to its rigidity. A bridge with a very flexible stiffening truss, or without any, may be much heavier and more expensive than one several times more rigid, as already shown.

Dr. Rieppel and Mr. Mayer both attempt to deduce a proper height for stiffening trusses. Dr. Rieppel finds a height of about one-sixtieth of the span, without a center hinge, and Mr. Mayer one-eighteenth of the span, for stiffening trusses hinged at the center.

Dr. Rieppel believes that the proper height is for that condition when the average chord strains and sections in the stiffening truss from live load are equal to their maxima from the same live load in an ordinary truss of equal span. But why should these chord sections be equal? It is difficult to see the relevancy or logic of that rule, because the proportion of chords to web, and the conditions under which the maxima of bending and shear occur in the two kinds of trusses, are very dissimilar. Equally unfounded appears to be the rule of Mr. Mayer, which presumes trusses assisted by cables, instead of cables assisted by trusses. Mr. Mayer is in favor of stiffening trusses having a height of one-eighteenth of the span, or one-ninth of the half span, when hinged at the center, and refers to a design of his for a bridge over the Hudson River as having been approved by a commission of eminent engineers. Aside from some sketches, no account giving sections and weights of such a design has been published, as far as known, so that his theories and results are not available for comparison. Dicta, without the basis of strain sheets, are not convincing.

The fact that the rules of Dr. Rieppel and Mr. Mayer diverge so widely is a sufficient indication of their unsoundness. They take no cognition of the relation of the sag of the cable to the dead weight of the suspended structure, both of which are of vital importance with stiffening trusses, as shown by Professor Melan's deductions, by the writer's Equation 1, and by the formulas given in Mr. Morison's paper.

It makes a great difference in the height and weight of the stiffening truss, whether the sag of the cable is one-eighth or one-twelfth, or whether the dead load is large or small. A chain bridge for the same live load may, on the whole, be heavier than a wire-cable

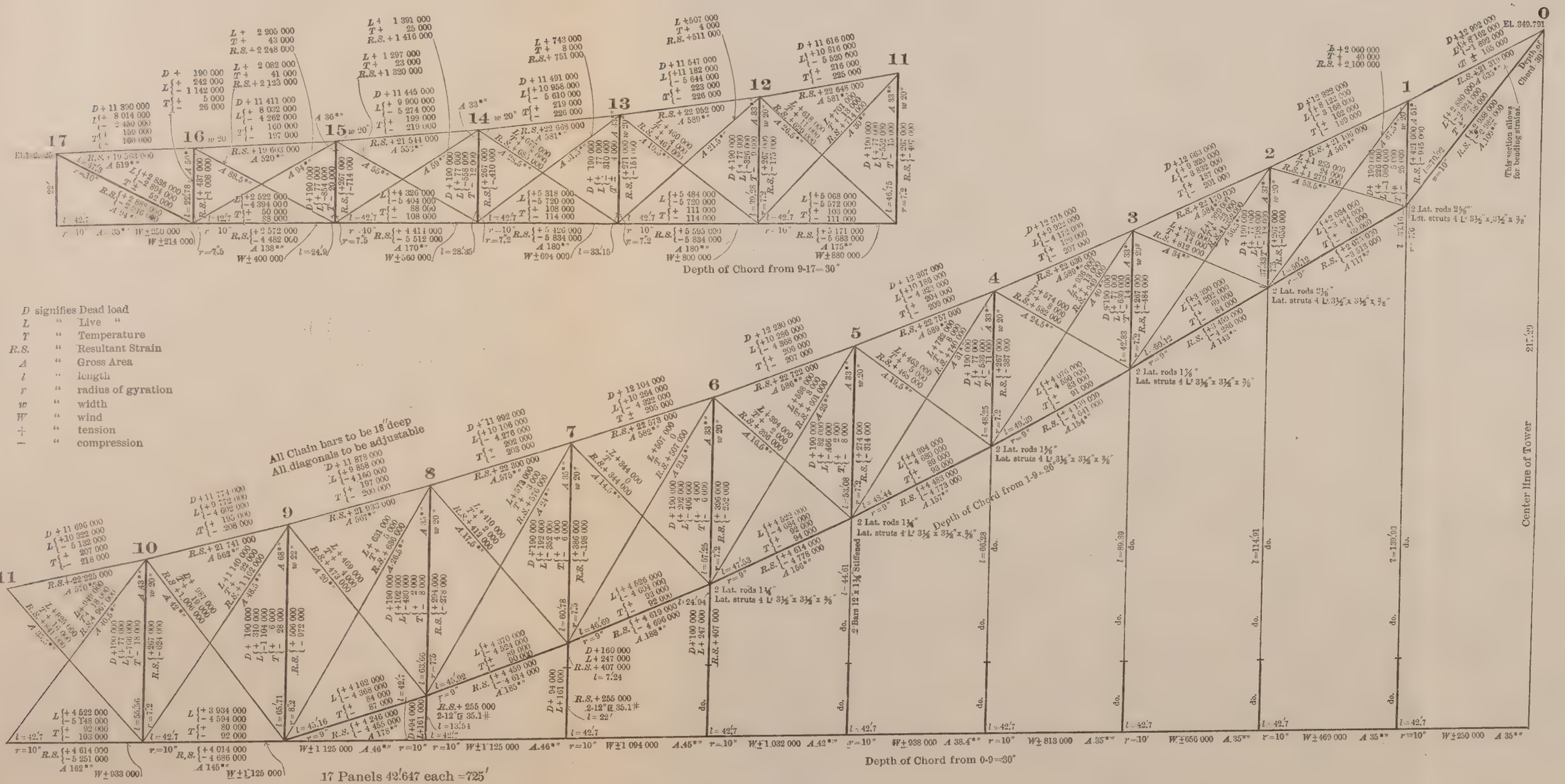
CONGESTED LOAD STRAINS {16 000 lb. per lin. ft. LIVE (for the 4 trusses) / 33 000 " " " " DEAD (" " " ")

SIDE SPAN—STRAIN AND SECTION SHEET.

STRAINS ARE FOR ONE TRUSS

Cross-sections of all members adapted, also, to a working live load of 8000 lb. per lin. ft. of bridge

See Plate IX.


D signifies Dead load
L " Live "
T " Temperature
R.S. " Resultant Strain
A " Gross Area
l " length
r " radius of gyration
w " width
W " wind
+ " tension
− " compression
All Chain bars to be 18″ deep
All diagonals to be adjustable
Depth of Chord from 9-17=30″
Depth of Chord from 1-9=20″
Depth of Chord from 0-9=30″
17 Panels 42.′647 each =725′
Center line of Tower
2 Lat. rods 2⅝″
Lat. struts 4 L 3½″x3½″x⅜″
2 Lat. rods 2⅛″
Lat. struts 4 L 3½″x3½″x⅜″
2 Lat. rods 1⅞″
Lat. struts 4 L 3½″x3½″x⅜″
2 Lat. rods 1⅝″
Lat. struts 4 L 3½″x3½″x⅜″
2 Lat. rods 1¼″
Lat. struts 4 L 3½″x3½″x⅜″
2 Bars 12″x 1⅛″ Stiffened
This section allows for bending strains.

bridge, but the stiffening trusses for the chain bridge will, for that very reason, as the equations show, be lighter than those for the wire bridge. Mr. Lindenthal.

The proposition is brought forth that in the case of three spans a more or less uniformly distributed load should be assumed, so that the stiffening frame may be reduced in weight. This proposition discloses the weakness of the case of the suspension bridge, when competing with the cantilever. The conditions of rigidity are particularly severe in a design with three spans: a middle and two side spans, as that for the Quebec Bridge or for the Manhattan Bridge. Some of the maximum stresses occur when one span is loaded and the other unloaded. That does not seem to be a good reason for claiming an exception for the suspension bridge on the ground that such loading will rarely occur. If the suspension cannot win on the same specifications of loads, etc., then it should not be used. The spandrel-braced suspension is better suited, as Mr. Schneider also observes, to meet the competition of the cantilever, than the suspension with stiffening trusses over three spans.

Dr. Rieppel, in his valued discussion, states that an extraordinary amount of wind bracing is required between the bottom chords near the towers, there being, he claims, four "enormous areas," one behind the other. If he had looked at the strain sheet of the proposed Manhattan Bridge, he would have observed that the enormous areas he imagines are each just 30 in. wide, which is a width of top chord not at all unusual with ordinary trusses. The extraordinary amount of wind bracing he would have found from the strain sheet to be two struts and four rods, weighing together 150 lb. per lin. ft. of bridge, less than ½% of the dead load.

But there is really no necessity even for that bracing, since the great tension upon the suspenders prevents bending or buckling of the bottom chord under the worst conditions. For that reason there is no wind bracing in the design for the Quebec Bridge. The Grand Avenue Bridge, in St. Louis, has likewise no wind bracing between the chains and bottom chords, and it seems to be perfectly secure in a region of tornadoes.

Dr. Rieppel, after unnecessarily expressing the opinion, repeatedly stated by the writer in his paper, that, with the spandrel-braced stiffening, the side spans should have greater depth than the central span, or should be supported from below (having evidently overlooked in the strain sheets the fact that in the Quebec Bridge and in the proposed Manhattan Bridge there is that greater depth), arrives at the not very logical conclusion that stiffening trusses with parallel chords accomplish the same purpose in the case of a "highway bridge," presumably meaning a bridge

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designed on the theory of police regulations, as already mentioned. That, of course, was not the issue of the writer's paper, which deals with a rational form of suspension bridge equally applicable to highway and railroad purposes.

One of the characteristics of such rational form is the fact that, other things being equal, the quantity of material for stiffening will be less than that for the cables, be they of wire or eye-bars, and, besides, it will be less than that required for separate stiffening trusses, be their height and proportions what they may.

There is no evidence existing, in the form of strain sheets or otherwise, that, at present prices for cable and chain material, not only the cable, but the entire superstructure with cables, will be materially cheaper than the chain and the chain bridge, as Dr. Rieppel claims. His own arguments contain no proof. On the other hand, we have the completely worked out designs for the Cologne Bridge by the Harkort Company, one for a wire-cable bridge and the other for a chain bridge, it being stated by that Company that the estimated cost for each is the same. This question will have a further conclusive answer when the strain sheets and bids for the Manhattan wire-cable bridge are published.

On the subject of architectural appearance, it may be fairly said that designers who have by their work given proof of artistic taste are generally believed to be more competent to give judgment than those having no such warrant, and they do not agree with the views expressed by some of the engineers on the appearance of the spandrel-braced chain.

The writer's thanks are due, and are herewith expressed, to all the participants in the discussion.

APPENDIX.

Mr Lindenthal.

REPORT OF THE COMMISSION OF BRIDGE EXPERTS APPOINTED BY HON. SETH LOW, MAYOR OF THE CITY OF NEW YORK, TO EXAMINE AND PASS UPON THE PLANS FOR MANHATTAN BRIDGE.

"NEW YORK, March 9, 1903.

"HON. SETH LOW, Mayor of the City of New York:

"DEAR SIR: Having been appointed by your letter of February 10th a Commission to pass upon the plans which the Commissioner of Bridges is about to submit to the Art Commission for the Manhattan Bridge across the East River, we have the honor to submit a preliminary report covering the principal features of the design, leaving our final report till the experiments which are to be made on special steel eye-bars have been completed. This report covers everything except matters dependent on the quality of the steel, and includes all features which can be of importance to the Art Commission.

"The Manhattan Bridge will be a suspension bridge, having a central span 1 475 feet long between centres of towers, and two side spans of equal length each 725 feet long measured from centre of tower to anchorage connection. Both central and side spans are somewhat shorter than the corresponding spans of the old East River Bridge.

"The general design consists of four cables supported by steel towers or bents, each cable being a steel eye-bar chain hanging in a vertical plane. The two centre cables are forty feet between centres and the side cables 28 feet between centres, making the total width 96 feet between centres of outside cables. Each tower or bent consists of four vertical posts, one under each cable; these posts having pin bearings at the bottom, the pins being carried on metal shoes of sufficient size to distribute the weight properly over the masonry.

"Although the design is a true suspension bridge, the method adopted for stiffening against irregular loads is by the use of trusses, of which the cable forms the upper member, instead of stiffening trusses at the floor level, as commonly used.

"Suspended from each pair of cables is a double-track, double-deck steel structure, carrying two elevated railroad tracks on the upper deck and two trolley tracks on the lower deck. The central space between the two lower decks is connected by a solid floor, which forms the highway floor. Two footways are carried on brackets, extending out from each lower deck. This arrangement gives a lower floor 126 feet wide over all, at the centre of which is a clear roadway entirely open, 35 feet 6 inches wide between wheel guards, and at each side a footway, also entirely open, with a clear width of about 11 feet. The arrangement seems likely to prove satisfactory. We have no suggestions to make for its improvement.

"The maximum congested load which could possibly be brought upon this bridge would consist of a continuous train of rapid transit cars on each of the four elevated tracks, of a continuous line of trolley cars on each of the four trolley tracks, of a crowd of heavy teams on the roadway and of a crowd of people on each footway. The heaviest

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rapid transit train which may run over this bridge is that adopted by the Interborough Company for the subway; such a train, in which two-thirds of the cars are motor cars (corresponding to the practice of the Manhattan Railway), with 120 passengers on each car, is estimated to have a possible weight of 1,700 pounds per linear foot and an extreme axle load of 26,000 pounds. The estimated maximum weight of a continuous line of trolley cars is 1,000 pounds per linear foot. The estimated greatest possible congested weight per linear foot on this bridge would then be as follows:

"4 rapid transit trains, at 1,700 pounds	6,800	pounds.
4 lines of trolley cars, at 1,000 pounds	4,000	"
35–5 feet of roadway, at 100 pounds per square foot	3,550	"
22 feet of footway, at 75 pounds per square foot	1,650	"
Total	16,000	"

"This is a possible load which could never occur unless special pains were taken to produce it. The conditions which would block the tracks with continuous lines of cars in one direction would probably prevent cars entering the bridge from the other direction. The weight on the sidewalks would correspond to about twelve people per linear foot, or something like 35,000 people on the bridge. The congestion of the roadway would be such that teams could not move. Under these conditions we consider that one-half of this amount may be taken as a maximum working load; this would make the working load 8,000 pounds per linear foot, which is three times that provided in the plans of the Brooklyn Bridge and 40 per cent. greater than that taken for the Williamsburgh Bridge.

"The provision of 2,000 pounds per linear foot for wind pressure proposed by the Commissioner is ample.

"In calculating the effect of temperature, provision has been made for an extreme variation of 110 degrees Fahrenheit, which we consider sufficient.

"We consider that the bridge should be so proportioned that with the congested load of 16,000 pounds per linear foot, covering the whole bridge, combined with dead load and wind pressure, no stresses would be produced anywhere reaching the elastic limit of the material or impairing the stability of the anchorages. In other words, it should not be possible for such extraordinary congested load to do any permanent injury to the bridge.

"We consider that the working load of 8,000 pounds per linear foot should be used in designing the main members of the structure. The congested load should be used in proportioning the hangers. The floor systems of the railroad and trolley tracks should be computed for concentrated loads of 26,000 pounds on an axle. The floor systems of the roadway should be proportioned for 100 pounds per square foot, with provisions for axle concentration. The floor systems of the footways should be proportioned for 100 pounds per square foot. We think it best to defer our opinion as to the unit

stresses which these different loads should be allowed to produce on different parts of the structure, until our final report, when we shall have before us the results of the experiments already referred to. We can only say now that the loads specified and the stresses which we are likely to recommend will make no great change in the weight of the structure. Mr. Lindenthal.

"The design contains three features, which, though not properly novel, are departures from the more common practice with suspension bridges. They are the cables, the stiffening trusses and the metal towers, each of which may be considered by itself.

"*Cables.*—The use of eye-bar chains is older than the use of wire in suspension bridges. It is proposed to make these eye-bars of special steel having an elastic limit of 50,000 pounds per square inch, and to subject them to unit stresses about one-half those put on the wire cables of the Williamsburgh Bridge. As the cables must carry their own weight, as well as that of other parts of the bridge and the moving load, their section must be somewhat more than double the section of wire cables. The quality of this steel is the only novel feature about these cables, and, while the indications are that a steel will be obtained which will fully meet the requirements of the case, we ask to defer any final opinion on this subject for the present. If it should be found impracticable to get this high grade of steel, similar chains can be made of steel which is in daily use, involving an increase in size of chains, but not modifying the general features of the design. The chains have decided advantages in the accessibility of all parts for inspection and protection, as well as in economy and rapidity of erection. They are to be preferred to wire cables whenever the cost of the chains is not materially greater. The cost of eye-bar chains and wire cables in this bridge would be about the same.

"*Stiffening Truss.*—Each chain is stiffened by three trusses, one for each span, the upper members being the curved chains, the lower members being in line with the floor for about half the length of each span and inclined upward, from such horizontal lines, to the top of each tower. The web consists of vertical posts, which serve also as suspenders for the floor, with diagonal tension members, the length of panels corresponding with the length of the bars in the chains. Each truss is an articulated structure, the strains in which can be definitely calculated, although its peculiar outline makes these calculations more than ordinarily complicated. The great depth of the truss at the middle of each side span and at the quarters of the central span, where the tendency to disturbance is greatest, will produce a very rigid structure, the deformations being so little that the strains can be determined with all necessary accuracy without considering changes of shape. This form of stiffening truss is desirable for the greater stiffness it produces and the more positive determination of strains under all variations of loads. The eye-bar chain cables render its use possible.

"*Towers.*—The tower adopted is, perhaps, the most novel feature in the design, but the novelty consists in its application to a structure of this magnitude. Each tower is really a bent and consists of

Mr. Lindenthal.

four vertical posts which are built steel members each supported on a pin at the bottom, the chains being rigidly fastened to the top with pins. The four posts are connected by cross struts and bracing into a rigid frame more than 300 feet high and 96 feet wide between centres, which is held vertically by the chains, but is free to rock on the pin bearings below to accommodate such changes in length as may take place in these chains under strains and temperature. The design is correct in principle and secures more uniform pressures and results more closely coinciding with the calculations than may be obtained from any form of rigid tower. Wherever metal towers are used we consider that the best practice is to fasten the cables rigidly to the top of the towers providing for changes of position by the spring of the tower rather than by movable saddles, the increased strain in the metal caused by such deflections of the towers being easily kept within safe limits. The device of a frame supported on pin bearings below eliminates even these strains.

"*Anchorages.*—The general plan of the anchorages, as shown by the preliminary studies, seems to us satisfactory, although the permissible pressures on the foundations cannot be finally determined until the material in the bottom is exposed.

"In reply to the four specific questions contained in your letter of appointment we beg to advise you that:

"1. The plans are in accordance with advanced knowledge of suspension bridge designing. They are likely to be as economical in construction as other forms of suspension bridges. They provide fully for temperature stresses. They provide for a structure of unusual rigidity under concentrated loads. Ample provision is made for wind pressure. They are consistent with the best protection from corrosion. Our report on the quality of the steel must be deferred to await the results of tests not yet made.

"2. The strength, stability and carrying capacity of the bridge will be adequate for any congestion of traffic which may occur on the railroad tracks, roadways and promenades if the provisions for loads laid down heretofore are followed.

"3. The structure as designed will be incombustible.

"4. The design favors speedy erection of the superstructure after the masonry is ready.

"Very respectfully, your obedient servants,

"Geo. S. Morison,
C. C. Schneider,
Henry W. Hodge,
Mansfield Merriman,
Theodore Cooper."

Final Report.

"New York, June 29, 1903.

"Hon. Seth Low, Mayor of the City of New York.

"Dear Sir: Having been appointed by your letter of February 10th a Commission to pass upon the plans prepared under the direction of the Commission of Bridges for the Manhattan Bridge

Mr. Lindenthal.

across the East River, we submitted on March 9th a preliminary report covering the principal features of the design. We deferred our final report, relating to the quality of the material to be used for the eye-bar cables, until additional tests could be had and we should be satisfied, by correspondence with the leading manufacturers of structural steel, that the desired material could be obtained at reasonable price and in open competition.

"Three different manufacturing plants have manufactured and tested nickel-steel bars for the purpose of this investigation. From the results of the tests submitted to us, we are satisfied that material which will meet the requirements hereinafter outlined can be obtained for this bridge.

"Our investigation relating to material has been restricted to the special steel best suited for use in eye-bar cables, as this feature of the design under consideration is the main departure from the usual practice in the design of suspension bridges. This investigation satisfied us that a steel of the following qualities, in full size annealed eye-bars, will be satisfactory:

"Ultimate strength, pounds per square inch.....	85,000
Actual elastic limit, " " " "	48,000
Percentage of elongation in 18 feet.....	9
" " reduction at fracture.....	40

"The maximum stress in these eye-bars, under actual dead load combined with the working load of 8,000 pounds per linear foot given in our preliminary report, together with effect of temperature, should not exceed 30,000 pounds per square inch.

"The tests so far made would indicate that the above results can be obtained by a steel made by the open hearth process, containing 3¼ to 3½ per cent. of nickel; containing not over .05 per cent. of sulphur; not over .06 per cent. phosphorus, if made in an acid furnace, nor over .04 per cent. if made in a basic furnace.

"Three manufacturers, with established plants, state that they are now prepared to furnish steel to fulfill the above requirements; and others state that they can arrange to furnish it in a reasonable time, if the demand warrants them in so doing.

"The strength of steel eye-bars having the above qualities will be 50 per cent. greater than standard bars now used in the best class of bridge construction; the working strains can therefore be increased in the same proportion.

"In our preliminary report of March 9th, we reported favorably on all features relating to the design of this bridge, subject only to the uncertainty of obtaining this quality of material. This uncertainty is now removed.

"In this final report, we unanimously recommend the adoption and execution of the proposed design of the Manhattan Bridge, as submitted to us by the Commissioner of Bridges.

"Very respectfully, your obedient servants,

"GEO. S. MORISON,
C. C. SCHNEIDER,
MANSFIELD MERRIMAN,
HENRY W. HODGE,
THEODORE COOPER."

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 999.

THEORY AND FORMULAS FOR THE ANALYTICAL COMPUTATION OF A THREE-SPAN SUSPENSION BRIDGE WITH BRACED CABLE.*

By Leon S. Moisseiff, Assoc. M. Am. Soc. C. E.

With Discussion by
Messrs. I. P. Church, George F. Swain and Leon S. Moisseiff.

The scarcity of published information on fully-worked-out, practical applications of the theory of suspension bridges to long-span structures is well known to engineers. The writer hopes that the present paper will furnish some of the desired information, and will help to fill part of the gap in engineering literature, as far as it applies to bridges of the type described.

When, in the course of the writer's professional duties, the computation of a three-span suspension bridge with braced cables came up, he could not find, in engineering literature, the theory of this type developed in any way. All there was on hand for the problem consisted of the general principles of the elastic theory. The deduction of the required equations was made still more troublesome by the fact that the outlines of the stiffening trusses, as shown in Fig. 1, do

* Presented at the meeting of September 21st, 1904.

not follow continuous curves convenient for integrations, and summations had to be used instead.

It is the writer's hope that the theory and formulas deduced in this paper are presented with sufficient clearness to enable the reader to follow their development without having to resort to mathematical textbooks.

These formulas were applied to the computation of a first-class city bridge, of long span, and of a capacity for city traffic greater than that of any already constructed for the same city. While the design of bridges of this type presents many interesting features, the writer does not believe the present time to be ripe for discussion, and will not treat of the merits and demerits of the bridge in general. Therefore, the theory alone will be presented, and only as many of the results as may be of general interest and may help to illuminate the subject.

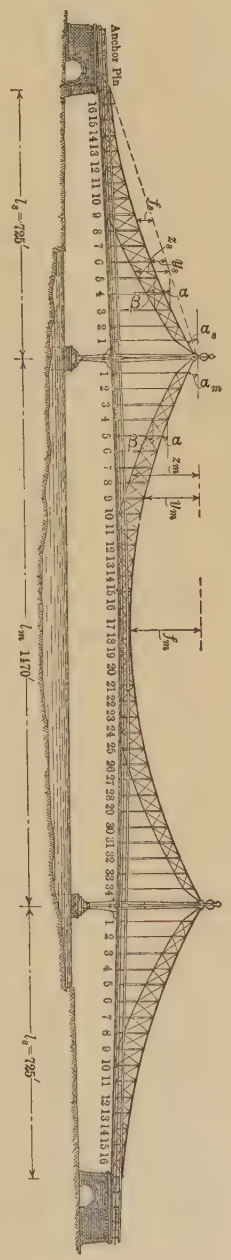


FIG. 1.

In considering the design of a long-span suspension bridge, it is well to remember the main features by which it differs from bridges of other types. The suspension bridge, together with its antipode, the arch, produces on its abutments reactions the lines of action of which, unlike those in common beam and truss bridges, are not vertical, but, generally, more or less inclined to the horizontal. This, as is well known, is the distinctive characteristic of arch and suspension bridges, and it determines their design, construction, and behavior under load, and under the action of other causes. In fact, the existence of the horizontal components of the reactions is the great reason for the use of these bridges. While the com-

mon beam and truss are in a sense self-contained, and their reactions on the abutments are independent of the geometric form of the truss, the material of which they are made and its elastic behavior, arch and suspension bridges are not self-contained, and their reactions are dependent on all these conditions. To make them self-contained, an additional member would be required, which would act as a tie between the supports, in the case of the arch, and as a strut, in the case of the suspension bridge. As it is, the rigidity of the earth supplies the required connection. The great importance of a slip in the abutments can readily be seen: It is equivalent to an undue change in length of the imaginary connecting member.

This imaginary connecting member would take up the horizontal components of the reactions; and the vertical components, only, would be transmitted to the abutments. The so-formed truss could then be computed practically as a simple beam. Actually, the function of this imaginary member is performed by the horizontal component of the reaction supplied by the abutments or anchorages.

As long as vertical forces, only, are acting on an upright or inverted polygon, it is evident, from the first principles of statics, that the horizontal component of the stress in the polygon will be constant, from point to point throughout the span, for the given system of forces. Naturally, the horizontal components of the reactions will then be equal to each other and to the constant component of the stress. This is a well-known property of the arch, and is common to all its possible geometric figures. It forms the very foundation of all rational arch theories. When confused by mathematical symbols and unknown variables, it is the constancy of the horizontal component which guides the engineer to a knowledge of moments and shears.

This is true for arches as well as suspension bridges, and, in so far, the theory of the arch applies also to its inverted form. In fact, the theory of these structures is practically the same, and to understand one means to know the other. But the computations based on their theory vary considerably because of considerations of an engineering and practical character.

The upright arch has its supports below its center of gravity, and, therefore, is in unstable equilibrium. The suspension bridge, on the contrary, has its supports above the center of gravity, and is in stable equilibrium.

If, on an upright arch of a given span and rise, a given system of loads is acting, and if the arch is assumed to consist of a number of members connected by frictionless hinges, there is only one geometrical polygon which will correspond to the requirements of equilibrium. This is its equilibrium polygon. If by any cause this polygon be disturbed from its original position, it will not tend to come back, but will depart from it and collapse. If the same arch is inverted, its equilibrium polygon will be identical with the former, but also inverted. But an important change has taken place meantime; when similarly disturbed, the inverted polygon will go back to its original position. To keep a flexible upright polygon in position would require great care in its design, and the provision of a number of outside forces to prevent its displacement. On the contrary, the inverted polygon must only be able to resist the stresses caused in it, and the force of gravity will guide it safely to its final position. The effect of the above properties on the erection of arches and suspension bridges is self-evident. All that is necessary, from this point of view, in the erection of the main member of a suspension bridge is to suspend it so that it will have the desired rise or versine when under a certain load. This enables the engineer to suspend all the fixed load of the bridge from the cable, without requiring any stiffening to keep it in proper shape.

Convenient as the stability of a suspended polygon may be, for erection, and safe as it may be, this movability does not satisfy the requirements of bridge traffic. Traffic is dependent on tractive power and grades, and the limits of the movability and deflection of a bridge are determined by the steepness of the grades which its traffic can endure. To reduce the movability and deflection of a suspension bridge, its equilibrium polygon is "stiffened." This can be accomplished in several ways. The most common of these are: The stiffening of the original equilibrium polygon by attaching to it a truss, one chord of which is frequently formed by the polygon itself, and the suspension from the flexible polygon of a sufficiently rigid truss. The deflections and distortions of a cable or chain are caused by changes in temperature and by loads traveling over the bridge, and it is the function of the stiffening truss to limit these distortions.

As we see, the fixed or dead load does not enter directly as a cause of distortion from the curve assumed by the cable under the weight

of the bridge. On the contrary, due to its stability of form, it tends to reduce the distortion produced by the other causes, and its beneficial influence is much felt in heavy and flexible bridges. Hence the stiffening truss must be designed primarily to resist the distorting effect which will be caused by a given moving load in the various positions which it may assume. In addition, the effect of changes in temperature must be included. The cable itself, as the main supporting member, need only be designed for the greatest total load which may come on it. If sufficiently strong for the latter, and resting on towers of proportionate strength and well anchored, the bridge is safe from danger, even should some of the stiffening members buckle or break. The knowledge of this is greatly inducive to the sound sleep of the engineer; but structures are not built to break, even without danger of complete failure, and, in important city bridges, the buckling of a stiffening chord or the breaking of a few suspenders may frighten the public unnecessarily.

At no stage of the design of an important suspension bridge is good judgment and a thorough knowledge of its functions required more than in determining the moving loads for which to provide and the unit stresses to be allowed for the stiffening trusses. The moving load being decided, the choice of the depth and form of truss and of the allowable unit stresses fixes the sectional areas and deflections which will follow necessarily from the assumed values.

To eliminate the effect of the fixed load on the stiffening truss, the cable is erected together with the truss suspended from it, the floor system, paving, etc., and at a certain temperature, assumed as normal, the truss is adjusted for its final connections. The fixed load on the bridge before the closing of the truss, therefore, cannot cause any stresses in the latter. Any additional load which may come on the bridge after this will causes stresses in the stiffening truss. So, also, will a change of temperature from the normal. From now on, the truss and the cable must act together; they form one elastic system. The action of this system will be dependent on the geometrical configurations of the component parts and on their elastic behavior. This will be seen plainly in the equations which will be developed in the following:

As stated before, the three-span bridge, the theory of which is presented here, is of the braced-cable type, and of the outlines

shown in Fig. 1. The upper curve represents the main cable, and follows the equilibrium polygon of the weight of the bridge. It also forms the upper chord of the stiffening truss, which is attached directly to it. Under the action of the fixed load, and at an assumed normal temperature, there will be no stresses in the members of the stiffening truss. Any additional fixed or moving load which may come on the bridge, or any departure from the normal temperature, will cause stresses in the truss, which will then act as an inverted, braced arch over three spans. As shown in Fig. 1, the ends of the main- and side- span trusses come on pins or hinges, while they are continuous between these points. It is obvious that they form three two-hinged arches, and, as such, are statically indeterminate in the first degree. This means that the equations furnished by the conditions of static equilibrium are not sufficient to determine the stresses, and that one more equation of condition is required. It is furnished by the conditions of the elastic behavior of the truss. In one of its forms it is well known to engineers under the name of the "Principle of Least Work," and has been discussed repeatedly in the *Transactions* of this Society.

Algebraically, the general expression for the principle of least work is written:

$$\frac{\delta W}{\delta X} = \int \frac{P}{E A} \frac{\delta P}{\delta X} ds + \int \frac{M}{E I} \frac{\delta M}{\delta X} ds = 0 \ldots\ldots\ldots (1)$$

in which,

W = the total elastic work in the system when free from vibration;
X = an unknown force, assumed as an independent variable;
P = the direct axial force acting in any part of the system;
M = the bending moment acting in any part of the system;
s = the length of the member considered;
E = its coefficient of elasticity;
A = its cross-sectional area;
I = its moment of inertia about its neutral axis.

It has generally been found convenient to take the horizontal component of the pull in the cable as the unknown variable indicated by X in Equation 1. As stated before, the horizontal component, which may be denoted by H, represents the characteristic mark of the arch or suspension bridge. It forms a function of the stress in any part of the structure, and is constant at any section for a given condition

of loading. These properties make it convenient to express the stress and work of the several parts in terms of H, as was done in the case described.

The general method used for the derivation of the formula for the horizontal pull is common to most applications of the least-work principle, and is the same as developed by Professor J. Melan in his book on "Arches and Suspension Bridges." The deduction of the equations for a three-span suspension bridge of the type described is original with the writer.

After the lengths of the spans, the required clearances for the purposes of navigation, and the grades have been fixed, and the desirable versine of the cable has been chosen, the curve of the cable which forms the upper chord of the truss has thereby been determined. In the main span the unstiffened cable will naturally assume the form of the equilibrium polygon of the fixed load on it. As the latter is practically uniformly distributed over the whole of the span, its equilibrium polygon will approximate a parabola. For all preliminary computations, and even for most final computations, it has been found that the curve of the cable may be assumed to be parabolic.

The form of the curves of the side spans are determined by that of the main span, because the three spans must balance, under the fixed load and normal temperature. In other words, the horizontal component of the pull in the cable must be the same throughout the bridge. For a uniform load over the whole main span on the unstiffened cable, it is, as is well known:

$$H_1 = \frac{p_m l^2_m}{8 f_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which,

p_m = the load on the main span, per linear foot of bridge;

l_m = the length of the main span, from the centers of the towers; and

f_m = the versine of the main-span cable.

The expression for the horizontal pull in the side-span cable is, similarly:

$$H_2 = \frac{p_s l^2_s}{8 f_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which the suffix, s, indicates the similar notation for the side spans. As stated in the foregoing, the balancing of the three spans requires that $H_1 = H_2$, or

$$\frac{p_m l^2_m}{f_m} = \frac{p_s l^2_s}{f_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Whence the versine of the side-span parabola:

$$f_s = \frac{p_s}{p_m}\frac{l^2{}_s}{l^2{}_m} f_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

As can be seen from this expression, the value of f_s is determined by the data.

If the uniform weight of the bridge, including the cables, is the same for the main and side spans, $p_m = p_s$, the expression for the side-span versine becomes:

$$f_s = \frac{l^2{}_s}{l^2{}_m} f_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Or, in other words, the versines are to each other as the squares of their spans. For all preliminary computations, the use of this relation is perfectly justified. If the fixed load is not uniformly distributed, the versines will be to each other as the greatest moments due to the loads on a simple beam. It should be noted, however, that the versine of the side spans, as well as all curve ordinates, must be measured at the center of the span from the straight line drawn through the center of the pin at the top of the tower to the center of the end pin at the anchorage as indicated in Fig. 1.

It will be noticed that the versine of the side-span curve, f_s, is independent of the elevation of the anchor pin relative to the tower. The latter elevation is determined by considerations as to the steepest grade allowable and the least cost of the anchorage masonry.

The curves of the cable having been determined and drawn, the outlines of the stiffening trusses are laid out. The form of the lower chord, the depths of the truss and the panel arrangement are governed by considerations of economy and good looks. The type of the truss discussed in this paper is adaptable to such depths of truss as will generally follow the growth of the bending moments.

The floor system can now be designed and its weight computed, together with the weight of railings, pavement, rails, etc. In the present case, which is that of a long-span city bridge with solid buckle-plate floor, the weight of the foregoing items constituted one-half of the total fixed load on the bridge. With a thorough understanding of the influences and effects of the several parts of the structure, it is now comparatively easy to determine approximately the fixed load on the bridge, the probable areas of the cable and stiffening chord at the center of the main span, the average areas of

the cable and stiffening chord, and the approximate moments of inertia at any panel point.

At normal temperature and under fixed load, only, the stiffening chord will, by the assumptions of the design and erection, be free of stress. With the advance of an additional load, the trusses will act as inverted arches, and the load will be distributed between cable and stiffening chord. For purposes of computation, it is important to determine approximately the relative share of the load sustained by each of them. The following formula has been found to give closely approximate results.

If the two chords of the truss are assumed to be suspended without any diagonal stiffening members between them, a load on the bridge would be sustained by the two chords in proportion to their vertical deflection under the stress caused. Let us denote by H the sum of the horizontal components in both chords and by $m\,H$ and $n\,H$ the proportion of H taken up by the cable and by the lower chord, respectively; so that,

$$H = m\,H + n\,H, \text{ and } m + n = 1 \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

If, further, A_1 denotes the average area of the cable in the span considered, A_2 the average area of the stiffening chord, L_1 the length of the cable, L_2 that of the chord, f_1 the versine of the cable and f_2 that of the chord, the following relation will hold true:

$$\frac{H_1}{H_2} = \frac{A_1 L_2 f_2}{A_2 L_1 f_1} = \frac{m}{n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The Determination of the Horizontal Pull in Cable and Chord Due to Moving Load.—

Let

H = the total horizontal pull in the cable and stiffening chord;
$m\,H$ = " horizontal pull in the main-span cable;
$n\,H$ = " " " main-span stiffening chord;
$m_1\,H$ = " " " side-span cables;
$n_1\,H$ = " " " " stiffening chord;
A_c = " cross-sectional area of the cable at any point;
C_m = " " " main-span stiffening chord;
C_s = " " " side-span " "
I = " moment of inertia of the truss at any point;
E = " coefficient of elasticity of the material;
h = " height of the tower;

y_m and y_s = the ordinates of the cable, in main and side spans, respectively;

z_m and z_s = " ordinates of the stiffening chord, in main and side spans;

α = " angle the cable makes with the horizontal at any point;

α_m and α_s = " angles it makes with the horizontal at the tower pin, in main and side spans, respectively;

β = " angle the stiffening chord makes with the horizontal at any point;

s = " length of the cable curve;

a = " panel length.

The bridge is symmetrical about an axis through the center of the main span.

To express the stress in the several parts of the structure in terms of the unknown horizontal pull, H, is quite simple. The portion of H acting in the main cable is $m\,H$, and, since the stress in the cable varies as the secant of its angle, α, with the horizontal, it is:

$$m\,H \sec.\ \alpha = m\,H\frac{d\,s}{d\,x}.$$

Similarly, for the side spans:

$$m_1\,H\frac{d\,s_1}{d\,x}.$$

The direct stress in the stiffening chords as suspended members, similarly to that in the cables, is expressed by:

$n\,H \sec.\ \beta$, for the main span,

and

$n_1\,H \sec.\ \beta$, for the side spans.

The stress caused in the stiffening truss, as such, by bending moments due to moving loads is somewhat less simple. However, the writer believes that the action of the stiffening truss is explained clearly by the simple algebraic form in which the bending moment at any section of the truss can be expressed.

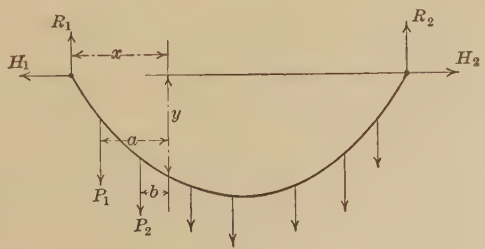


FIG. 2.

Let Fig. 2 represent

an inverted rigid arch of any form which is subjected to the action of a number of vertical forces, P_1, P_2, etc.

The reaction at the points of support can be decomposed into the vertical reaction, R_1, and the horizontal pull, H. As the external forces on the arch are all vertical, they have no horizontal component. The horizontal reaction, H, therefore, is the same at both points of support, and the horizontal component of the direct stress in the arch is constant throughout for the given system of loads. The vertical component, on the contrary, varies from the point of application of each load; and the vertical reactions, R_1 and R_2, follow the law of the lever, as for simple beams. If we now take moments about any point, x, y, we get:

$$M_x = R_1 x - (P_1 a + P_2 b + \ . \ . \ . \ ) - Hy \ldots\ldots\ldots (9)$$

But it will be observed that the first two expressions on the right side represent the bending moment of a simple beam resting freely on its supports. If we denote this bending moment by $\mathfrak{m}$ we may write:

$$M_x = \mathfrak{m} - Hy \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

This is the general expression for the bending moment at any point, for a suspension bridge, as well as for an upright arch. In plain words, it states that, as far as bending is concerned, both the upright and the inverted arch act as simple beams which are relieved by a moment, Hy, which is due to the horizontal reaction, H, acting at the distance, y, from the point, x, y. It is well to remember the general form and meaning of this expression for the moment, because it represents the fundamental idea of the arch, and, in the most complicated cases, it will serve as an excellent method of orientation.

For the truss discussed in this paper, the general form of the moment, expresed in H, becomes:

For the main span, $M_x = \mathfrak{m} - m H y_m - n H z_m \ldots\ldots$ (11)

" " side spans, $M_x = \mathfrak{m} - m_1 H y_s - n_1 H z_s \ldots\ldots$ (12)

The general expression for the principle of least work, Equation 1, is:

$$\frac{\delta W}{\delta H} = \int \frac{P}{E A} \frac{\delta P}{\delta H} d s + \int \frac{M}{E I} \frac{\delta M}{\delta H} d s = 0.$$

Since the coefficient of elasticity will be practically the same for all parts of the structure, it may be omitted altogether. Table 1, Plate VII, contains the tabulation of the foregoing expression extended over the several parts of the system.

TABLE 1.—THE LEAST WORK FOR MOVING LOAD.

Member.	Direct axial stress or bending moment.	$\frac{\delta P}{\delta H}$ or $\frac{\delta M}{\delta H}$	Area, or moment of inertia.	Length of member.	$\int \frac{P}{A}\frac{\delta P}{\delta H} ds + \int \frac{M}{I}\frac{\delta M}{\delta H} ds = 0.$
Cable, main span	$m H \frac{ds}{dx} = m H \sec. \alpha$	$m \frac{ds}{dx}$	A_c	$\Sigma a \sec. \alpha$	$m^2 \sum_{o}^{lm} \frac{a \sec.^3 \alpha}{A_c} H$
Cable, side spans	$m_1 H \frac{ds}{dx} = m_1 H \sec. \alpha$	$m_1 \frac{ds}{dx}$	A_c	$2 \Sigma a \sec. \alpha$	$2 m^2_1 \sum_{o}^{ls} \frac{a \sec.^3 \alpha}{A_c} H$
Stiffening chord, main span	$n H \sec. \beta$	$n \sec. \beta$	C_m	$\Sigma a \sec. \beta$	$n^2 \sum \frac{a \sec.^3 \beta}{C_m} H$
Stiffening chord, side spans	$n_1 H \sec. \beta$	$n_1 \sec. \beta$	C_s	$2 \Sigma a \sec. \beta$	$2 n^2_1 \sum \frac{a \sec.^3 \beta}{C_s} H$
Stiffening truss, main span	$\mathfrak{m} - m H y_m - n H z_m$	$-m y_m - n z_m$	I	Σa	$\left[m^2 \sum \frac{y^2_m a}{I} + n^2 \sum \frac{z^2_m a}{I} + 2 m n \sum \frac{y_m z_m a}{I} \right] H - m \sum \frac{\mathfrak{m} y_m a}{I} - n \sum \frac{\mathfrak{m} z_m a}{I}$
Stiffening truss, side spans	$\mathfrak{m}_1 - m_1 H y_s - n_1 H z_s$	$-m_1 y_s - n z_s$	I	$2 \Sigma a$	$\left[2 m^2_1 \sum \frac{y^2_s a}{I} + 2 n^2_1 \sum \frac{z^2_s a}{I} + 4 m_1 n_1 \sum \frac{y_s z_s a}{I} \right] H - 2 m_1 \sum \frac{\mathfrak{m} y_s a}{I} - 2 n_1 \sum \frac{\mathfrak{m} z_s a}{I}$

If for A_c the area at the center be approximated and the cable be assumed to vary in section as its secant, we get for parabolic curves:

$$\frac{m^2}{A_c} l_m \left(1 + \frac{16}{3}\frac{f^2_m}{l^2_m}\right) H \text{ and } 2\frac{m^2_1}{A_c} l_s \left(1 + \frac{16 f^2_s}{3 l^2_s}\right) H.$$

Summing up and equating to zero, the following expression is obtained for H:

$$H = \frac{m \sum \frac{\mathfrak{m} y_m a}{I} + n \sum \frac{\mathfrak{m} z_m a}{I} + 2 m_1 \sum \frac{\mathfrak{m} y_s a}{I} + 2 n_1 \sum \frac{\mathfrak{m} z_s a}{I}}{\left[m^2 \sum_{o}^{lm} \frac{a \sec.^3 \alpha}{A_c} + 2 m^2_1 \sum_{o}^{ls} \frac{a \sec.^3 \alpha}{A_c} + n^2 \sum \frac{a \sec.^3 \beta}{C_m} + 2 n^2_1 \sum \frac{a \sec.^3 \beta}{C_s} + m^2 \sum \frac{y^2_m a}{I} + n^2 \sum \frac{z^2_m a}{I} + 2 m n \sum \frac{y_m z_m a}{I} + 2 m^2_1 \sum \frac{y^2_s a}{I} + 2 n^2_1 \sum \frac{z^2_s a}{I} + 4 m_1 n_1 \sum \frac{y_s z_s a}{I} \right] = D} \quad .. (13)$$

It will be noticed that the work of the web members of the truss is not included in the summation of Table 1. The effect of these members is very small, and, since they had to be made adjustable, it did not appear necessary to include them.

A glance at the denominator of the expression deduced for H, in Equation 13, which we will denote by D, shows that it is independent of any load on the bridge and dependent on such values only as form the geometric and elastic characteristics of the structure. For a given structure, of the sections and dimensions assumed, the denominator, D, is constant, whatever the load on the bridge. Therefore, it need be computed only once, which reduces greatly the labor of computation.

From the form of the equation,

$$H = \frac{1}{D}\left[m \sum \frac{\mathfrak{m}\, y_m\, a}{I} + n \sum \frac{\mathfrak{m}\, z_m\, a}{I} + 2\, m_1 \sum \frac{\mathfrak{m}\, y_s\, a}{I} + 2\, n_1 \sum \frac{\mathfrak{m}\, z_s\, a}{I}\right] \quad \ldots\ldots\ldots\ldots \quad (14)$$

it is seen that the expressions in the numerator which represent the effect of the load can be treated independently; the first expression in the brackets being the effect of moving load on the main span, and the second that on the side spans. The coefficients, 2, in the latter are indicative only of the existence of two symmetrical side spans. In the case of an identical load on both side spans, the coefficients would represent the actual effect of the load on H.

The expressions contained in Table 1 are general, and are true for any form of cable curve and variable panel length, and the summations may be much simplified for regular curves and equal panel lengths.

H for Uniform Load from Anchorage to Anchorage.—To obtain the greatest reactions on the towers and at the anchorages, the bridge must be loaded with its full uniform load from anchorage to anchorage. For a uniform load, p, per unit of length, the simple beam moment:

$$\mathfrak{m} = \frac{p}{2}\, x\, (l - x) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

in which l indicates the length of span. For a parabolic curve of the cable, the ordinate of the parabola,

$$y = \frac{4f}{l^2}\, x\, (l - x) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

for the origin of the co-ordinates at the center of the tower pin. f in-

dicates the versine of the curve. Substituting these values for $\mathfrak{m}$ and x, it becomes:

$$m \sum \frac{\mathfrak{m}\, y_m\, a}{I} = m \frac{p\, l^2_m}{8\, f_m} \sum \frac{y^2_m\, a}{I},$$

$$n \sum \frac{\mathfrak{m}\, z_m\, a}{I} = n \frac{p\, l^2_m}{8\, f_m} \sum \frac{y\, z\, a}{I},$$

and similarly for the side spans. The total horizontal pull in the cable and chord, due to a full uniform load, p, per horizontal unit of length on the main span only, we then get as:

$$H_m = \frac{p\ l^2_m}{8\, f_m} \times \frac{m \sum \frac{y^2_m\, a}{I} + n \sum \frac{y\ z\ a}{I}}{D} \quad \ldots\ldots\ldots (17)$$

For a full load on one of the side spans:

$$H_s = \frac{p\ l^2_s}{8 f_s} \times \frac{m_1 \sum \frac{y^2_s\, a}{I} + n_1 \sum \frac{y_s\, z_s\, a}{I}}{D} \quad \ldots\ldots\ldots (18)$$

For the load extending from anchorage to anchorage, the effects of each span must be added, so that

$$H_{full.} = H_m + 2\, H_s \quad \ldots\ldots\ldots (19)$$

To obtain the proportions of H sustained by cable and stiffening chord, the foregoing values of H are multiplied by the proportions, m and n, deduced before.

H for a Single Concentration.—If a single concentrated load, P, acts on a simple beam, of a span, l, at a distance, v, from the point of support, the expression for the bending moment due to it will take the form

$$\mathfrak{m}_{o-v} = \frac{P\,(l - v)\, x}{l} \quad \ldots\ldots\ldots (20)$$

for any section, x, between o and v, and

$$\mathfrak{m}_{v-l} = \frac{P\,(l - x)\, v}{l} \quad \ldots\ldots\ldots (20a)$$

for any section, x, between v and l.

Substituting these expressions for $\mathfrak{m}$ in the numerator of the general equation for H:

$$\sum \frac{\mathfrak{m}\, y\, a}{I} \text{ and } \sum \frac{\mathfrak{m}\, z\, a}{I},$$

we obtain for a concentrated load, P:

$$H_P = \frac{P}{D}\Bigg[m \sum_o^v \frac{a\, x\, y}{I} - \frac{m\, v}{l} \sum_o^l \frac{a\, x\, y}{I} + m\, v \sum_v^l \frac{a\, y}{I} + n \sum_o^v \frac{a\, x\, z}{I} - \frac{n\, v}{l} \sum_o^l \frac{a\, x\, z}{I} + n\, v \sum_v^l \frac{a\, z}{I} \Bigg] \quad \ldots\ldots\ldots (21)$$

This equation for H represents the analytical expression for the influence line of H for a load, P, traveling over the bridge. It is equally true for the main and for the side spans, for which, of course, the corresponding values and summations must be used.

H for a Uniform Load Continuous from Point of Support to any Section.—If a uniform load, p, per unit of length, advances continuously from one end of a simple beam, of a span, l, to any point distant u from it, the expression for the bending moment due to this load will be:

$$\mathfrak{m}_{o-u} = \frac{p\,u}{2\,l}(2\,l - u)\,x - \frac{p\,x^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

for any section between o and u, and

$$\mathfrak{m}_{u-l} = \frac{p\,u^2}{2\,l}(l - x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22a)$$

for any section between u and l.

Substituting these expressions for the simple-beam bending moment, $\mathfrak{m}$, in the numerator of the general equation for H, we get, for a continuously advancing load, p:

$$H = \frac{p}{2\,D}\left[m\,u\left(2 - \frac{u}{l}\right)\sum_{o}^{u}\frac{a\,x\,y}{I} - m\sum_{o}^{u}\frac{a\,y\,x^2}{I} \right.$$

$$+ \frac{m\,u^2}{l}\sum_{u}^{i}(l - x)\frac{a\,y}{I} + n\,u\left(2 - \frac{u}{l}\right)\sum_{o}^{u}\frac{a\,z\,x}{I}$$

$$\left. - n\sum_{o}^{u}\frac{a\,z\,x^2}{I} + \frac{n\,u^2}{l}\sum_{u}^{l}\frac{(l - n)\,a\,z}{I} \right] \quad \ldots\ldots\ldots (23)$$

To obtain the value of H due to a partial uniform load between any two points, we need only subtract, from the value computed from the above equation for the load extending from the support to the farther point, the value computed due to the load to the nearer point. As the values for H are tabulated from panel point to panel point, this is very simple. The equation deduced for H applies to either main or side spans for the corresponding values and summations.

With the several equations which have been established in the foregoing, the effect of any kind of moving load on the bridge can be computed.

The Determination of the Horizontal Pull in Cable and Chord Due to Temperature Changes.—The stiffening truss is free of stress at an assumed normal temperature which is the temperature of the final

adjustment of the bridge under its fixed load only. Any change of temperature from the normal will cause stresses in the stiffening truss members. If ω denotes the coefficient of expansion of steel per 1° Fahr., and t the number of such degrees from the normal temperature which any member, of length, s, has departed, the change in length of this member will be $\omega t s$. If this member is under stress, P, a change in its length in the sense opposite to the stress will produce the work $P \omega t s$. The least-work expression for the effect of a uniform change of temperature will then be

$$\pm \int \omega t \frac{\delta P}{\delta H} d s,$$

which must be added to the summation of Table 1. Applying this expression to the several parts of the system, we get, by referring to Table 1:

Due to cables in main span: $m \omega t \Sigma_o^{lm} a \sec.^2 \alpha$;

" " side span: $2 m_1 \omega t \Sigma_o^{ls} a \sec.^2 \alpha$;

" stiffening chord in main span: $n \omega t \Sigma_o^{lm} a \sec.^2 \beta$;

" " " side " $2 n_1 \omega t \Sigma_o^{ls} a \sec.^2 \beta$.

It will be noticed that none of the above expressions is a function of H, and, therefore, they will appear in the numerator of the H equation, not disturbing the value of the denominator, D. The expression resulting for the effect of a change of t degrees in the temperature of the structure is:

$$H_t = \mp \frac{E \omega t}{D} [m \omega t \Sigma_o^{lm} a \sec.^2 \alpha + 2 m_1 \omega t \Sigma_o^{ls} a \sec.^2 \alpha + n \omega t \Sigma_o^{lm} a \sec.^2 \beta + 2 n_1 \omega t \Sigma_o^{ls} a \sec.^2 \beta] \ldots\ldots (24)$$

Moments and Stresses in the Side-Span Trusses.—It will be found, generally, that the side spans in three-span suspension bridges are too rigid to get the greatest bending moments, under the action of a partial moving load on them. The greatest positive moment, by which we will denote the moment giving a downward deflection, will be caused by the full moving load on the side span and no load on the remainder of the bridge. Going back to the general expression for moments, Equation 12,

$$M_x = \mathfrak{m} - m_1 H y_s - n_1 H z_s.$$

The value of H for a full load on one side span has been deduced in Equation 18, and the moment of a simple beam for a uniform load over the full length of the span,

$$\mathfrak{m} = \frac{p x (l_s - x)}{2}.$$

The expression for the moment thus becomes:

$$\text{Maximum positive } M_x = \frac{p\,x\,(l_s - x)}{2} - m_1 H y_s - n_1 H z_s \ldots\ldots (25)$$

The greatest negative moments in the side span will be produced by the full load on the main span and far side span and no load on the span considered. It is given by

$$\text{Maximum negative } M_x = -m_1 H y_s - n_1 H z_s \ldots\ldots\ldots (26)$$

H represents here the value of the horizontal component due to full load on the main span and one side span. Equations 17 and 18.

The moments produced by a uniform change in temperature of t degrees either way follows at once from the general expression for moments:

$$\text{Temperature moment } \pm M = \mp m_1 H_t y_s \mp n_1 H_t z_s \ldots\ldots (27)$$

but it will be remembered that in Equation 24 H has a negative sign, so that the temperature moment for an increase in temperature will be:

$$\pm M = \pm m_1 H_t y_s \pm n_1 H_t z_s \ldots\ldots\ldots\ldots\ldots (28)$$

After the moments have been computed for the several conditions of loading and temperature giving the greatest positive and negative moments, it is very simple to find the chord stresses in the stiffening truss produced by the bending moments, by dividing the latter by the corresponding lever arms. The chord stresses thus obtained must be added to the direct axial stress in the chord, which is given by the corresponding value of H multiplied by the secant of the angle which the chord makes with the horizontal. The sum of both represents the total stress in the chord at the given point. For the upper chord, of course, the stress due to the fixed load is included.

The stresses in the web members of the stiffening trusses due to the several conditions of load and temperature are determined by the method of moments, the same as for any truss with curved chords, except that the horizontal reaction, H, as well as the vertical reaction, must be considered.

Moments and Stresses in the Main-Span Trusses.—For the full uniform load on the main span and no load on the side spans, the bending moment will, similarly to Equation 25 deduced for the side span, be given by:

$$M_{tot.} = \frac{p}{2} x\,(l_m - x) - m H y - n H z \ldots\ldots\ldots\ldots (29)$$

H represents here the value of H for the full load on the main span, as given by Equation 17.

But in most sections, the greatest moments and stresses are produced by partial moving loads. The positions of loading giving the greatest positive and negative bending moments and shears have been determined graphically by the so-called "locus-line" or *Kaempfer-drucklinie.*

If a concentrated load, P, travels along the span of an upright or inverted arch, the lines of action of the reactions produced at its ends will intersect in one point with the line of action of the load.

The curve generated by the continuous succession of this point as the load travels over the span, which is the locus of the points of intersection, may be denoted as the locus-line. The deductions, both analytical and graphical, of the thrust-line and its application to determine the positions of moving load for greatest chord and web stresses have been discussed fully in the works of Mueller-Breslau, Weyrauch and Melan.

The greatest negative moment at any point, due to partial load on the main span, is caused by the load extending from the far tower to the limiting section found for this point. As there is no load from the near tower to the limiting section, the value, $\mathfrak{m}$, of the general equation for moments, represents the simple beam reaction, R_1, on the unloaded side, by the distance of the section from it. The equation for the greatest negative bending moment at x thus becomes:

$$M_{min.} = R_1 x - m H y_m - n H z \ldots\ldots\ldots\ldots (30)$$

For a uniform load, p, per unit of length, extending from the limiting section, distant u from the left tower to the right tower, the moment may be written:

$$M_{min.} = \frac{p (l - u)^2 x}{2 l} - m H y_m - n H z_m \ldots\ldots\ldots (31)$$

The values of H in this equation, of course, must represent the values corresponding to the loading used.

The greatest positive moments will be found by subtracting algebraically the corresponding values obtained from Equations 30 or 31, from those obtained from Equation 29:

$$M_{max.} = M_{tot.} - M_{min.} \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

The greatest negative moments on the main span which will be produced by loads on the side spans will take place with the latter fully loaded. If, as before, H_s (Equation 18), represents the total horizontal pull produced by the full load on one side span, the

PLATE VIII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 999.
MOISSEIFF ON
THREE-SPAN SUSPENSION BRIDGE.

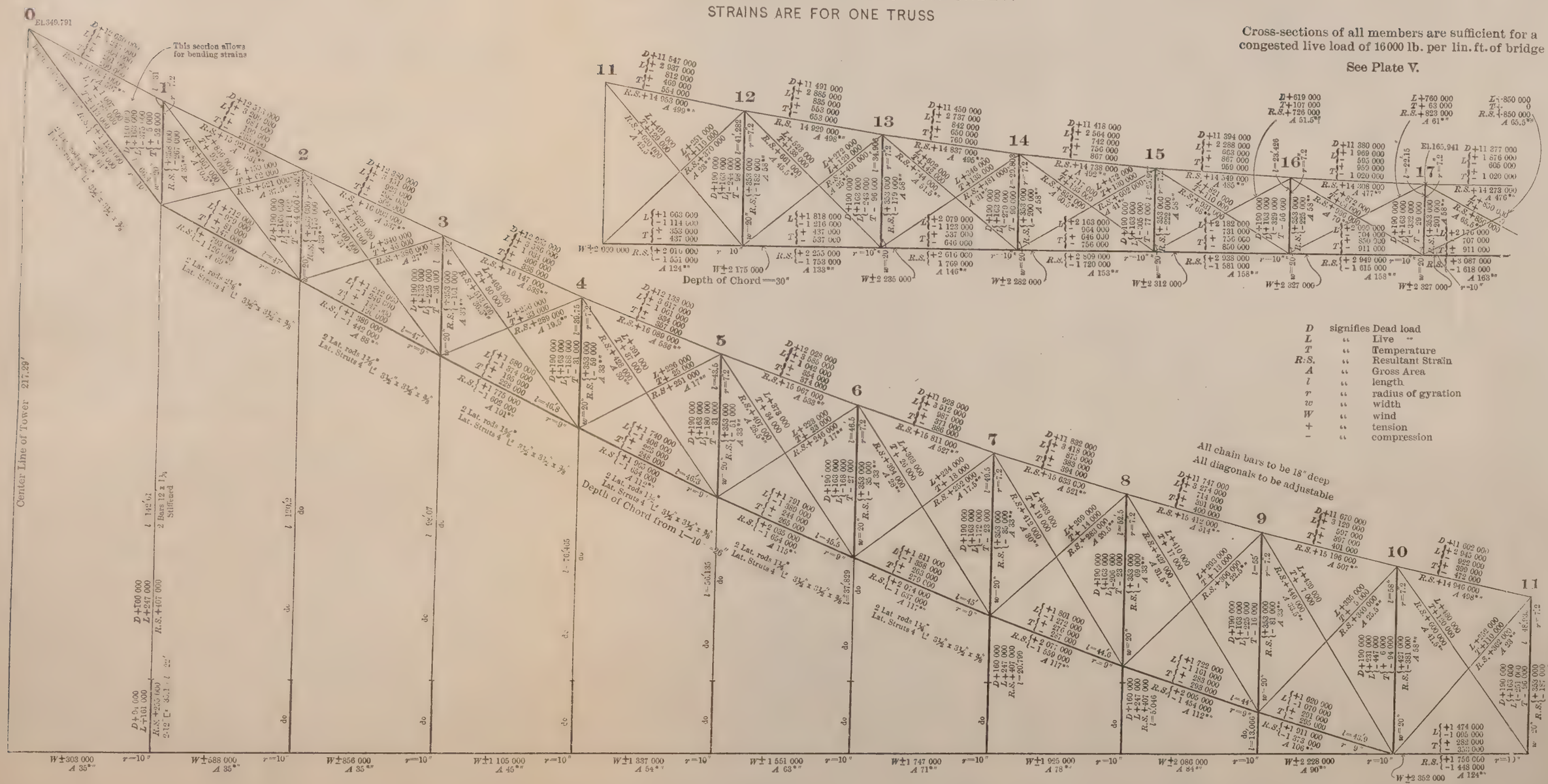
WORKING LOAD STRAINS { 8 000 lb. per lin. ft LIVE (for the 4 trusses.) / 31 000 " " " " DEAD (" " " ") }

MAIN SPAN — STRAIN AND SECTION SHEET.

STRAINS ARE FOR ONE TRUSS

Cross-sections of all members are sufficient for a congested live load of 16000 lb. per lin. ft. of bridge

See Plate V.



greatest negative moment due to both side spans being loaded will be:

$$M_s = -2\,m\,H_s\,y_m - 2\,n\,H_s\,z_m \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

The two negative moments, Equations 31 and 33, must be combined to obtain the greatest possible negative moment.

In combining the chord stresses due to bending with those due to the corresponding axial stress, care must be taken in determining the absolute maxima.

The stresses in the web members were computed by the method of moments, after the position of load giving the greatest stress had been determined by the aid of the locus-line.

The Deflections of the Trusses.—The deflections of the stiffening truss of a suspension bridge should not be considered as to the effects of the moving loads only. The condition of a suspension bridge balanced under the fixed load is that of an equilibrium polygon under stress. As stated in the foregoing, its stability is such that if disturbed by the advent of the moving load it will tend to get back into equilibrium, and this the more so the heavier the bridge. If, under the fixed load, the bridge is balanced with the horizontal pull in the cable, H_f, and the moving load afterward causes an additional pull, H_m, with a resulting deflection, δ_x, the bending moment produced by this condition in the stiffening truss will be:

$$M^1_x = \mathfrak{m} - H\,y - (H_f + H_m)\,\delta_x \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

The first two expressions on the right-hand side of Equation 34 represent the moment as determined by the formulas deduced in the foregoing, while the third expression represents what we may term the "relief moment." The latter is caused by the disturbance in balance of the bridge by the deflection, which is equivalent to an increased versine. The effect of the fixed load on the stiffness of a suspension bridge is shown clearly in Equation 34. The heavier the bridge, the greater, of course, becomes the value of H_f, and the greater is the relief moment. The effect of this moment should be computed and the moments and stresses corrected correspondingly.

The resulting deflection, δ_x, of course, will be the deflection of the stiffening truss due the moment, M_x. Now, the deflection of any beam, by the common theory of flexure, may be written:

$$\frac{d^2\,\delta_x}{d\,x} = \frac{M_x}{E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Integrating the same twice, we get for the deflection

$$\delta_x = \frac{1}{E} \sum \Sigma \frac{M_r a^2}{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

The greatest deflection of the main span, due to moving load, will occur with the main span fully loaded and the remainder of the bridge unloaded. Substituting for M_c^1, in Equation 36, the moment due to uniform load on a simple beam, we get, for the deflection at the center of the span,

$$\delta_{\frac{l}{2}} = \frac{M^1}{f} \times \frac{1}{E} \sum_{0}^{\frac{l}{2}} \sum_{0}^{\frac{l}{2}} \frac{a^2 y}{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

where M^1 is the moment resulting from Equation 34.

Equations 34 and 37 determine the values of both the moment and the corresponding deflection.

It will be noticed that no account was taken of the distortion of the polygon from its original curve. This has been done because, while for a cable not stiffened directly, the distortion of the original curve is of some importance, in the case described, its effect is negligible.

The Application of the Formulas Deduced to the Design of a Three-Span Suspension Bridge.

As stated, the formulas deduced in the foregoing were applied to a three-span suspension bridge of the form shown in Fig. 1. The bridge is a city bridge designed for a traffic capacity unparalleled in the annals of modern bridge building for similar purposes. It is intended to carry four trolley lines, four elevated railroad tracks, a wide roadway and promenades, and a working load of 8 000 lb. per lin. ft. of bridge, or a congested load of 16 000 lb. is provided for to take care of this traffic. A solid floor of buckle-plates is provided, and the total weight of the floor system, including pavement, rails, railings, etc., amounts to 16 300 lb. per lin. ft. of bridge. The cables and stiffening trusses were designed for the use of nickel-steel bars and built members. The total estimated fixed load on the bridge, including the floor, amounts to 31 000 lb. per lin. ft. on the main span and about 33 000 lb. on the side spans. The spans are 725, 1 470 and 725 ft., as shown on Fig. 1. A versine of 184 ft. was decided on for the main span, which, by the application of Equation 5, resulted in a side-span versine of 48.4 ft.

PLATE IX.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 999.
MOISSEIFF ON
THREE-SPAN SUSPENSION BRIDGE.

PROPOSED MANHATTAN BRIDGE

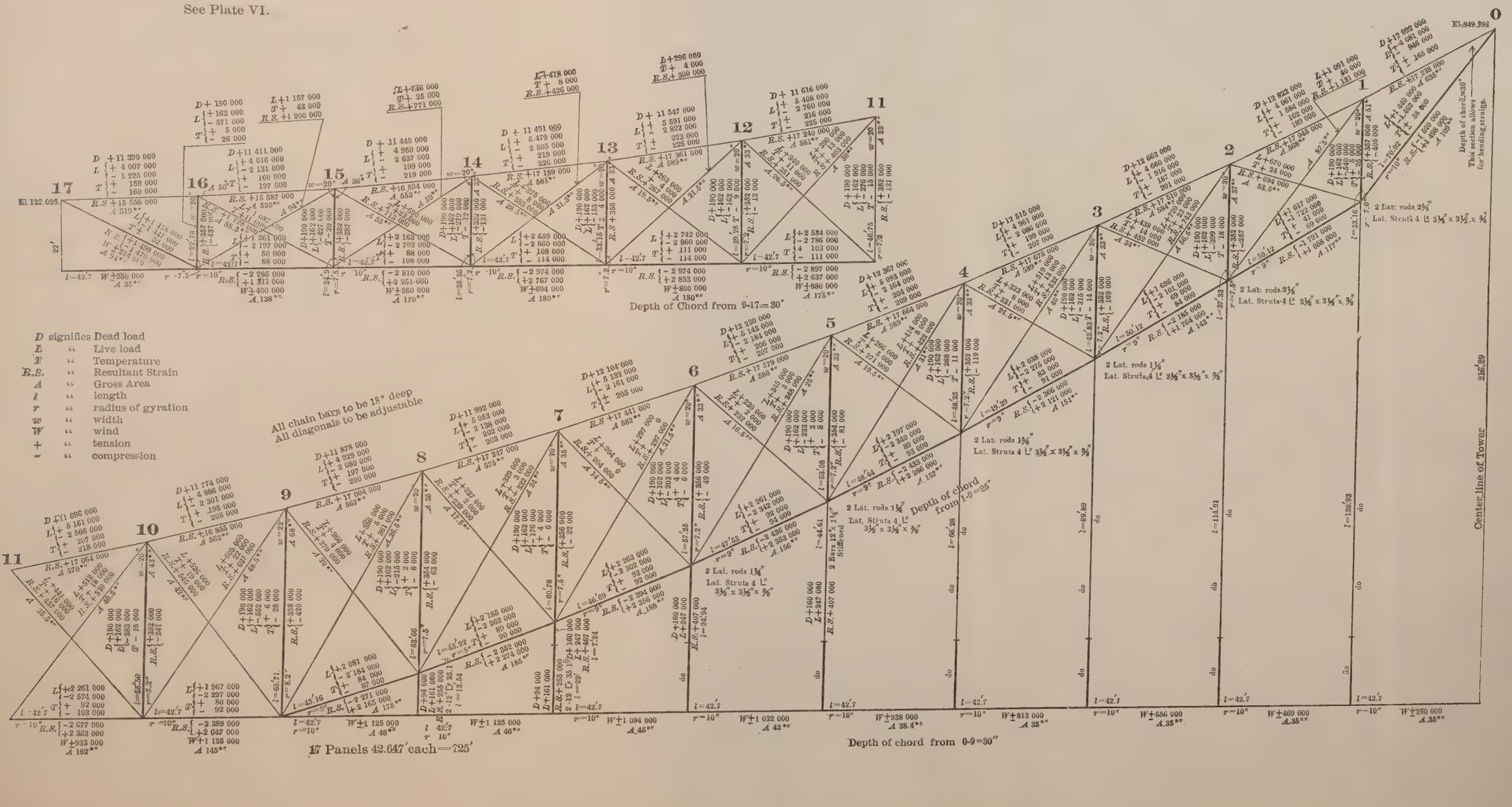
WORKING LOAD STRAINS { 8 000 lb. per lin. ft. LIVE (For the 4 trusses) / 33 000 " " " " DEAD (" " " " ") }

SIDE SPAN STRAIN AND SECTION SHEET.

STRAINS ARE FOR ONE TRUSS

Cross-sections of all members are sufficient for a congested live load of 16000 lb. per lin. ft. of bridge

See Plate VI.


D signifies Dead load
L " Live load
T " Temperature
R.S. " Resultant Strain
A " Gross Area
l " length
r " radius of gyration
w " width
W " wind
+ " tension
− " compression
All chain bars to be 18" deep
All diagonals to be adjustable
Depth of Chord from 9-17=30"
Depth of chord from 0-9=30"
17 Panels 42.647' each=725'
Center line of Tower
El. 132.025
El. 349.291
2 Lat. rods 2⅝"
Lat. Struts 4 L 3½" x 3½" x ⅜"
2 Lat. rods 2⅛"
Lat. Struts 4 L 3½" x 3½" x ⅜"
2 Lat. rods 1⅝"
Lat. Struts 4 L 3½" x 3½" x ⅜"
2 Lat. rods 1¼"
Lat. Struts 4 L 3½" x 3½" x ⅜"
2 Bars 12" x 1¼" Stiffened

The proportions of the total horizontal pull, due to moving load or temperature, taken by the cable and stiffening chords, are:

Main span: $m = 0.82$; $n = 0.18$;

Side spans: $m_1 = 0.89$; $n_1 = 0.11$.

It was found that, with a uniform moving load on the whole of the main span, about 8.5% of the load is carried by the stiffening truss acting as a beam, and the remaining 91.5% by the chains and chords acting as suspended polygons. The greater stiffness of the side spans, due to their short length and great depth at the center, shows its effects in the results obtained for the same kind of loading on them.

The side-span truss carries 98.6% of the load as a truss, and transfers 1.4% only to the cables.

The work on the computations presented no special difficulties. The summations which appear in the formulas deduced in this paper were computed and tabulated for every panel point, and the actual work was less formidable than the long equations would make it appear. The computation of the moments and stresses was simple work, and care only had to be exercised in obtaining the greatest resulting stresses. The analytical method displayed its advantages during the work of computation. The formulas being once established, the actual labor of tabulation can be carried on by several persons at the same time, each taking a part of the work, and, while it is desirable that each computer should be thoroughly acquainted with the formulas and their deductions, as was actually the case, such knowledge is not absolutely necessary. The work can thus be pushed ahead quite rapidly should the time allowed require it.

The stress sheets, Plates VIII and IX, and also Plates V and VI, in the paper* by Gustav Lindenthal, M. Am. Soc. C. E., may serve to show the relative amounts of the stresses and their variations, and also what may be expected under the given conditions.

* *Transactions*, Am. Soc. C. E., Vol. LV, p. 1.

DISCUSSION.

Mr. Church. IRVING P. CHURCH, ASSOC. AM. SOC. C. E. (by letter).—It is not without some diffidence that the writer presents this discussion on Mr. Moisseiff's paper, from the fact that some of the features which seem to him to be at variance with the principles of mechanics may involve errors of small practical importance and thus be fairly negligible; and that he has been unable to find time to ascertain by numerical trial whether or not such is the case.

The first result which the writer is unable to confirm is on page 102, in the paragraph beginning with "If the two chords of the truss, etc.," and ending with Equation 8. Here the statement is made that a load on the bridge would be sustained by the two chords (now supposed suspended without any diagonal stiffening members between. in proportion to their vertical deflection under the stress caused) Mathematical work follows, and, finally, claim is made for the truth of Equation 8, but without demonstration. This paragraph seemed so obscure to the writer that he has attempted to throw light on it, in his own mind, by working out the following simple case:

In Fig. 3, $O\,B_1$ and $B_1\,C$ are two elastic bars, of equal length, l_1, and sectional area, A_1, jointed to each other at B_1; their upper extremities being supported on fixed pins, O and C. $O\,B_2$ and $B_2\,C$ are two other bars, but of greater length, l_2, and of different sectional area, A_2, pivoted to each other at B_2 and to the fixed pins, O and C.

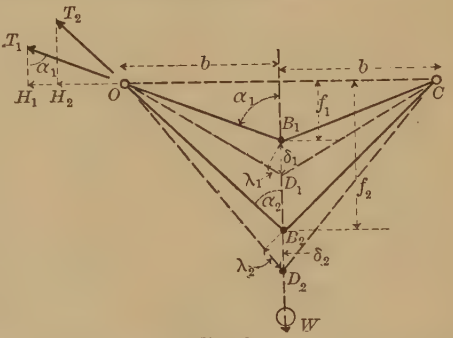


FIG. 3.

These "chords," of two rods each, sustain a weight, W (although one chord would be sufficient), suspended on a single vertical suspension rod (dotted in the figure). The suspension rod is supposed to have been fitted originally to the pins at B_1 and B_2 in such a way that all four bars are in tension when the load has settled into place. Let the vertical deflection, $B_1\,D_1$, of the pin, B_1, be denoted by δ_1 and $B_2\,D_2$, that of the pin, B_2, by δ_2. Let f_1 and f_2 represent the respective "versed sines" of the two chords, and b the common half-span. These are marked in the figure, as also the angles, α_1 and α_2. Let T_1 and T_2 denote the respective total tensile stresses induced in the bars, $O\,B_1$ and $O\,B_2$. The values of T_1 and T_2, and also those of their horizontal components, H_1 and H_2, will depend on the two vertical

deflections, and upon other quantities, in accordance with the follow- Mr. Church.
ing relations (δ_1 and δ_2 are much exaggerated in the figure):

Since the extremity, B_1, has sunk to a new position, D_1, vertically underneath B_1, and since the extremity, O, is considered fixed, the elongation of the bar has a value, λ_1, which may be obtained by projecting $B_1 D_1$, or δ_1, upon the line, $D_1 O$; that is,

$$\lambda_1 = \delta_1 \cos. \alpha_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

But if E is the modulus of elasticity of the material of the four bars (the same for all), we have, from a familiar relation in the "Mechanics of Materials,"

$$\lambda_1 = \frac{T_1 \, l_1}{A_1 E},$$

and hence Equation 38 becomes

$$T_1 = \frac{E A_1 \delta_1 \cos. \alpha_1}{l_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Now $H_1 = T_1 \sin. \alpha_1$; hence, with $\frac{f}{l_1}$ for $\cos. \alpha_1$, and $\frac{b}{l_1}$ for $\sin. \alpha_1$, we have

$$H = \frac{E A_1 \delta_1 f_1 b}{l^3_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Similarly, for the bar, or member, $O B_2$, in its elongated condition under stress,

$$H_2 = \frac{E A_2 \delta_2 f_2 b}{l^3_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

whence, by division, writing L_1 for $2 l_1$, and L_2 for $2 l_2$ (L_1 and L_2 being thus the total lengths of the two "chords," respectively),

$$\frac{H_1}{H_2} = \frac{A_1 L^3_2 f_1 \delta_1}{A_2 L^3_1 f_2 \delta_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

It is also easily seen from Equation 39 that

$$\frac{T_1}{T_2} = \frac{A_1 L^2_2 f_1 \delta_1}{A_2 L^2_1 f_2 \delta_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

If the suspension rod is fitted accurately to the pins, B_1 and B_2, before loading, all five rods being then straight but under no stress, then, after loading, on account of the stretching of the part, $B_1 B_2$, of the suspension rod, δ_2 will be greater than δ_1.

But if the elongation of the part, $B_1 B_2$, of the suspension rod is considered negligible under the circumstances, $\delta_1 = \delta_2$, and Equation 42 reduces to

$$\frac{H_1}{H_2} = \frac{A_1 L^3_2 f_1}{A_2 L^3_1 f_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

If, now, the proportion, $\frac{L_2}{L_1} = \frac{f_2}{f_1}$, were true, we should immediately, from Equation 44, derive the relation stated by the author in his Equation 8, *viz.*,

$$\frac{H_1}{H_2} = \frac{A_1 L_2 f_2}{A_2 L_1 f_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Mr. Church. But such a proportion ($L_2 : L_1 :: f_2 : f_1$) is evidently very far from the truth when the versed sine is small compared with the length of the chord or cable, as is the case in the spans of most suspension bridges. The writer, therefore, is forced to the conclusion that Equation 8 is not even approximately correct (though it is possible that the author had in mind some different relation between δ_1 and δ_2 than that of equality).

If the pins, *O* and *C*, move horizontally to a slight extent during the gradual loading of the structure in Fig. 3, the mathematical outcome, of course, is more complicated, the amount of such movement being partly dependent on the design of the "side-span cables."

If the author had presented the detail of treatment of a simple specific case, like that in Fig. 3, his meaning, doubtless, would have been clearer, and the degree of approximation more evident.

Next take up the method of expressing the stresses in the "cables," or chords, when the stiffening truss is in action (middle of page 103). In Fig. 4, let *O k* represent a portion of the main cable (*i. e.*, the upper chord of the truss), extending from the tower-pin at *O* to any segment or "member," *k*. The forces acting are: The tension, T_o, in the segment next to the tower-pin (its horizontal component can be denoted by H_1, or $m\,H$); the tension in the bar, or segment, *k*, *viz.*, *T*, the value of which is to be expressed; certain vertical forces, P_1, P_2, etc., at the intervening joints; and also certain oblique pulls or thrusts at these joints, coming from diagonal members which are in action, *viz.*, Q_1, Q_2, etc. Summing the horizontal components of the forces for the equilibrium of this portion of the upper chord (no longer a cable under purely vertical loads), we obtain

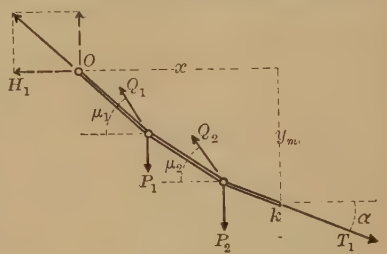


FIG. 4.

$$T \cos.\ \alpha = H_1 + Q_1 \cos.\ \mu_1 + Q_2 \cos.\ \mu_2 \ldots\ldots\ldots\ldots (45)$$

or,

$$T = m\,H \sec.\ \alpha + (Q_1 \cos.\ \mu_1 + Q_2 \cos.\ \mu_2) \sec.\ \alpha \ldots\ldots (46)$$

instead of the result obtained by the author, *viz.*, $T = m\,H$ sec. α; which result, therefore, would seem to be contrary to the laws of mechanics.

A similar treatment of a portion of the main-span stiffening chord gives a similar outcome. To assign to this lower chord the properties of a cable under purely vertical loads (*i. e.*, to claim that the stress in any part is equal to $n\,H$ sec. α) seems quite erroneous; since the stresses existing in the diagonal members are of great importance in the treatment of this lower chord, which is not supposed to be under any stress at all (aside from that due to its own weight),

unless the diagonal members are called into action (by a moving load or change of temperature), in which case we find an oblique Q at each joint. Mr. Church.

As regards forming an expression for the "bending moment" for any cross-section of the stiffening truss, when regarded approximately as a curved beam, or inverted arch rib, it should be remembered that, in the case of a beam or rib, where the forces in a section are equivalent, not to a stress couple and a shear simply (as in a simple beam), but to a couple, a shear, and a thrust, the moment of the couple (*i. e.*, the bending moment) is not equal to the algebraic sum of the moments of the forces acting on the portion of the beam on one side of the cross-section, unless the axis of moments is taken through the center of gravity of the plane figure formed by the sections of the two chord members of the truss (or very nearly so).

Thus, in Fig. 5, we have a representation, as a "free body," of a portion of the stiffening truss situated between the tower pin, O (the pressure on which is replaced by its two components, horizontal component, H, and vertical component, R), and a cross-section made near the middle of any panel. In the three members of the truss thus cut (*viz.*, k, r, and v), we find acting the stresses, T_1 (in upper chord), Q (in the diagonal member), and T_2 (in the lower chord). The point, n, of the cross-section is taken at the center of gravity (in side view) of the plane figure formed by the sections of the two chord members.

The sum of the moments of the forces on the left, taken about n, will be the value of M, or "bending moment," to be used for the purpose for which M would be used when a treatment involving the analogy of a curved beam is adopted. On this basis, we derive (see Fig. 5 for symbols)

$$M = R\,x - P_1\,a - H\,w \ldots\ldots\ldots (47)$$

The writer is unable to see any warrant for replacing the term, $H\,w$, by $m\,H\,y_m + n\,H\,z_m$, as the author has done in Equation 11; even supposing that correct values of m and n are available. It is indeed true that the product, $H_1\,y_m$ (*i. e.*, $m\,H\,y_m$) would occur in forming moments for the forces in Fig. 4 about the point, k, of the cable or upper chord. But the body shown in that figure, being a portion of the upper chord, is not a stiff beam, and the bending moment in the section, k, is zero.

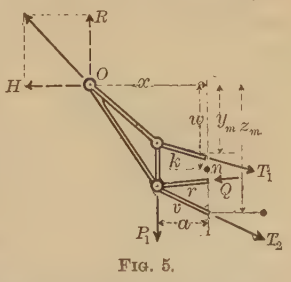


Fig. 5.

The writer finds it difficult to recognize any necessity for the invention and use of the artificial quantities, m and n of this paper. A straightforward application of the Principle of Least Work to the entire assemblage of bars or members would seem to answer every purpose; since the stress in each member is capable of being expressed in terms of known quantities and the one unknown, H.

Mr. Church. There seems to be in the paper no specific declaration of the important feature that the tower-pins have free horizontal movement; though it is implied, of course, in the evident assumption that the "*H*" is the same for all three spans.

Mr. Swain. GEORGE F. SWAIN, M. AM. SOC. C. E. (by letter).—The three papers on Suspension Bridges, by Messrs. Cilley, Lindenthal, and Moisseiff, are interesting and timely. Mr. Cilley's paper* is a correct and valuable discussion, and gives a correct and, in its fundamental principle, simple method of computing the structure of the Manhattan Bridge. Mr. Lindenthal's paper is valuable, and the form of stiffened suspension structure which he has adopted appears to be economical and well proportioned in its general dimensions, besides being agreeable to the eye.

Mr. Moisseiff's paper, however, suggests some criticism. The general equation (Equation 1) applies to solid beams. Strictly speaking, it only applies to such beams if they are straight, for if plane cross-sections are assumed to remain plane, the formula for the fiber stress in curved solid beams involves more than two terms. It is unnecessary to say that the formula supposes the stress on a cross-section to be at right angles to the cross-section. There are two ways in which the formula may be applied to the suspension structure in question. The first is by applying it to the structure as a whole, P being the direct force at right angles to each section, and M the bending moment on each section, the sections being taken at right angles to the neutral axis. This application cannot be made with accuracy in this case, for this is a framed structure of varied section, with the neutral axis discontinuous (as the sections of chords change suddenly at the joints), with a curved axis, and with the chords along which the stresses act (or one, at least) not at right angles to the section.

The second way of applying Equation 1 to this case would be by applying it to each piece, P being the direct stress in the piece, and M the bending moment, the latter being zero in the case of a framed structure with the pieces subject only to direct stress.

The second of these methods would be correct, and would give the results obtained by Mr. Cilley. Mr. Moisseiff, however, does not apply this method, but apparently uses Equation 1 according to the first method, though, as it would seem, incorrectly.

He first uses the empirical formula (Equation 8) for the ratio of the horizontal components in cable and stiffening chord, assuming that, no matter what the loading is, the horizontal components in the cable and in the stiffening chords are constant throughout the span. Equation 8 says that, no matter what the loading, there is always the same ratio between these horizontal components. These

* *Transactions*, Am. Soc. C. E., Vol. LIII, p. 413.

assumptions cannot be true. Equation 8 seems to be wholly empirical, and its use can hardly be justified until its correctness is demonstrated. Will not the author give some better demonstration than the bare statement that "it will hold true"? Until such proof is given, no method based upon this equation should be used, especially since—as already stated—simple and accurate methods are available. Mr. Swain.

Assuming this equation, however, if, for any given loading, H is the true horizontal reaction, then $m\,H$ and $n\,H$ are the true horizontal components in the cable and stiffening chord, and, neglecting

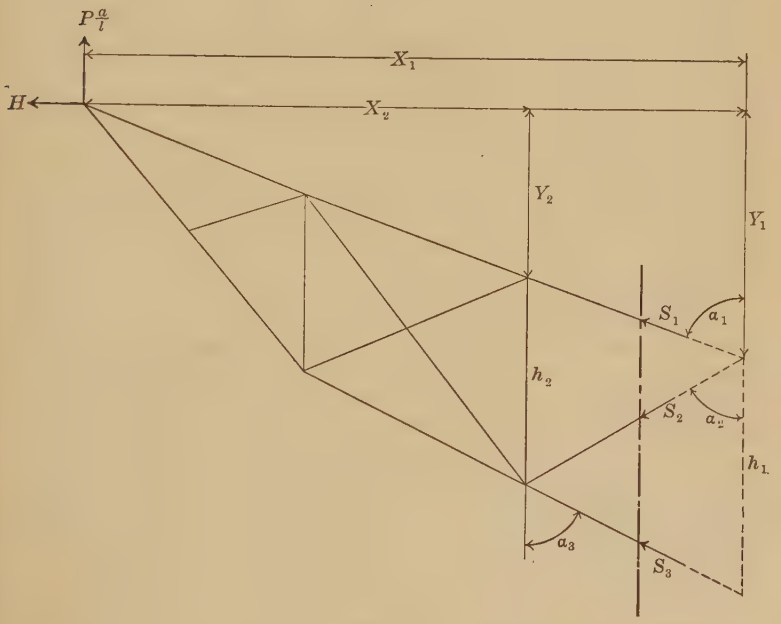


FIG. 6.

the web, there would seem to be no reason for taking further account of the moment. This, however, the author proceeds to do, in Equations 11 and 12, and in the table of least work. It is easily observed that if $m\,H$ and $n\,H$ are the real horizontal components in the cable and chord, the moment, , as given in Equation 11, is zero, by considering it as a moment taken about a point in the horizontal through the points of support, web stresses being neglected.

On the other hand, taking the first method of applying the general equation, if A_1 and A_2 be the areas of top and bottom chords,

Mr. Swain. respectively, the ordinate to the center of gravity of a vertical section will be $Y_{c.\ g.} = \frac{A_1 y}{A_1 + A_2} + \frac{A_2 z}{A_1 + A_2}$, or approximately $Y_{c.\ g.} = m\ y + n\ z$, since m and n may be considered according to Equation 8, to be, respectively, $\frac{A_1}{A_1 + A_2}$ and $\frac{A_2}{A_1 + A_2}$, though not exactly so. In this case, the moments in Equations 11 and 12 may be taken as approximately the moments about the centers of gravity of the sections; but it will follow that $m\ H$ and $n\ H$ are not the true stresses in the chords. In this case the term with the moments should be used in connection with the other term with P, in which P is the total direct force on the entire cross-section, as already stated, but which Mr. Moisseiff does not compute.

In other words, if the summation of the terms for direct stress on chords, $m\ H$ sec. α and $n\ H$ sec. β are used as in the table, the moments should be omitted, and, if the summation of terms with moments is used, then the terms for the direct stress should be modified and made to agree correctly with Equation 1, in which P is the total direct force on the section; but we have already stated the reasons for believing this application of Equation 1 or the first method, to be at best only rough.

In other words, the general equation (Equation 1) is neither applied to the structure as a whole, in which case P would be the direct normal force on any section, and M, by Equations 11 and 12, would be approximately the moment about the center of gravity (and which, as already stated, would be not a strictly correct application); nor to each piece, in which case P and M would be the direct force and bending moment on each piece separately, and in which case, if $m\ H$ and $n\ H$ were the true horizontal components in cable and stiffening chord, no other quantities would enter. Moreover, the effect of web is entirely neglected and the empirical formula, Equation 8, is assumed.

Furthermore, it may be shown that if Equation 8 is assumed to give the correct ratio of the horizontal component in the chords, this equation alone will suffice for a determination of the structure, without any recourse to the method of least work. The structure is, in fact (as Mr. Cilley has shown*), a singly indeterminate structure, *i. e.*, only one additional condition is needed, besides the statical equations. This condition is given by Equation 8. It may also be shown in this manner that Equation 8 must be incorrect. For let a vertical section be taken at any point in the main span, and let there be a single load, P, at a distance, a, from the right pier, then we shall have four unknown quantities to find, *viz.*, S_1, S_2, S_3, and H, and we shall have four equations, *viz.*:

* *Transactions*, Am. Soc. C. E., Vol. LIII, footnote on p. 415.

$$\Sigma V = 0;\ \Sigma H = 0;\ \Sigma M = 0;\ \text{and}\ \frac{S_1 \sin.\ \alpha_1}{S_3 \sin.\ \alpha_3} = \frac{m}{n} = c.$$ Mr. Swain.

By writing out these equations and solving, we shall find the value of H to be

$$H = \frac{Pa}{l}\left[\frac{h_1 x_2 + ch_2 x_1}{ch_2 y_1 + h_1 y_2 + h_1 h_2}\right]$$

Now if we take a section in any panel, we ought to obtain the same value of H_1, if the method is correct; but it is clear that the quantity in parentheses will not be constant when expressed in each panel, except for some one particular shape of structure.

Considering these facts, the writer cannot but conclude that the author's method is incorrect. It is to be regretted that simpler, clearer and more direct methods were not used. It would be interesting to know what degree of correspondence was obtained between the author's computations and those described ir Mr. Cilley's paper, the latter being unquestionably correct in principle.

It is to be observed, further, that even an approximate agreement, between Mr. Moisseiff's method and Mr. Cilley's practically correct methods, in any one case, would not prove the former to be correct or approximately correct, inasmuch as it appears to involve errors in principle. It probably involves a number of errors, some of which may partially counteract each other. It does not seem to be worth while to endeavor to discuss these errors quantitatively.

Leon S. Moisseiff, Assoc. M. Am. Soc. C. E. (by letter).—The criticism directed against the writer's paper relates, first, to the correctness of the approximate equation indicated in the paper by Equation 8, and, secondly, to the procedure in applying the "least work" principle. Mr. Moisseiff.

In his paper the writer stated that:

"For purposes of computation, it is important to determine approximately the relative share of the load sustained by each of them. The following formula (Equation 8) has been found to give closely approximate results."

It did not appear to the writer to be necessary to give any deduction of this equation, since it is given as approximate only and does not form an essential part of the method described. More accurate and more complicated equations could be deduced without, in the opinion of the writer, producing results much nearer the truth.

The neat example given by Professor Church, representing the case of distribution of stress, as illustrated in Fig. 3, does not well apply to a long-span suspension bridge. An example much nearer to the actual conditions of the problem is presented by treating a crescent formed of two parabolas, one the parabola of the chain and the other having the same span and the versine of the chord.

Mr. Moisseiff. If we denote the span of a parabola by l and its versine by f, the length of the parabola is, as is well known, approximately

$$L = l + \frac{8}{3}\frac{f^2}{l}.$$

A small change in the length of the curve will be given by

$$\Delta L = \frac{16}{3}\frac{f}{l}\,\Delta f.$$

A load suspended from the parabola, no matter how distributed, will produce an elongation in the elastic material of the curve, which, as long as the curve, by assumption, maintains its shape, may be expressed by ΔL. If the horizontal component of the stress produced be H, the stress at any point will be $H\frac{ds}{dx}$, and the total elongation of the curve will be

$$\Delta L = \int_o^l \frac{H}{E\,A}\frac{ds}{dx}\,ds = \frac{H}{E\,A}\int_o^l \frac{ds^2}{dx}.$$

where A denotes the sectional area of the member, E the coefficient of elasticity of its material, and ds a length element of the curve.

Since $ds^2 = dx^2 + dy^2$,

$$\int \frac{ds^2}{dx} = l + \int_o^l \left(\frac{dy}{dx}\right)^2 dx$$

The equation of the parabola, as given in Equation 16, is

$$y = \frac{4f}{l^2} \times (l - x),$$

and

$$\frac{dy}{dx} = \frac{4f}{l^2}(l - 2x).$$

Hence

$$\int_o^l \left(\frac{dy}{dx}\right)^2 dx = \frac{16f^2}{3l},$$

and, substituting, the total elongation, due to H, is:

$$\Delta L = \frac{H}{E\,A}\left(l + \frac{16}{3}\frac{f^2}{l}\right).$$

From the relation given in the above, $\Delta f = \frac{3}{16}\frac{l}{f}\Delta L$, and substituting for ΔL the right hand expression in the last equation, we get:

$$\Delta f = \frac{H}{E\,A}\left(\frac{3}{16}\frac{l^2}{f} + f\right).$$

In the case of two curves forming a crescent, if we assume the de-

flections to be the same, $\Delta f_1 = \Delta f_2$, and denote the corresponding values by the suffixes, 1 and 2, we get: Mr. Moisseiff.

$$\frac{H_1}{H_2} = \frac{\frac{3\,l^2}{16\,f_2} + f_2}{\frac{3\,l^2}{16\,f_1} + f_1} \cdot \frac{A_1}{A_2}.$$

The numerical solution of this proportion gives $m = 0.80$ and $n = 0.20$, while that of Equation 8 gives, as stated, $m = 0.82$ and $n = 0.18$. The effect of this variation, on the resulting stresses, is negligible, and the actual value is apparently nearer the latter than the former. It should be remembered that H represents here the horizontal component of the thrust only, and not of the whole stress in the member.

As to the procedure in the application of the least work principle and the determination of the resulting stresses, the criticism is well directed, as far as the mathematical rigor of the method is concerned, but the effect of the latter on the practical results is very small, as will be shown later.

The method adopted was to consider the live-load and temperature stresses in the trusses as the result of the effect of two causes, one a bending moment or couple and the other a force acting at any point, x, with the angle, φ; the effect of the former being much the more important. It is true, as Professor Swain has pointed out in his discussion, that the neutral axis of the truss could not be located exactly, but the effect of the moment of inertia on the stresses is not in direct proportion, and, whatever slight inaccuracies are due to this, their effect is reduced still more in the results. The bending moments were thus derived, as shown in the paper, in a perfectly correct way as to "principle." The effect of the force, acting at an angle, φ, on the thrust in the chain and chord is very small, the effect of elongation for the full load was 2% of the total, H. The lack of rigor in the method pursued is in the determination of this effect: the work of the thrust having been assumed as equivalent to the stress due to its horizontal component as distributed between chain and chord. Evidently, the variation must be very small for continuous moving loads such as have to be provided for in large suspension bridges. The total effect of the thrust on the work of deformation being only 2%, a variation even of 20% of this effect will be insignificant, from a practical point of view.

Fortunately, the carefully worked check calculations described in Mr. Cilley's paper* afford an excellent opportunity to evaluate the relative degree of accuracy obtained by the two methods.

With this in view, the writer undertook to determine finally the variation between the graphic method, as applied in this case for the

* *Transactions*, Am. Soc. C. E., Vol. LIII, p. 413.

Mr. Moisseiff.

determination of the stresses, and the analytical computations, and performed the laborious task of bringing the results of the analytical computations to a common basis with those of the graphical method so as to be able to compare the numerical results and their variations. For this purpose, all the assumptions had to be the same, and it has probably been noticed already, by the reader who has followed both the analytical and graphical methods, that a diagonal system was assumed in the graphical check computations, which consists of single diagonals sloping toward the center of span and designed to resist both tension and compression. But the actual design provides for adjustable and intersecting diagonals designed to act as tension members only, the same as in ordinary railway bridges. This is of small importance, as far as the effect of the web deformations on the stresses is concerned; and being very small, it has, in fact, been neglected altogether in the analytical computations.

But, in determining the stress in a member of the chain or chord, it makes a very considerable difference whether one or the other of the two diagonals of a panel is acting, because the point around which the moment is to be taken and the corresponding lever arm are both different. In other words, this means that, barring other differences, the bending moments at each point, if known, would be comparable in their numerical values as to the agreement between the two methods. But while the bending moments at each point are given by the analytical method, the graphical procedure gives the stresses directly. The moments obtained by the analytical method, therefore, were used to obtain the stresses for a truss with a diagonal system, as used in the graphical computation. This, together with some minor corrections, furnished the results tabulated in Tables 1 to 4 in the columns headed "Analytic." The results of the graphical computation are placed in the adjacent columns, as well as the variation of both methods. The numbers tabulated are in thousands of pounds. The stresses given in Tables 1 to 4 do not, it is repeated, represent the actual stresses in the design of the bridge (these are given in the stress sheets, Plates V,* VI,* VIII and IX), but the stresses of the same truss with a diagonal system corresponding to the one used for the graphical computations.

Since the stresses in the web members were computed, as stated in the paper, in the ordinary way, by the method of moments, and because of the additional labor involved, no comparison of these stresses was made.

It will be seen from Tables 1 to 4 that the variations between the two methods affect the stresses in the chains of the side spans 0.9% at the highest and in those of the main spans 0.5%, while the

* Plates V and VI may be found in the paper by Gustav Lindenthal, M. Am. Soc. C. E., entitled "A Rational Form of Stiffened Suspension Bridge," *Transactions*, Am. Soc. C. E., Vol. LV, p. 1.

Mr. Moisseiff.

average is still lower. The greatest variation in the stresses of the side span chords is 2.8%, with an average variation of 2.5 per cent. The greatest variations in the main span chords are 3.7 and 4.0%, but these are doubtful, because they form a marked irregularity in the run of the differences. As it is, the average variation is less than 1.8 per cent. The specifications for this bridge allowed "a variation in cross-section or weight, of any piece of steel, of 2½% from that specified."

TABLE 1.—Comparative Stresses in Chains of Side Spans. Working Load.

Member.	Live Load.		Variation in Tension.	Variation in Compression.	Dead and Live Loads.	
	Analytic.	Graphic.			Analytic.	Graphic.
0 to 1	+4 163 — 937	+4 054 — 930	—2.7%	—0.7%	+17 155	+17 046
1 " 2	+4 750 —1 576	+4 652 —1 564	—2.2%	—0.8%	+17 572	+17 474
2 " 3	+5 080 —1 902	+4 984 —1 896	—1.9%	—0.2%	+17 743	+17 647
3 " 4	+5 215 —2 078	+5 080 —2 066	—2.6%	—0.6%	+17 730	+17 595
4 " 5	+5 260 —2 156	+5 124 —2 144	—2.6%	—0.5%	+17 627	+17 491
5 " 6	+5 255 —2 176	+5 112 —2 164	—2.7%	—0.5%	+17 485	+17 342
6 " 7	+5 196 —2 153	+5 054 —2 138	—2.7%	—0.7%	+17 300	+17 158
7 " 8	+5 104 —2 090	+4 968 —2 078	—2.7%	—0.6%	+17 096	+16 960
8 " 9	+4 990 —2 005	+4 854 —1 992	—2.7%	—0.4%	+16 868	+16 732
9 " 10	+4 950 —1 986	+4 816 —1 978	—2.7%	—0.4%	+16 724	+16 590
10 " 11	+5 225 —2 278	+5 082 —2 262	—2.7%	—0.4%	+16 921	+16 778
11 " 12	+5 476 —2 538	+5 330 —2 528	—2.7%	—0.4%	+17 092	+16 946
12 " 13	+5 663 —2 687	+5 508 —2 702	—2.7%	+0.5%	+17 210	+17 105
13 " 14	+5 708 —2 790	+5 558 —2 784	—2.6%	—0.2%	+17 199	+17 049
14 " 15	+5 530 —2 628	+5 378 —2 616	—2.8%	—0.5%	+16 975	+16 823
15 " 16	+4 960 —2 124	+4 864 —2 112	—1.9%	—0.6%	+16 371	+16 275
16 " 17	+4 013 —1 218	+3 950 —1 210	—1.6%	—0.7%	+15 403	+15 340

The greatest variation between the analytic and graphic methods is nine-tenths of 1 per cent.

Attention is called to the uniformity in the variations, indicating the close parallelism of the two methods. The degree of accuracy is certainly close enough for an engineering structure, and the writer feels justified in the application of his method. Any approximations it may contain are, as demonstrated, of the nature of minor secondary stresses.

Mr. Moisseiff.

TABLE 2.—Comparative Stresses in Chords of Side Spans. Working Load.

Member.	Live Load.		Variation.	Live Load.		Variation.
	Analytic.	Graphic.		Analytic.	Graphic.	
			Per Cent.			Per Cent
0 to 1	+ 1 440	+ 1 434	— 0.7	— 1 481	— 1 442	— 2.6
1 " 2	+ 1 017	+ 1 014	— 0.3	— 1 047	— 1 020	— 2.6
2 " 3	+ 1 695	+ 1 688	— 0.4	— 1 715	— 1 696	— 1.4
3 " 4	+ 2 033	+ 2 028	— 0.2	— 2 095	— 2 038	— 2.7
4 " 5	+ 2 197	+ 2 188	— 0.4	— 2 260	— 2 200	— 2.6
5 " 6	+ 2 261	+ 2 250	— 0.5	— 2 323	— 2 262	— 2.6
6 " 7	+ 2 263	+ 2 254	— 0.4	— 2 330	— 2 266	— 2.7
7 " 8	+ 2 255	+ 2 216	— 1.6	— 2 290	— 2 228	— 2.7
8 " 9	+ 2 144	+ 2 142	— 0.1	— 2 210	— 2 154	— 2.5
9 " 10	+ 2 257	+ 2 254	— 0.1	— 2 323	— 2 266	— 2.5
10 " 11	+ 2 529	+ 2 524	— 0.2	— 2 607	— 2 536	— 2.7
11 " 12	+ 2 692	+ 2 732	+ 1.5	— 2 820	— 2 746	— 2.6
12 " 13	+ 2 809	+ 2 820	+ 0.4	— 2 898	— 2 806	— 3.2
13 " 14	+ 2 659	+ 2 648	+ 0.4	— 2 733	— 2 662	— 2.6
14 " 15	+ 2 163	+ 2 163	0	— 2 185	— 2 154	— 1.4
15 " 16	+ 1 261	+ 1 256	— 0.3	— 1 293	— 1 262	— 2.4
16 " 17	+ 1 396	+ 1 394	— 0.2	— 1 440	— 1 402	— 2.6

TABLE 3.—Comparative Stresses in Chains of Main Span. Working Load.

Member.	Live Load.		Variation.	Dead and Live Loads.	
	Analytic.	Graphic.		Analytic.	Graphic.
			Per Cent.		
0 to 1	+ 3 237	+ 3 194	— 1.3	+ 15 887	+ 15 844
1 " 2	+ 3 479	+ 3 428	— 1.5	+ 15 992	+ 15 941
2 " 3	+ 3 614	+ 3 560	— 1.5	+ 15 994	+ 15 940
3 " 4	+ 3 638	+ 3 592	— 1.3	+ 15 893	+ 15 847
4 " 5	+ 3 612	+ 3 560	— 1.4	+ 15 750	+ 15 698
5 " 6	+ 3 538	+ 3 594	— 1.6	+ 15 566	+ 15 622
6 " 7	+ 3 454	+ 3 390	— 1.9	+ 15 382	+ 15 318
7 " 8	+ 3 296	+ 3 250	— 1.4	+ 15 128	+ 15 082
8 " 9	+ 3 151	+ 3 090	— 1.9	+ 14 898	+ 14 837
9 " 10	+ 2 963	+ 2 912	— 1.7	+ 14 633	+ 14 582
10 " 11	+ 2 957	+ 2 894	— 2.1	+ 14 559	+ 14 496
11 " 12	+ 2 912	+ 2 850	— 2.1	+ 14 459	+ 14 397
12 " 13	+ 2 764	+ 2 760	— 0.2	+ 14 251	+ 14 255
13 " 14	+ 2 596	+ 2 580	— 0.6	+ 14 046	+ 14 030
14 " 15	+ 2 324	+ 2 312	— 0.5	+ 13 742	+ 13 730
15 " 16	+ 2 016	+ 2 040	+ 1.2	+ 13 410	+ 13 434
16 " 17	+ 1 875	+ 1 800	— 4.0	+ 13 255	+ 13 180
17 " 18	+ 1 876	+ 1 800	— 4.0	+ 13 253	+ 13 177

The greatest variation between the analytic and graphic methods is about one-half of 1 per cent.

TABLE 4.—Comparative Stresses in Chords of Main Span. Working Load. Mr. Moisseiff.

Member.	Live Load.		Variation.	Live Load.		Variation.
	Analytic.	Graphic.		Analytic.	Graphic.	
			Per Cent.			Per Ce nt.
0 to 1	+1 011	+1 016	+0.5	— 785	— 802	+2.2
1 " 2	+ 716	+ 718	+0.3	— 556	— 568	+2.2
2 " 3	+1 249	+1 260	+0.9	—1 002	— 984	—1.8
3 " 4	+1 590	+1 600	+0.6	—1 257	—1 236	—1.7
4 " 5	+1 752	+1 776	+1.2	—1 331	—1 354	—1.7
5 " 6	+1 809	+1 832	+1.3	—1 385	—1 374	—0.8
6 " 7	+1 825	+1 866	+2.2	—1 402	—1 372	—2.1
7 " 8	+1 818	+1 854	+2.0	—1 356	—1 338	—1.3
8 " 9	+1 740	+1 784	+2.4	—1 267	—1 254	—1.0
9 " 10	+1 683	+1 718	+2.1	—1 200	—1 172	—2.3
10 " 11	+1 521	+1 544	+1.5	—1 040	—1 014	—2.5
11 " 12	+1 729	+1 123	+1.8	—1 123	—1 096	—2.5
12 " 13	+1 965	+2 016	+2.6	—1 170	—1 194	+2.0
13 " 14	+2 275	+2 230	—2.0	—1 222	—1 222	0
14 " 15	+2 323	+2 410	+3.7?	—1 185	—1 198	+1.1
15 " 16	+2 408	+2 514	+4.0?	—1 110	—1 094	—0.5
16 " 17	+2 479	+2 542	+2.5	— 931	— 946	+1.6
17 " 18	+2 515	+2 510	—0.2	— 800	— 798	—0.2

It has been the writer's aim to treat the subject from an engineering point of view. As an actual engineering problem, the design and computation of the bridge came before the writer, and as such it had to be solved. The methods adopted and followed out were developed on lines mainly determined by the engineering requirements of the case. It has not been the object to furnish the most elegant solution of the stresses in a hypothetical framework, but to prepare the design of a bridge intended to be built. The procedure adopted, and described in the paper, proved to be convenient of application and ready of execution in the office. The effects of the several parts of the structure on the stresses can be easily separated, and the formulas are readily adapted to convenient approximations. Preliminary estimates, therefore, can be made in short time, with sufficient accuracy, which, from the constructor's point of view, is of great importance. In fact, the first set of stresses, based on assumed areas and moments of inertia, will, generally, in the case of lack of time, be found to be all that is required for obtaining satisfactory bids for the steel superstructure on the unit-price basis.

As to the accuracy of the results which may be expected from the method of computation used, the following observations should be remembered: In the deduction of the equations a series of assumptions have been made. A constant and identical coefficient of elasticity has been assumed for all members of the truss, or, what amounts to the same thing, what has been considered a true average

Mr. Moisseiff. value of the former has been taken. The effect of the stress in the web members has been neglected. The truss has been assumed to maintain its geometrical figure undistorted, and, finally, the principle of linear stress has been taken for granted, which says that if a number of various forces act on a truss the sum of the stresses caused by each individual force will represent the total stress caused by all the forces.

In the computation, proper, the distribution of the horizontal component of the pull at the tower pin between the cable and the stiffening chord has been introduced, and an approximate formula given for its determination. All these assumptions will be found to be well within the limits of accuracy sought in engineering structures of this character, with the exception of the principle of linear stress. The latter certainly does not apply correctly to heavy long-span suspension bridges, and, as stated in the paper, corrections of the stresses should be made, allowing for the deformations of the trusses. The truth of this assertion becomes apparent if it is remembered that a suspension bridge with a heavy dead load will certainly deflect less under a given live load than the same bridge with a light dead load.

In working on the computations the tabular method was used, and proved very convenient; and the formidable looking equations took very simple aspects in the actual work, and very little drafting work was required.

It is the writer's belief that whenever a similar case presents itself to an engineer he will find the procedure, as laid out, to be convenient of application and furnishing an easy survey of the case.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1000.

A NEW SWING BRIDGE AT COPENHAGEN, DENMARK.*

By H. C. V. Moeller, Esq.†

With Discussion by
Messrs. John C. Moses, L. J. Le Conte and H. T. Forchhammer.

At the end of the past century it was resolved to renew an old wooden draw bridge crossing the southern part of Copenhagen Harbor. The cost of the new bridge was divided between the municipality and the harbor authorities in the proportion of 2 to 1. In the summer of 1901 the piers of the new bridge were completed so far as to allow the erection of the superstructure. All work on the bridge was finished in the spring of 1903, just in time to replace the old structure, which had been seriously crippled in a collision with a steamer.

The bridge is a combined highway and double-track railway bridge, consisting of one center-bearing draw span and six deck plate-girder spans, with one roadway and two sidewalks. The roadway is paved for heavy street traffic, and is provided with two sets of rails, one set of double-track rails for street-railway service and

* Presented at the meeting of April 5th, 1905.
† Chief Engineer of the Port of Copenhagen, Denmark.

another set adjacent to them for steam-railway traffic. The steam-railway tracks approach the bridge on a curve with a radius of 300 ft. Each of the two openings of the draw span have a clear width of about 72 ft.

General Dimensions of the Bridge.—The whole length from shore to shore in this part of the harbor is 514.6 ft. (156.9 m.), of which the length of the movable part is 220.3 ft. (67.17 m.). The roadway is 27.2 ft. (8.29 m.) wide, except at the ends, where it is widened on account of the railway tracks forming a Y on the bridge so as to connect with the tracks in the adjoining quay streets. On both sides of the roadway there are sidewalks 10.3 ft. (3.14 m.) wide, of which about 2 ft. in width is occupied by the trusses of the movable span. At the shore piers the roadway is 8.75 ft. (2.67 m) above mean water level, and rises from here, on a grade of 1 in 70, to the center of the swing span. The clear height below this part of the bridge is 8.75 ft. between mean water level and the lowest part of the bridge in the center of each sailing passage.

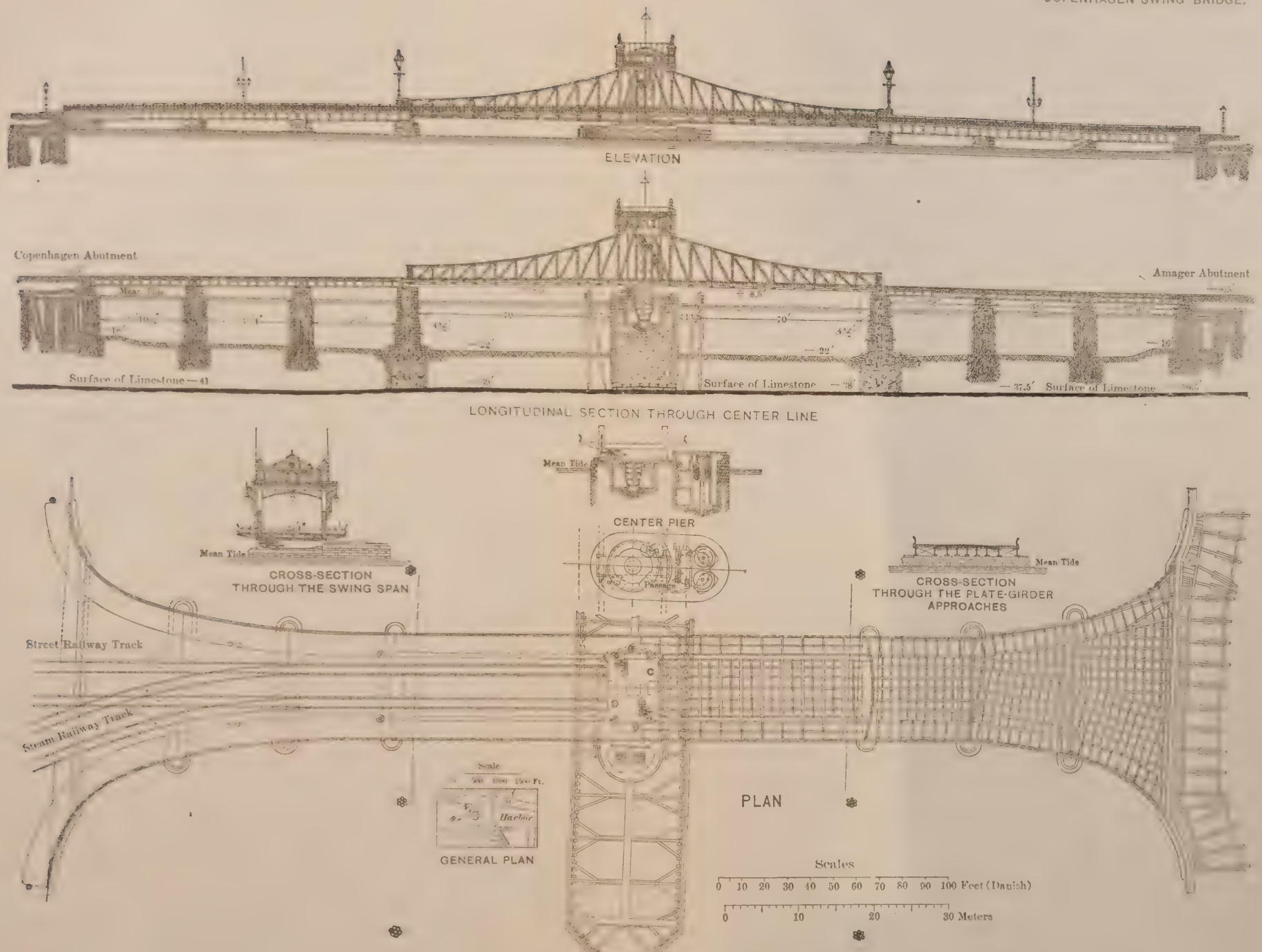
The Substructure.—The bridge rests upon nine piers, *viz.*, two abutments, four piers for the fixed spans, two end piers and one center pier for the swing span.

Solid limestone rock extends under the whole city of Copenhagen, and its surface rises to about 42.21 ft. (12.87 m.) below mean water level at the west abutment; from there the surface of the rock rises gently upward to about 37.59 ft. (11.46 m.) below mean water level at the east abutment. The rock is covered with a layer of hard clay mixed with sand and limestone about 10 ft. thick. On the eastern side this is covered with a layer of gravel, which had to be dredged away, so that there was everywhere a depth of 20.6 ft. (6.28 m.) of water below the bridge and 22.7 ft. (6.9 m.) in the sailing passages.

While this layer of clay was deemed sufficiently solid to carry the six piers for the fixed spans, it was necessary, on account of the proposed deepening of the sailing passages, to place the foundations for the center pier and the two end piers for the swing bridge on rock bottom.

The Center Pier.—Besides serving as a support for the swing span, the center pier is designed to give room for the hydraulic operating machinery. This explains its oval shape, 30.9 ft. (9.41 m.)

PLATE X.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1000.
MOELLER ON
COPENHAGEN SWING BRIDGE.



in width and 63.83 ft. (19.46 m.) in length, up and down stream. The pier has semi-circular ends of the same diameter as the width of the pier. The center pier thus provides space for the pivot-room below the vertical axis of the bridge, the accumulator-room in the southern part of the pier, with its floor 15.94 (4.86 m.) below mean water level, and for a passageway, 5.15 ft. (1.57 m.) wide, between these rooms.

The center pier had to be placed in a very exposed position, as ships passing through the old bridge were brought close up to the western side of the center pier for the new bridge; dry work inside the coffer-dams, therefore, was entirely out of the question. The pier was built "floating," at a convenient point, by which means navigation was disturbed as little as possible, and the danger due to collisions between boats and the pier was reduced to a minimum.

At first a strong chamber-shaped iron platform was built on a slip, this platform forming the bottom of the floating, hollow pier during its construction. Later, as the outer walls of the pier were raised, it sunk deeper and deeper, the weight of the brickwork and the iron bottom always balancing the buoyancy.

As it was necessary that the masonry be placed so as to weigh down the pier in equilibrium, and at the same time resist the greatest pressure caused by the water, and eventually the impact of blows against the sides of the pier, the masonry was arranged as a ring wall, consisting of horizontal arches supported by three transverse walls, *viz.*, one at the center and two through the center of the circular ends. The ring wall, on both sides of the pier, was divided by the transverse walls into two flat arches of brickwork. The ring wall was of clinker bricks, and the transverse walls and the rear part of the masonry were of concrete.

The pier was constructed in the northern part of the harbor. When the floating pier had sunk to a depth of 16.47 ft. (5.02 m.), it was towed to the site and lowered into place. The material above the rock had been excavated by a dredge and leveled off as far as possible by exploding small blasts. The pier was then held by strong moorings and the masonry work continued until the lower edge of the pier was within about 5 ft. of the rock surface. This surface again was cleaned of mud, which had flowed down into the dredged hole during the further construction of the ring wall and

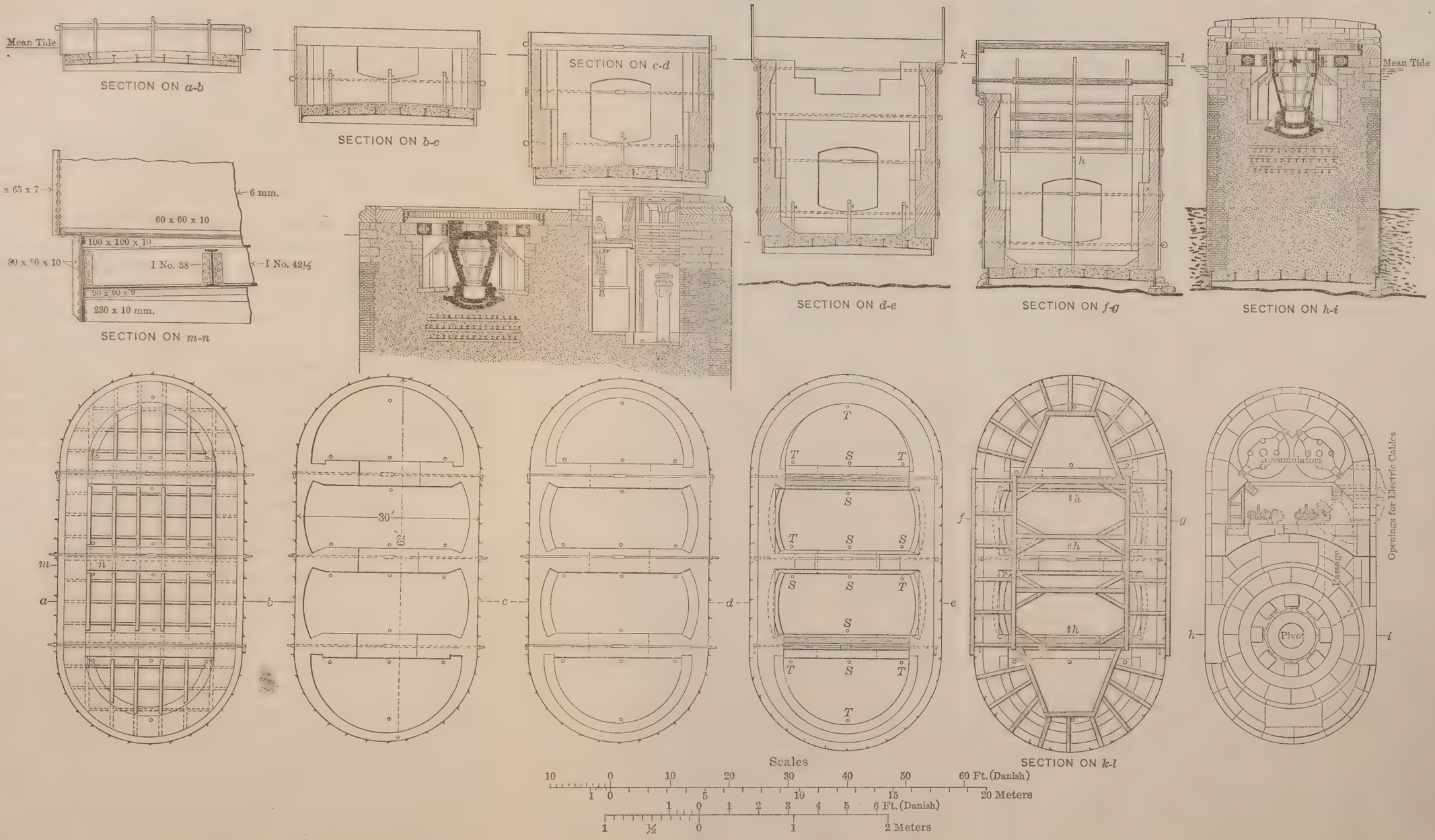
the transverse walls. The floating pier was then placed in its correct position and sunk to the bottom in less than an hour by filling it with water. Clay and sand were deposited outside and around the foot of the pier to a height of about 12 ft., and the hollow space between the iron bottom of the pier and the rock was filled with neat cement mortar, which was led down through a number of pipes, while the water was led away through vertical pipes. The space inside the ring wall was filled with concrete after the water had been pumped out. The entire weight of the finished center pier is about 11 000 000 lb.

The Two End Piers for the Swing Span.—The end piers are also placed on the surface of the limestone rock, at depths of 38.08 ft. (11.61 m.), and 40.15 ft. (12.24 m.) below mean water level, respectively, for the eastern and western piers. The length of each pier is 58.7 ft. (17.89 m.), and their thickness is 9.25 ft. (2.82 m.) at the top, increasing to 14.4 ft. (4.39 m.) at the bottom. They are composed of five horizontal courses of blocks, placed on a concrete foundation cast inside a submarine cylinder placed upon the rock. The different blocks of concrete, with a granite coating, are grooved together to make the whole pier one solid body.

The Four Smaller Piers, Supporting the Fixed Spans.—The four smaller piers are built on a high pilework driven into the firm layer of clay in about 21 ft. of water. The lengths of these piers are, respectively, 60.22 ft. (18.36 m.) and 76.7 ft. (23.38 m.), with rounded ends. The construction is the same for all four piers; from mean water level upward the piers have a thickness of 8.2 ft. (2.51 m.), and there they are of granite, resting upon a 2-ft. layer of concrete, cast over a system of wooden piles, in which there are three rows of vertical 12-in. piles and two rows of inclined piles. Outside the concrete layer and the piles a 10-in. pile-planking is placed, inside of which a filling of clay surrounds the bearing and staying piles.

The Two Abutments.—The abutments are constructed on a high pilework, on the same principle as the four smaller piers. They are anchored into the ground by a bulkhead anchorage.

The Superstructure.—The fixed spans of the bridge are supported by a system of shallow girders about 3 ft. (1 m.) in height. Each of the sidewalks is carried by two girders of the same height, 3 ft., upon which steel I-beams are placed. These beams again


Mean Tide
SECTION ON a-b
SECTION ON b-c
SECTION ON c-d
SECTION ON d-e
SECTION ON f-g
SECTION ON h-i
65 x 65 x 7
6 mm.
60 x 60 x 10
100 x 100 x 10
80 x 80 x 10
I No. 38
I No. 42½
90 x 90 x 9
230 x 10 mm.
SECTION ON m-n
k
l
h
Mean Tide
a
b
c
d
e
f
g
m
n
30′
62′
T
S
Accumulators
Openings for Electric Cables
Passage
Pivot
SECTION ON k-l
Scales
10 0 10 20 30 40 50 60 Ft. (Danish)
1 0 5 10 15 20 Meters
1 0 1 2 3 4 5 6 Ft. (Danish)
1 ½ 0 1 2 Meters

support a deck of corrugated-plate flooring, filled in with clinkers, and covered by a layer of concrete with an asphalt wearing surface.

On the roadway, the buckle-plates are also covered with a layer of concrete carrying a 2-in. layer of compressed asphalt bricks, with granite curbing along the edge of the sidewalks. The rails of the railway and tramway are fastened to the steel framework of the bridge by underlying bearings, and along both sides of the tramway rails the asphalt is reinforced by a double row of blocks of Australian wood, as these may stand the wear of the wagon wheels better.

The Swing Span.—The swing span is supported by two trusses, the lower parts of which are gently ascending from the ends toward the center, while their upper chords form parabolic curves ascending more rapidly from the ends toward the center. The upper and lower chords are about 2 ft. (63 cm.) wide, and the cross-section is built up of steel plates and angles.

The roadway between the trusses is supported by floor-beams of riveted steel plates and angles, and placed at the verticals of the main girders. These floor-beams are connected by longitudinal beams supporting the buckle-plates. On top of the buckle-plates there is a layer of concrete supporting an oak pavement.

The engine and watch-house is placed on top of the trusses, and contains the necessary machinery for operating the swing span.

General Arrangement of the Machinery for the Swing Span.—When the bridge is closed, the swinging part rests upon six bed-plates, *viz.*, two on each of the two end piers and two on the center pier. The trusses are supported at the center by a heavy cross-girder, resting on a large hydraulic piston—the pivot upon which the bridge turns.

When the bridge is to be opened, the whole movable part, weighing about 850 tons, is lifted about 6 in. (15 cm.) in 15 seconds, resting entirely upon the pivot. It is raised by hydraulic power acting on the pivot, which at the upper end is guided by a collar-bearing, and travels in a low hydraulic cylinder (the pivot cylinder) filled with a mixture of half glycerine and half water. The lifting and lowering of the pivot and the bridge are done by connecting the pivot cylinder alternately, by hydraulic pipes, with heavily and lightly loaded accumulators, so that the fluid is either forced into or carried away from the pivot cylinder.

The bridge is turned by a pinion fixed to the movable part and geared to a circular rack fixed rigidly to the center pier. This pinion, through a train of gearing, of which part is below the roadway and part is in the engine-house (these two parts being connected with each other by a vertical shaft placed in the trusses), is driven by an electric motor with a normal capacity of 30 h. p., and a maximum capacity of 40 h. p. The motor is placed in the engine-house, on the top of the trusses. There are two independent sets of machinery, arranged symmetrically around the center of the bridge, each capable of turning the bridge one-quarter turn in one minute. Ordinarily, the two sets of machinery work together, thus neutralizing the reactions in opposite directions on the rack rim. The bridge can be turned 180° in each direction.

At each end of the swing span there is a hydraulic buffer fixed to the end piers and arranged in such a manner as to stop the bridge whether it be turned in one direction or the other. By striking against these buffers, the velocity of the bridge is gradually checked and the span adjusted in its right position; each buffer is designed to resist a force of 12 tons. At each end of the swing span there is also a locking gear, operated by the bridge tender in the engine-house, and this secures the bridge in its correct position.

As previously mentioned, the swing span is released from its bed-plates upon the center and end piers by being raised 6 in., while resting upon the hydraulic pivot. One of the two accumulators, situated in the center pier, is so heavily loaded that its downward movement is sufficient to cause the pivot to rise. When the bridge is to be lowered, connection is opened between the pivot cylinder and the other accumulator, the weight of which is adjusted so that it moves upward while the pivot moves downward. When the bridge on its downward movement comes to the point where its ends bear against the bed-plates upon the end piers, the reactions from these diminish the pressure so as to check the upward movement of the light accumulator, and the bridge is allowed to settle down the small distance (about ½ in.) still left between the bridge and the bed-plates upon the center pier by opening the connection between the pivot cylinder and a return-water tank. Electric pumps force the fluid from the light accumulator and from the return-water tank, respectively, over into the heavy and the light accumulators, in

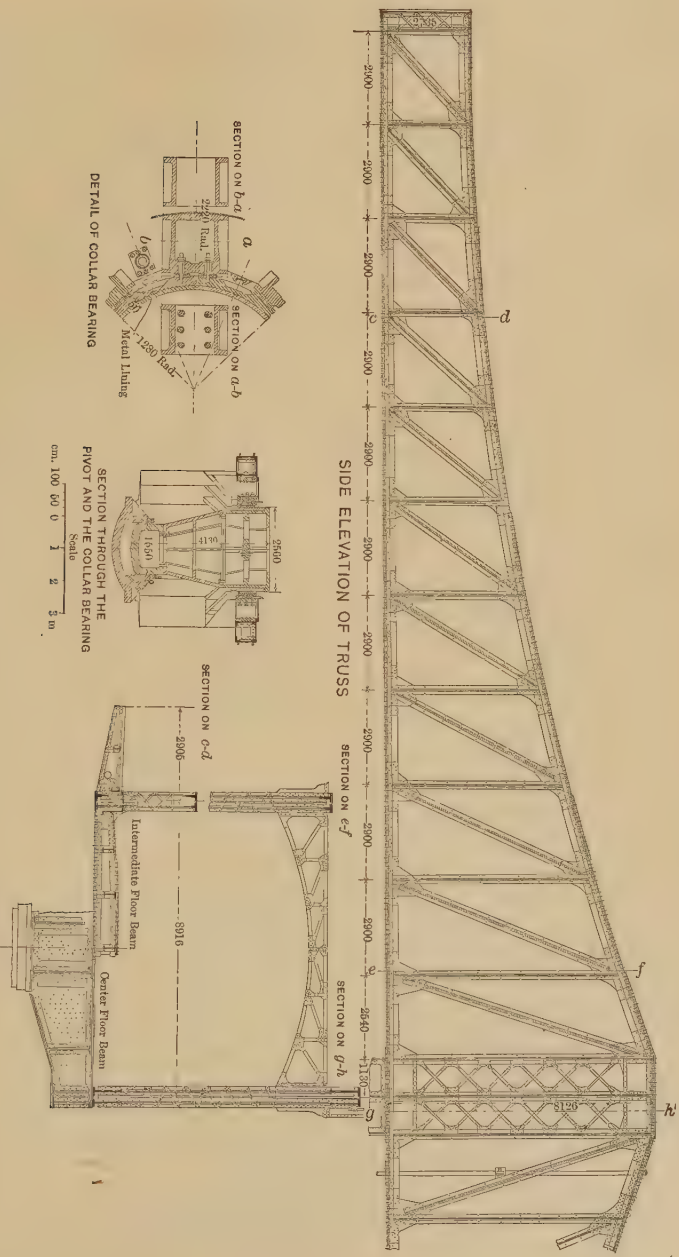

SIDE ELEVATION OF TRUSS
SECTION ON b-a
SECTION ON a-b
DETAIL OF COLLAR BEARING
Metal Lining
SECTION THROUGH THE PIVOT AND THE COLLAR BEARING
Scale
cm. 100 50 0 1 2 3 m
SECTION ON c-d
SECTION ON e-f
SECTION ON g-h
Intermediate Floor Beam
Center Floor Beam
CROSS-SECTION

FIG. 1.

order to make everything ready for raising and lowering the swing span again. As will be seen, the work performed in raising the swing span is utilized partly while lowering it; as the fluid-pressure in the pivot cylinder is used to raise an accumulator loaded with 56 tons (39 atmospheres), it being only necessary, by aid of the electric pump, to press the fluid from this accumulator to the other, which is loaded with 76 tons (52½ atmospheres). The small quantity of fluid which is led from the pivot cylinder into the return-water tank is pressed by the reserve pump into the light accumulator. Of course, this machinery works automatically as soon as the heavily loaded accumulator is lowered. The whole operation, until this accumulator is raised again, takes 3 minutes.

The Electric Installation.—The power for the manipulation of the bridge is electricity, delivered from one of the municipal electric stations. There are five cables which cross the western sailing passage and are led through five cast-iron pipes in the side wall of the center pier to a switch-board on the upper floor in the accumulator-room. Two of the electric cables have an electro-motive force of + 220 volts, two of — 220 volts, while the fifth is an 0-cable; (one plus and one minus cable are in reserve, but they are put in service automatically if the acting cables should fail). From the switch-board, the conductors branch off to the pump motors and the lamps in the accumulator-room, and to the pivot-room. The transmission of the current, from the conductors in the center pier to the swing span, is obtained by a system of contacts consisting of five copper contact-rails, shaped as horizontal rings, around the pivot and fixed to it by porcelain insulators, and trolleys running upon these rails, three upon each rail. The trolleys are fastened to arms which, by movable joints, are fixed to the walls of the pivot-room, thus allowing the trolleys to follow the raising and lowering of the pivot and its contact-rails. The upper two of these rails transmit + and — 220 volts to the electric motors for the swinging machinery, and to the arc lights on the bridge, the lower two rails transmit the current necessary for the telephone and alarm apparatus between the bridge tender in the engine-house and his assistant in the accumulator-room, while the center rail serves the circuit of the electric apparatus in the accumulator-room for manipulating the main valve on the connecting pipe between the pivot cylinder and the accumu-

DIAGRAM SHOWING THE HYDRAULIC MACHINERY
FOR RAISING AND LOWERING THE BRIDGE
850 Tons
S
Pivot Cylin-der
Valve
Valve
Heavy Accumulator
30
Tons
Pump
Light Accumulator
60
Tons
Pump
Return Water Tank

Fig. 2.

lators from the engine-house. For this transmission the earth serves as the return conductor. From the inner side of the hollow pivot the conductors are led as vulcanized rubber cables with iron armatures through the inner room of the main cross-girder, and from here the cables ascend along the trusses to the switch-board in the engine-house, from which they branch off to the different apparatus.

Loading.—The loads for which the bridge is constructed are as follows: Traffic by persons and carriages—100 lb. per sq. ft.—a truck weighing 20 tons, equally divided upon four wheels with an axle distance of 13.12 ft. (4 m.); the weight of two locomotives in opposite directions; besides the effect of wind pressure and braking. The weight of one locomotive and tender is 64 tons, the locomotive has three axles, the weight upon the center one being 13 tons and upon each of the two others 12 tons, the distance between the axles being 4.26 ft. (1.3 m.). The tenders also have three axles spaced 4.92 ft. apart (1.5 m.), the load upon each axle being 9 tons. The wind pressure is calculated to reach 34 lb. per sq. ft. when the bridge is loaded, and 54 lb. per sq. ft. when unloaded. During the turning of the swing span, this is calculated as being loaded by a 20-ton truck on one end and exposed to a wind pressure of 34 lb. per sq. ft.

Working Expenses.—The cost of electricity consumed at one opening and closing of the swing span for the passage of a boat has proven to be 4.3 cents, that is, 1.3 cents for the raising and lowering of the span and 3 cents for its turning. The municipal price for motor electricity per hectowatt per hour is about 0.4 cent.

DISCUSSION.

JOHN C. MOSES, M. AM. SOC. C. E. (by letter).—Swing bridges of two well-known types have been in use for many years. In one type the loads are carried by a circular drum resting on a ring of conical wheels which rests in turn on a track on the pier. A central pedestal holds everything in position laterally and carries a portion of the load, to insure its staying in place. In the other type the entire load is carried by the central pedestal, and balance wheels and track are provided to prevent the span from tipping when turning. In a large bridge of this type it is necessary to provide wedges or jacks at each end of the center cross-girders which will carry any transversely unbalanced live load, such as a train on one of two tracks. In both types the ends are lifted after closing the bridge. Mr. Moses.

Both types require a good deal of machinery when the bridge is a large one operated by power from a central house. The first cost is large, the work is expensive to erect in satisfactory adjustment, it is likely to be deranged subsequently, and it is difficult to protect the machinery from bad weather. Proper provisions for temperature changes and live-load deflections add much complication on a long span. Indicators, brakes, buffers and other safety devices are used to protect these exceedingly heavy pieces of moving machinery from self-destruction. Running expenses are high, repairs frequent, and complete break-downs not unknown. Nearly every new bridge designed is an attempted improvement on previous ones which have developed this or that defect. Therefore, it is of great interest to examine the design outlined by the author, as it presents such novel features, and is in successful operation.

Many of the old difficulties at once disappear. End-lifts and side-wedges, with their gearing, shafts, boxes, indicators, automatic brakes and safety devices, are entirely eliminated. Hinged pedestals with roller bearings can be provided at the ends to take care of the temperature and deflection changes, as in fixed spans. The ends can be raised the exact amount required by theory, and any necessary adjustment can be easily made by the use of shims. While end-locks and buffers may be advisable for a heavy bridge, they could be dispensed with on ordinary spans. **V**-shaped grooves in the end bearings would center and lock in place the trusses. A good hand-brake on the turning gearing and a controller such as used for crane work will quickly place, and hold in position while being lowered, a span weighing 500 tons. A possible objection to thus holding the bridge in place with the turning machinery might arise in this new type from the vertical motion of the pinion on the rack, or on

Mr. Moses. its shaft, when the bridge is lowered, but, with the ordinary types, it is entirely feasible.

The author has provided his turning machinery in duplicate. The usual practice is to rely on hand-gearing in case of accident. This means very slow work with a large bridge. While he, no doubt, has hand-gearing also, he is not likely to have to use it, and the extra cost of the duplicate equipment seems to be a wise investment.

The feature of chief interest in the design is the hydraulic pivot. The general scheme with the heavy and light accumulators is simple, and the return-water tank is an ingenious method of overcoming a difficulty. A full detail of the pivot would be of great interest, and the writer hopes it will be added. The success or failure of the entire scheme will depend on the efficiency of the detail used for packing the 5-ft. revolving plunger, with its load of 875 tons. No hand-gearing or duplicate machinery will remedy a leak here. Loss of pressure in the pivot, due to breakage in pipes or accident to accumulators, might be prevented by an automatic check-valve; but it is difficult to see how an accident to the pivot ram can be prevented from being very serious. The provision for holding the pivot vertical also seems to be elaborate, and probably cost as much as the usual track and wheel ring. The accumulators with their pumps, and the enlarged pier needed to hold them, will cost more than the ordinary end-lifts and side-wedges. If an assistant is required in the pier, the cost of wages for operation will exceed that of the ordinary types. Does the bridge then show a freedom from wear and break-downs that would justify the future use of the same plan? A fuller presentation of the details for study would help to anticipate the verdict of time. Further information and a report of any actual difficulties thus far met will add much to the author's interesting paper.

Mr. Le Conte. L. J. Le Conte, M. Am. Soc. C. E. (by letter).—It is really very difficult for an American engineer to understand and discuss fairly the heavy types of riveted bridges, such as are built so commonly in Europe.

The details of the plate-girder approaches will compare fairly well with those usually adopted by American bridge engineers; but, in the swing span, a very wide divergence is found, in the short panel lengths, only 9.5 ft.; the height of the center tower, only 28 ft.; the height of end posts, only 9.2 ft.; and, finally, a dead load of something like 8 100 lb. per running foot. These apparently abnormal conditions are all very hard to understand, yet, nevertheless, one is compelled to respect and admire such good construction. There is one feature about the swing span which is certainly praiseworthy, namely, the two solid bearings, 8 by 2 ft., one on each side of the

center pier directly under the main trusses. This detail, in some respects, is to be preferred to the ordinary turn-table with roller bearings. On the other hand, when it comes to operating the draw-span device with a hydraulic lift for raising the swing span clear of its supports, it is not to be commended. This device brings a terrible local strain on the bridge just over the center pier; in point of fact, it is nothing more than an extreme case of center-pintle bearing, with all its well-known serious disadvantages. American bridge engineers have practically abandoned the center-pintle bearing as being obsolete and condemned by experience, although, theoretically, it makes the easiest turning swing span. Mr. Le Conte.

If an American engineer were called upon to design such a draw span, he would, in all probability, select the pin-connected type as being best adapted to the case in hand.

The double tracks and floor system would be identically the same, in every particular, the main difference being the trusses and turn-table device. Such a span could be built, first-class in all respects, with a dead load not exceeding 5 000 lb. per running foot. This design, moreover, would be capable of withstanding a concentrated load and a railway live load far in excess of the loads mentioned by the author. The writer would not have it understood from this, however, that he is constitutionally opposed to riveted bridges; on the contrary, he is a firm believer in the wisdom of using heavy riveted work for all single-track bridges, up to spans of 200 ft. or thereabout. For double-track bridges heavy riveted work is best, up to spans of about 100 ft., as it furnishes more dead load and greater rigidity (two highly important features for smaller spans) and, at the same time, gives far better general satisfaction. For all spans of more than 100 ft. for double-track, and of more than 200 ft. for single-track bridges, the pin-connected bridge is by far the better and cheaper construction.

H. T. Forchhammer, Assoc. M. Am. Soc. C. E.—The speaker was Assistant Engineer to Mr. Moeller, at the time this bridge was built, and desires to add some further explanation of some of the details of this very interesting paper. Mr. Forchhammer.

A glance at the "section on *h*—*i*" of the main span, on Plate XI, will show how important it was to get the space between the steel bottom and the rock entirely filled with cement. The plan beneath the "section on *d*—*e*" (Plate XI) indicates the pipes, *T, T,* through which the cement was led down, and *S, S,* the pipes through which the water was led away. In passing through the water large quantities of cement were transformed into "laitance," but, when all the water was led away, it would have been impossible to raise the laitance through the pipes because its specific gravity, or the specific gravity of the mixture of water and laitance, was

Mr. Forchhammer.

greater than that of water. Therefore, in order to get rid of the laitance, three of the pipes marked "*S*" were provided with valves at half their height, and, when all the water had escaped these valves were opened and the filling with cement was continued until all the laitance was out.

Attention might be called to the hydraulic buffers on this bridge, as they operate successfully and save much time. The speaker does not know whether such buffers are in use in the United States. Their advantage is that they check the movement of the bridge at the same spot, whatever the velocity at which the bridge is revolving when it strikes them, and the impact is reduced to a resistant force of the same amount under the entire blow.

HYDRAULIC BUFFER

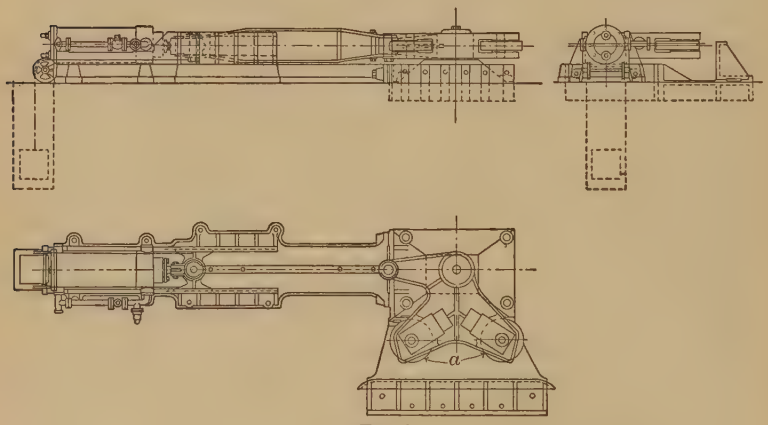


FIG. 3.

At Calais, France, the speaker has seen two draw-bridges with such buffers. One of these bridges was moved with full speed against the buffers, and they, with a movement of about 24 in., stopped the bridge in about 3 seconds, exactly in its central position, so that it could be locked without any waste of time.

The mechanism of the Copenhagen bridge is more complicated because the bridge is arranged to be swung in either direction. The principle, which was original with the Titan Machine Company, of Copenhagen, is shown clearly in Fig. 3. When a bar attached to the bridge strikes one of the rollers marked *a*, the fluid in the cylinder is compressed and creates a resistance as it passes from the front to the rear of the valve through some small openings in the latter.

Mr. Forchhammer.

Bridges of this general type seem to have attracted some attention. As far as the speaker knows, this type has only been used for a few smaller bridges in Germany. In England, hydraulic accumulators have been used for many swing bridges, to diminish the friction on the rollers, but hydraulic pressure has never been used to lift the bridge.

The collar bearing is shown on Fig. 1, but on a small scale. It is built as a sliding bearing and can be adjusted by the tap-bolts and wedges, as shown. This bearing, as well as the pivot, is built up of heavy steel castings, and is strong enough to resist the pressure of a heavy wind on the superstructure.

A diagram of the hydraulic machinery is shown on Fig. 2, with the accumulators, pumps, valves, pipes, etc. In addition, it should be mentioned that just outside the pivot and each accumulator is a safety valve which will prevent the heavy weights from falling suddenly in case any of the pipes should break.

This bridge has been criticised* as having too little depth, too great a number of members, excessive deflection, as lacking grace, etc. Considering the very shallow depth of the floor, which had to be maintained, the speaker believes that it would have been very difficult to build a more economic structure, and is sure that greater depth of truss and longer panels would not have added to its beauty.

†The author, Mr. Moeller, having read the discussions, has stated his views to the writer and requested him to reply.

Mr. Le Conte states that there would be "a terrible local strain on the bridge just over the center pier" when the bridge is being swung. The writer does not believe that such a strain would occur. When the bridge is swung, the trusses are carried on two bearings, 2 ft. wide and 8 ft. long, and even a very heavy wind pressure and a reasonable eccentric loading will not be sufficient to put in tension the bolts connecting the trusses to the main cross-girders. As to the trusses, there is no chance for the development of terrible, or not easily determined, stresses in them. It is otherwise with the main cross-girder, in which, in addition to the very heavy bending strain from concentric loading, there will be a heavy torsion strain, due to eccentric loading and wind pressure. This was fully considered in designing the girder, and it is amply strong, and rigid enough to take any torsion which will ever come.

Mr. Le Conte mentions the heavy dead load, the short panels and the shallow trusses, but it must be remembered that this is a city bridge, with a wooden pavement laid on concrete in the roadway, and with asphalt on concrete for the sidewalks. About three-eighths of the dead load is due to these—non-metallic—substances. The

* *The Engineering Record*, April 8th, 1905.

† This part of Mr. Forchhammer's discussion was contributed by letter after the previous discussions had been published.

Mr. Forchhammer.

weight of the trusses is about 2 100 lb. per ft. of bridge and the weight of the steel in the floor construction is about 1 900 lb. per ft. of bridge. As the depth of the floor construction was not great, it was necessary to have close spacing between the floor-beams, therefore there could have been little saving in steel by building the trusses deeper and by having wider panels, and, in addition, the æsthetic appearance would have been against it.

The bridge is rigid, and does not show excessive deflection. When swung, the deflection of the ends is about $1\frac{1}{4}$ in.; when resting on all three piers, the deflection at the center of each span is about $\frac{3}{8}$ in. due to dead load, and $\frac{1}{8}$ in. due to live load.

In reference to the cost, a bridge of this type is likely to be more expensive than the American types, but, on the other hand, the running expenses are very small. It is not necessary to have an assistant in the pier, because all the machinery may be operated safely from the pilot-house. Furthermore, if the æsthetic appearance were not to be considered, the cost could be reduced very much by building the center pier only half as large and placing the hydraulic machinery on a separate and lighter foundation.

One great advantage, in a bridge of this type, is the rapidity with which it can be operated, which, certainly—for a city bridge—is worth some expense. For the passage of a small ship, traffic is stopped for 2 min. 20 sec., and for a large ship 3 min. 10 sec. The motion of the bridge is very gentle, and is free from the shaking generally observed on swing bridges.

The bridge has not been in operation long enough to enable one to form any opinion as to its freedom from wear and breakdowns, but it has been used successfully since it was opened for traffic, in January, 1903, and, thus far, there have been no difficulties.

In reference to the request of Mr. Moses, for a more complete presentation of details: The pivot is built up of heavy steel castings. The upper third is cylindrical, 2 560 mm. in diameter and 1 480 mm. high, the thickness of the steel being 80 mm. It consists of two symmetrical pieces. Tap-screwed to the outer surface there is a cast-steel band, 80 mm. thick, which is turned to an exact cylinder.

The middle third is conical, and is 1 830 mm. high. This, also, is built up of two symmetrical pieces.

The lower third is in one cylindrical piece, 1 550 mm. in diameter, 1 010 mm. high, and 200 mm. thick. This piece, also, is turned to an exact cylinder.

All the several pieces of the pivot are reinforced with ribs, and connected with heavy tight-fitting screw-bolts.

The dimensions of the pivot were determined after a thorough investigation of the distribution of compressive and bending stresses due to vertical, as well as horizontal, loading on the bridge.

The pivot cylinder is one solid steel casting, 200 mm. thick. The packing consists of five layers of hemp, 30 mm. thick, greased with tallow, and, as far as the writer knows, its efficiency has proved to be fully satisfactory; in fact, the pressure beneath the pivot is scarcely 700 lb. per sq. in. In many English bridges the pressure is the same or even greater. Mr. Forchhammer.

In case of accident, the bridge is provided with a hand-pump for the pivot and hand-gear for the turning machinery.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1001.

THE RECONSTRUCTION OF THE BALTIMORE AND OHIO RAILROAD BRIDGE OVER THE OHIO RIVER, AT BENWOOD, WEST VIRGINIA.*

By J. E. Greiner, M. Am. Soc. C. E.

With Discussion by O. E. Hovey, M. Am. Soc. C. E.

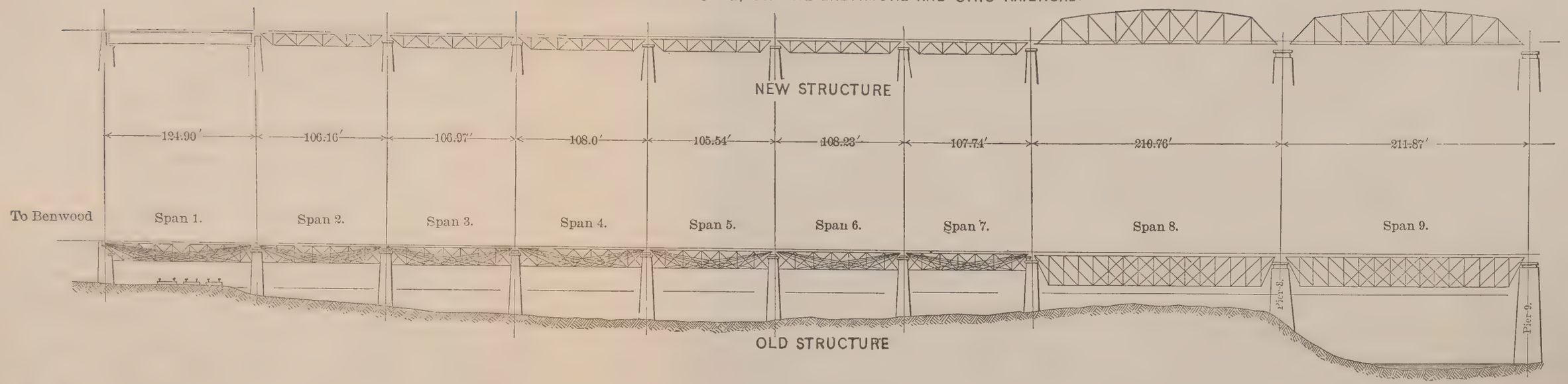
The Old Bridge.

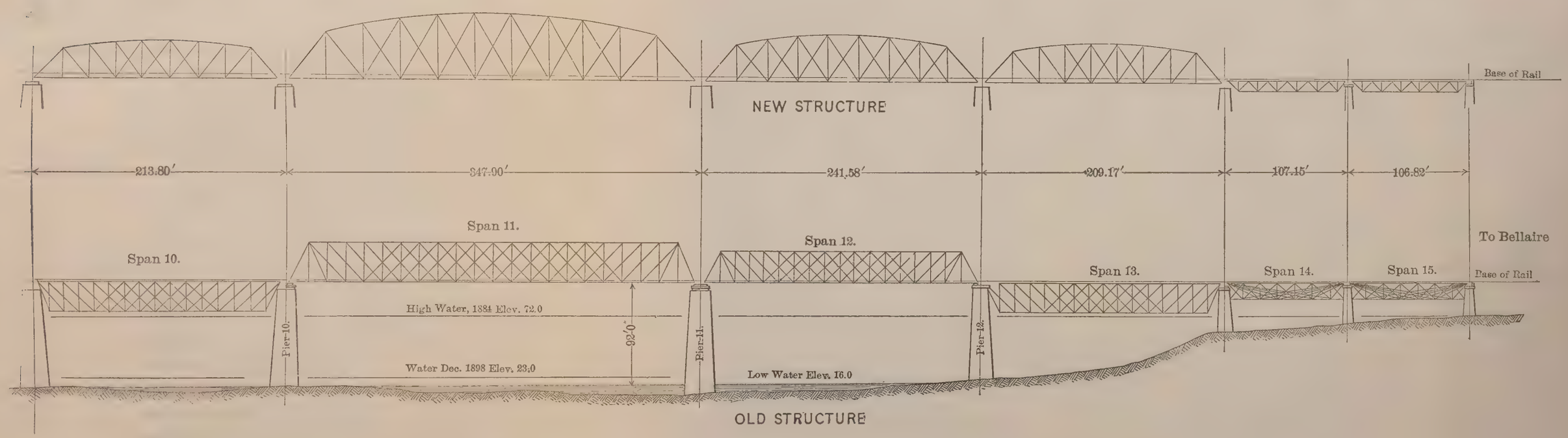
The original bridge at this place was built under the general authority given by the Act of Congress approved July 14th, 1862. This Act provided that bridges built over the Ohio River above the mouth of the Big Sandy should not have a less elevation than 90 ft. above low-water mark in the channel, nor a less span than 300 ft. over the main channel, the next adjoining span to be not less than 220 ft.

The bridge was composed of 9 Bollman deck spans, 4 deck spans and 2 through spans of Linville and Piper type, as shown on Plate XII, and, in addition thereto, there was an approach on the Ohio side consisting of 43 semi-circular stone arches, with 28 ft. clear span and a total length of approach of 1 490 ft. The length of the iron bridge, from center to center of abutments, is 1 435 ft. 6 in.,

* Presented at the meeting of April 19th, 1905.

THE BENWOOD BRIDGE, ON THE BALTIMORE AND OHIO RAILROAD.





and the total length of the entire bridge, including approach, from end to end of masonry, is 3 916 ft. 10 in.

The Bollman spans were built by the Baltimore and Ohio Railroad Company at its Mt. Clare shops, Baltimore. The other spans were built by the Keystone Bridge Company. The first stone was laid in the foundations on May 2d, 1868, and the bridge was opened for traffic on June 21st, 1871.

The structure was designed for a uniform load of about one ton per foot, with the unit strains in use at that time.

Periods of Reconstruction.

The rebuilding of this bridge was not a continuous operation, but was done in sections extending over a number of years, the policy being to replace those spans which were the most expensive to maintain, or which developed weakness under the existing traffic, in the order in which this weakness was made evident. Each section was designed in accordance with the specification in use on the Baltimore and Ohio Railroad at the time the design was made, the result being that the completed new structure is not of uniform strength.

The periods of renewal were as follows:

Year 1893.—Spans 14 and 15, designed for Consolidation engines weighing 107½ tons with tenders; all material, wrought iron; specifications of 1892; Union Bridge Company, builders.

Year 1900.—Spans 1 to 7, inclusive, designed for Consolidation engines weighing 125 tons with tenders; material, soft open-hearth steel; specifications of 1896; builders, Pencoyd Iron Works.

Year 1902.—Spans 8, 9, 10 and 13, designed for Cooper's E-50 engines, followed by a train load of 5 000 lb. per running foot; material, soft open-hearth steel; specifications of 1896; builders, Edgemoor Plant, American Bridge Company.

Year 1904.—Spans 11 and 12, designed for Cooper's E-50 engines; material, open-hearth structural steel; specifications of 1901; builders, Edgemoor Plant, American Bridge Company.

The Design.

The reconstruction of this bridge, except Spans 11 and 12, involved no work of particular interest, either in the design or in the erection, as there was no objection to the use of falsework. Spans 8, 9, 10 and 13 were made through bridges, instead of deck bridges, as the trusses and overhead bracing could be erected before disturbing the old structure, and in case the falsework had been washed out, there would have been the least possible interruption to traffic.

The design of Spans 11 and 12 was controlled by the proposed method of erection, the requirements being that the erection should be carried on without interrupting the railroad traffic or interfering with navigation in the main channel span (Span 11), there being no objection to falsework under Span 12.

The question of widening the channel was fully considered before the final adoption of the design upon which the structure was actually built. The War Department and the river men desired a much larger channel span than existed in the old structure, but there was a still further question as to who should pay for the difference in cost between rebuilding the bridge on the existing masonry and the lengthening out of the span as desired, the Railroad Company claiming under their charter the right to maintain the bridge on the existing masonry. The Railroad Company had obtained a special War Department permit during 1900 for the reconstruction of Spans 8, 9, 10 and 13 on the existing masonry, and, considering this fact, the War Department would probably have been satisfied with the removal of Pier 11 and the replacing of Spans 11 and 12 with one single span of a total length, from center to center of piers, of 589.48 ft., part of which could have been erected on falsework. This would have required extending Piers 10 and 12 and the removal of Pier 11. A study of this plan is shown in Fig. 1. It is estimated that this single span would have cost about $253 500 more than the construction on the existing masonry.

While the War Department would probably have agreed to a span of 589.48 ft., they preferred a span giving at least 700 ft. in the clear of the piers. This would make a length of 730 ft. from center to center of piers. This would have necessitated the removal of three piers, the lengthening out of one pier, the rebuilding

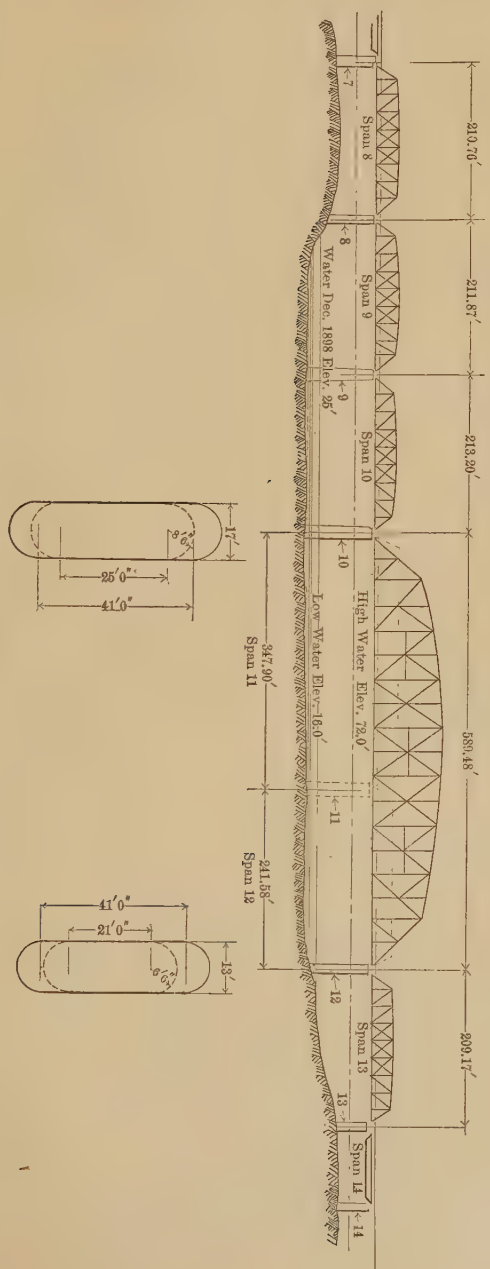

Span 8
Span 9
Span 10
Span 11
Span 12
Span 13
Span 14
Water Dec. 1898 Elev. 25′
High Water Elev. 72.0′
Low Water Elev. 16.0′
210.76′
211.87′
213.20′
589.48′
209.17′
347.90′
241.58′
17′
25′0″
41′0″
41′0″
21′0″
13′

FIG. 1.

of one pier, and the replacing of Spans 9, 10, 11 and 12 with one span of 284.55 ft. and one span of 730 ft. A study of this scheme is shown in Fig. 2. It was estimated to cost about $563 000 more than the cost of erecting on the existing masonry.

All the old bridge having been replaced by this time, with the exception of Spans 11 and 12, and the weakness of these spans being such as to prevent the Railroad Company from using the heavy cars and locomotives which were in operation on each side of the bridge, it became necessary for the Railroad Company, in order to get the benefit of their heavy power and equipment, to reconstruct these two spans as speedily as possible, and there being no prospect of the Government or the river men paying the additional cost for the increased channel span, and the Railroad Company not being inclined to bear this extra expense, it was decided to reconstruct these two spans on the existing masonry, care being taken not to interfere with navigation through the main channel. The method of erection, therefore, was the main feature which controlled the design of these spans. As Span 12 could be erected on falsework, this span presented no difficulty, but for Span 11 there were only three apparent methods of erection, as follows:

A.—Extend the piers by means of pile bents, erect the new span on floats, jack the old bridge to one side and float the new span into position, and then pick up the old span with the floats, move it to one side and dismantle it;

B.—Transfer the Baltimore and Ohio traffic over the Wheeling Terminal Bridge, and erect Span 11, using the existing structure in place of falsework;

C.—Erect Span 11 on the cantilever principle, Span 12 to be erected on falsework and designed as one shore arm and a temporary shore arm being placed on the outside of the existing structure in Span 10. In this method, the old Span 11 would be called upon to carry the railroad traffic and at the same time support the erection traveler. Between trains, this old span would support the traveler and such members as the traveler was handling.

Method *A* was abandoned as impracticable, on account of the frequent variation of the water level of the Ohio River.

Method *B* was at first considered favorably on account of the

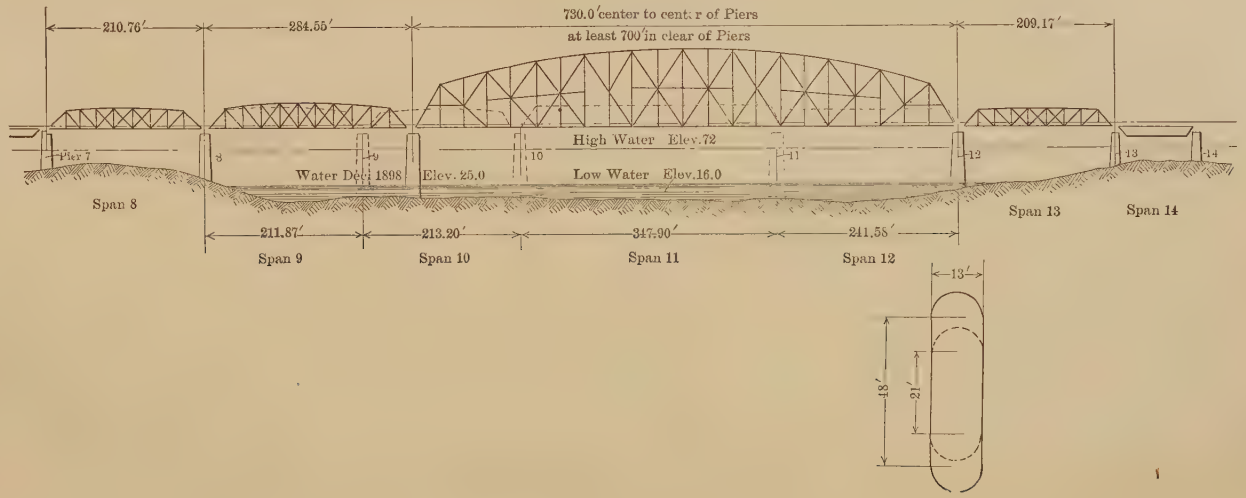

210.76′
284.55′
730.0′ center to center of Piers
at least 700′ in clear of Piers
209.17′
High Water Elev. 72
Pier 7
8
9
10
11
12
13
14
Water Dec. 1898
Elev. 25.0
Low Water Elev. 16.0
Span 8
Span 13
Span 14
211.87′
213.20′
347.90′
241.58′
Span 9
Span 10
Span 11
Span 12
13′
18′
21′

FIG. 2.

possibility of the Baltimore and Ohio Railroad transferring its traffic over the Wheeling Terminal Bridge for a period of 50 or 60 days while Span 11 was being replaced. This scheme was abandoned, on account of the congestion which would result on the Wheeling Terminal Bridge, due to the immense volume of business being done at that time.

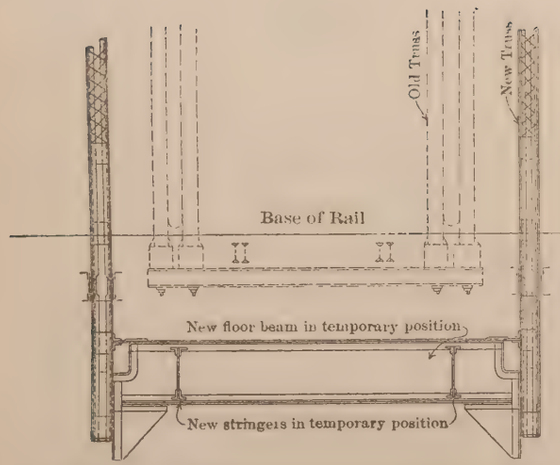
It was decided, therefore, to proceed with the erection in accordance with Method *C*. The structure was designed accordingly, and the work was carried through to completion. The following is a brief description of the main features in the design of these two spans:

Span 12 was designed and proportioned so as to take up reversal of stresses while the trusses were acting as cantilever arms during the erection of Span 11.

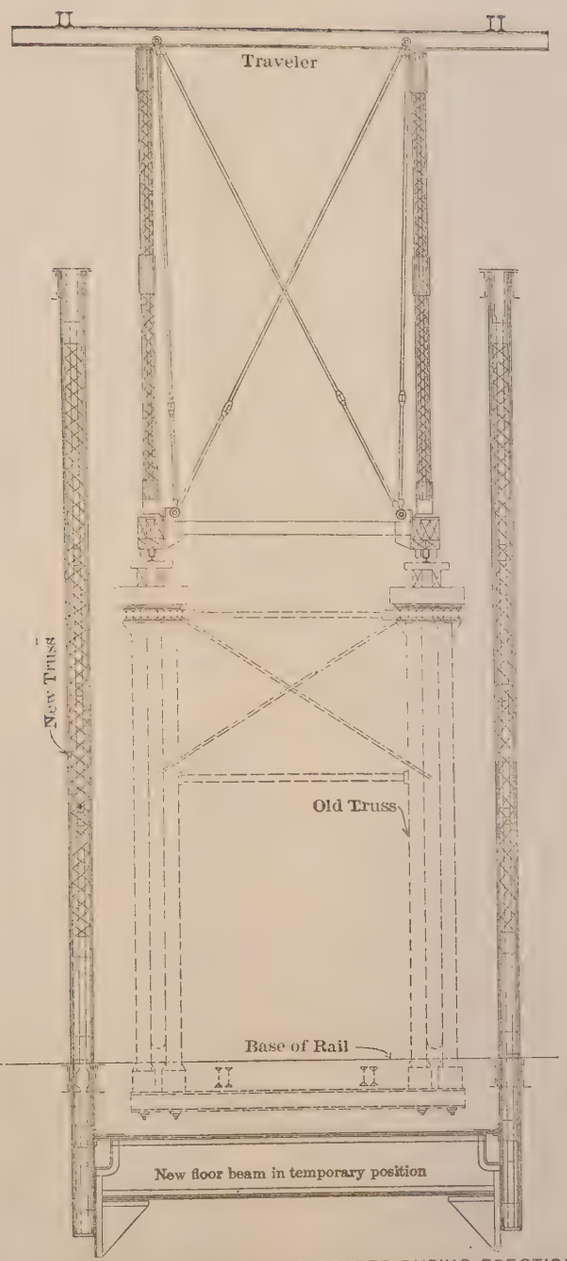
Span 11 was designed so that the trusses and upper bracing cleared the old span. The floor system and lower laterals, however, could not be made to clear, on account of the fact that the distance from the base of rail to the lowest part of the structure has to remain the same in both cases. The span had a stiff bottom chord throughout, all members, of course, being designed to meet the conditions of erection as a cantilever and as a free span after erection.

As the existing Span 10 was of a good, modern type of construction, it was not desired to replace it. It had eye-bar bottom chords, and could not be made to act as a shore arm during the erection of Span 11, and it became necessary to design special trusses, extending from Pier 9 to Pier 10, which could act as shore arms during the erection of Span 11. These trusses could be erected on the outside of the existing span, and were designed so that they could be inverted and used as a deck bridge after they had served their purpose during the erection of the Benwood Bridge. These erection trusses were to be clamped to the existing span in such a manner that they would be free to move vertically.

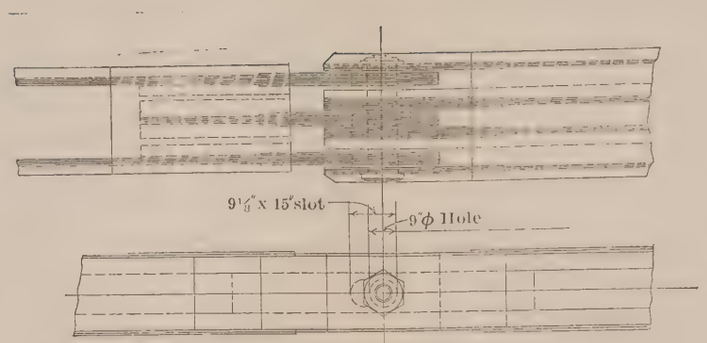
The east, or Benwood, end was arranged so that it could be raised or lowered with jacks, as was also the west, or Bellaire, end of Span 12. The erection of Span 11 was to proceed from the piers toward the center, the old truss being relied upon to carry the weight of the traveler and restricted traffic, and the individual members handled by the traveler. The west end of Span 11 was on



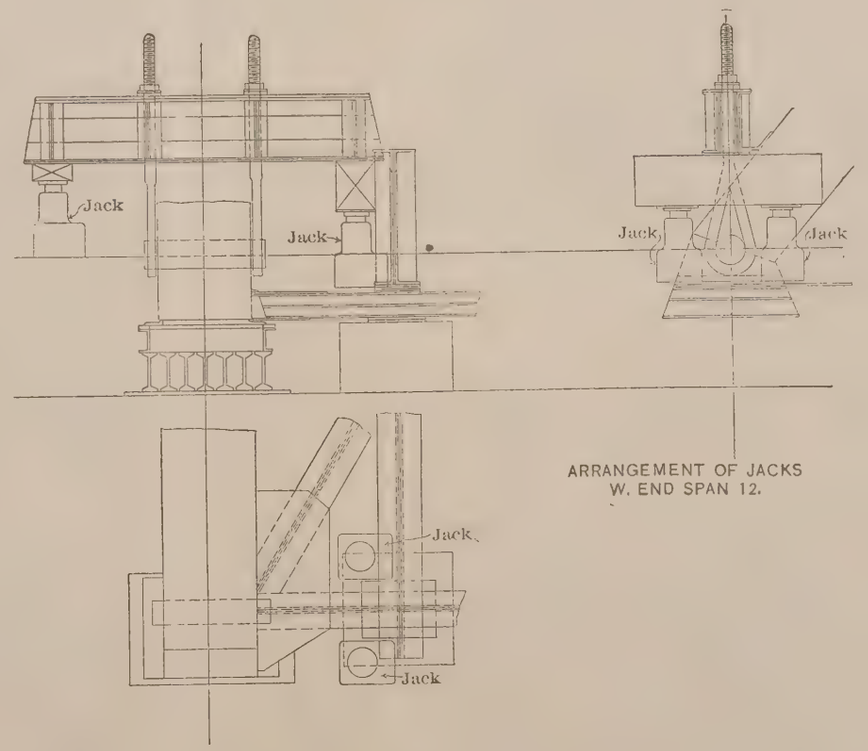
SPAN 11, CROSS-SECTION: NEW TRUSSES SWUNG



SPAN 11, CROSS-SECTION AT CENTER DURING ERECTION



SLIP-JOINT IN CENTER PANEL OF BOTTOM CHORD



ARRANGEMENT OF JACKS
W. END SPAN 12.

rollers. Attached to the end bottom pins at this end were struts which connected with cross-girders temporarily attached to Span 12. Each of these struts was in two sections, and these sections bore against a wedge arrangement. An overlap of $2\frac{1}{16}$ in. was provided in the bottom-chord center panel at low temperature, and about 10 in. clearance of the top-chord center panel, at high temperature. The west end of the center bottom chord was provided with a 9-in. slot, with a maximum movement of $4\frac{1}{2}$ in. each way. The wedge struts at the west end provided for a maximum movement of 8 in.

It will be seen from the foregoing description that, by jacking up the east end of Span 10 and lowering the wedges in the strut at the west end of Span 11, the top chords at the center of Span 11 could be lowered to their proper bearing; and, if, by some chance, there should not be sufficient movement attainable with the wedge arrangement there was still left an opportunity of jacking up the west end of Span 12 (see Plate XIII).

The Erection.

The method of erection of Spans 11 and 12 (see Plate XIV) was carried on as outlined in the following:

Span 12.—Falsework was placed under the old span, which was then taken down. The traffic was carried on this falsework while the new span was being erected. The falsework was not removed until Span 11 was swung, as it was required to support the live load, thereby avoiding vibration in the west half of Span 11 during erection.

The dead-load positive reaction per truss on Span 12 was 157 000 lb., and as the erection-stress negative reaction was only 56 000 lb., no counterweighting was required.

The East Shore Arm. Span 10.—For the east shore arm, the temporary trusses were erected outside of the trusses of Span 10, and on the same line as the trusses in Span 11. They were erected directly from the existing span, without the use of falsework, and were clamped to the existing trusses, the clamps being arranged so as to permit of movement up and down. The east ends of these erection trusses were dropped about 36 in. below the level. The dead-load positive reaction about equalled the erection-stress nega-

tive reaction, and, therefore, provision was made for about 25 tons counterweights for each truss.

Span 11.—The erection of this span was started at the east end, and, as the traveler moved toward the center of the span, the portals, top laterals and as much of the sway bracing as possible were removed. The floor-beams were placed upside down, in a temporary position below the bottom chord pins, and connected to the posts, which were lengthened for this purpose and which were cut off after the floor-beams were placed in their proper position. After the east half was completed as far as the center, and all bracing was placed, the west arm was erected in the same manner, connection being made in the center panel of the top and bottom chords.

No difficulty whatever was experienced in connecting the two halves of this span. The east end of Span 10 was jacked up and the wedges at the west end of Span 12 were withdrawn until the members came to their proper position. After this span was swung, the intermediate stringers were placed in their temporary position, under the old trusses, and connected by temporary timber laterals. The old trusses and floor were then blocked up and the trusses and bracing taken down, using the new span for support. The end panels of the laterals were placed in their permanent position, and temporary stringers provided, so as to clear these end laterals.

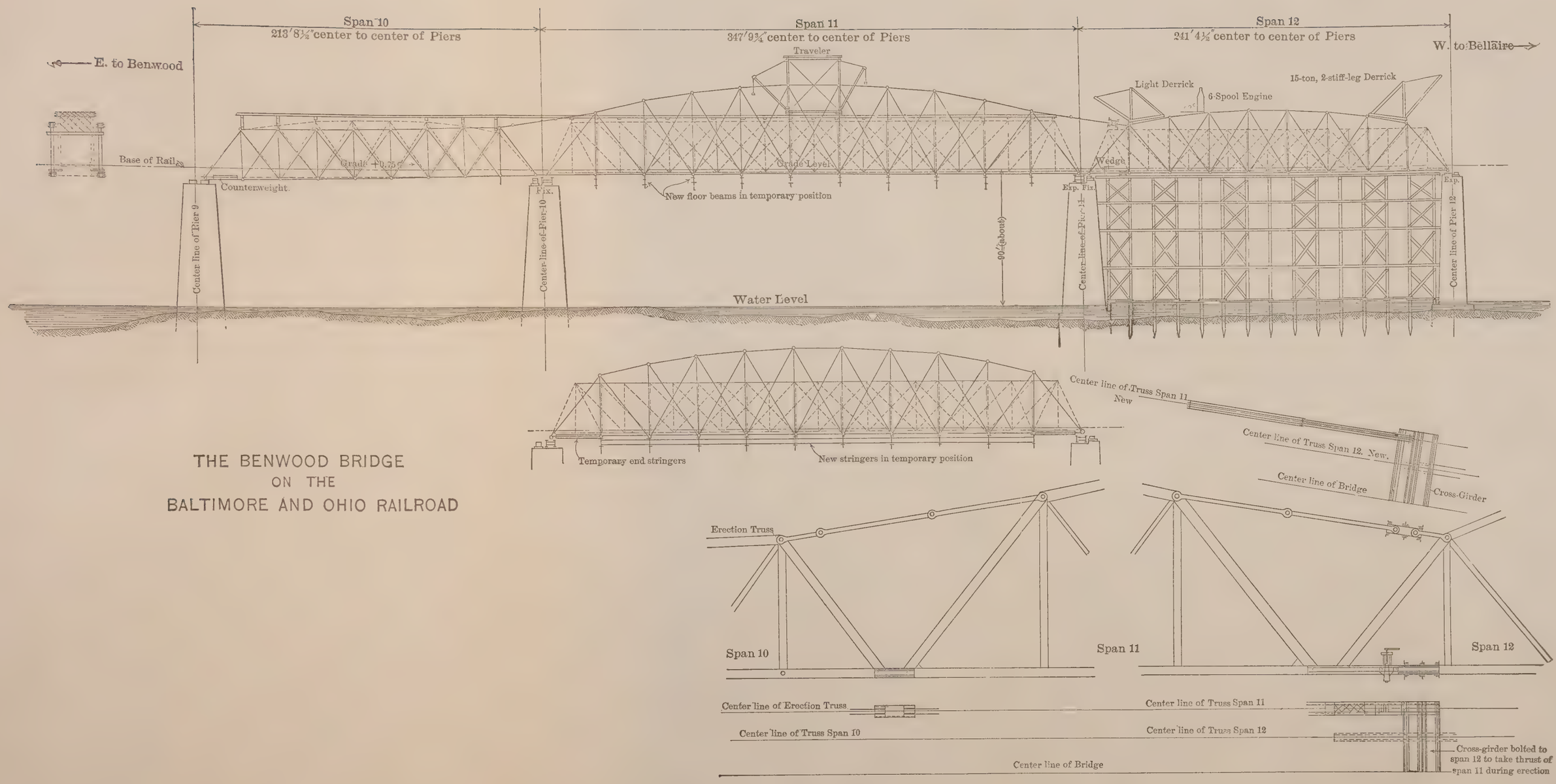
General.

The traveler used during this erection, as will be observed from the plan, was run on the top chords of the existing bridge. It weighed 40 tons. Nearly all the material was floated out under the spans on barges, then picked up and placed in position by the traveler.

During the erection of Span 11, it was necessary to take the greatest care in placing the traveler in a fixed position during the handling of material, and, before the erection proceeded, the old bridge was carefully adjusted and the posts braced with temporary wooden braces. The maximum tension strains in the old bridge during this erection did not exceed 16 000 lb. per sq. in., and the compression strains were in proportion.

As Span 11 was considerably wider than the existing bridge, and

THE BENWOOD BRIDGE
ON THE
BALTIMORE AND OHIO RAILROAD



as the bearings were rather closer to the ends of the piers than was desired, the masonry was cut down, under traffic, and I-beam grillages were placed on top of the piers, with a view of distributing the weight.

The entire work of erection was conducted in a most careful and satisfactory manner, there being no hitch or errors of any moment, either in workmanship or in judgment, and the successful completion of this dangerous piece of erection, which was of an unusual character, perhaps without precedent, is very creditable, not only to the Erection Department of the American Bridge Company, but to Foreman J. W. Sweet, who had direct charge of the forces in the field.

DISCUSSION.

Mr. Hovey. O. E. HOVEY, M. AM. SOC. C. E.—Spans 11 and 12, to be renewed at the same time, could be designed to suit the cantilever erection of Span 11, using the new Span 12 as one anchorage, with a temporary connection to Span 11. Span 10 was already built, and was of modern construction, with eye-bar bottom chords and ordinary top-chord splices, and could not be used as an anchorage without reconstruction, which was too expensive. It was proposed, therefore, to build a pair of temporary outside anchorage trusses for Span 10, with horizontal, stiff, bottom chords and uniformly inclined, stiff, top chords, designed to take anchorage tension; and a web system as light as consistent with safety. After this proposal was made, the Baltimore and Ohio engineers found that a deck bridge of the same length would be required on another location, and the light temporary anchor trusses for Span 10 were abandoned and deck trusses were designed to carry the regular Baltimore and Ohio Railroad loading when erected in final position. These trusses were designed to be erected upside down to serve as temporary anchorages for Span 11, and the necessary modifications in the sections were made. These deck trusses were erected outside of Span 10 and clamped to it, and were attached by temporary connections to Span 11.

The calculations of the relative positions of the old Span 11 and the new Span 11, erected as a cantilever, were quite laborious, and were made in a very satisfactory manner by W. M. White, Assoc. M. Am. Soc. C. E., under charge of the speaker, in the Pencoyd office of the American Bridge Company.

No difficulty was experienced in making the final connections in the field, as ample provision had been made for all necessary adjustments of the cantilevers formed by the two halves of Span 11.

No attempt was made to economize too much in the design of the various adjusting devices, as the experience of undue economy in the design of similar appliances for the erection of a large cantilever bridge several years ago was not satisfactory, and the delay in making connections was no doubt more expensive than complete adjusting devices would have been. The results attained at Benwood justified the expenditure for complete apparatus.

The bottom chord of the long span was stiff throughout. No eye-bars were used in the chord. It was not considered good practice to use a chord made up of a combination of riveted stiff members and eye-bars.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1002.

ROUND-HOUSE FRAMING.*

By R. D. Coombs, Assoc. M. Am. Soc. C. E.

It is not claimed by the writer that the following designs and outlines of specifications for round-houses are entirely new, but, by bringing before the Society what he considers desirable construction, it is hoped that the discussion may be of benefit in future work.

The writer recognises certain apparent inconsistencies: For instance, in the concrete-steel design (Plate XV), there is shown a wooden monitor. This might more appropriately be made of fireproof materials. However, the greater distance from the floor raises the question whether or not such additional expense is warranted. It might be added that the monitor shown should be dowelled or otherwise fastened to the sill girder. The cast-iron door posts, of course, may be made of structural steel design.

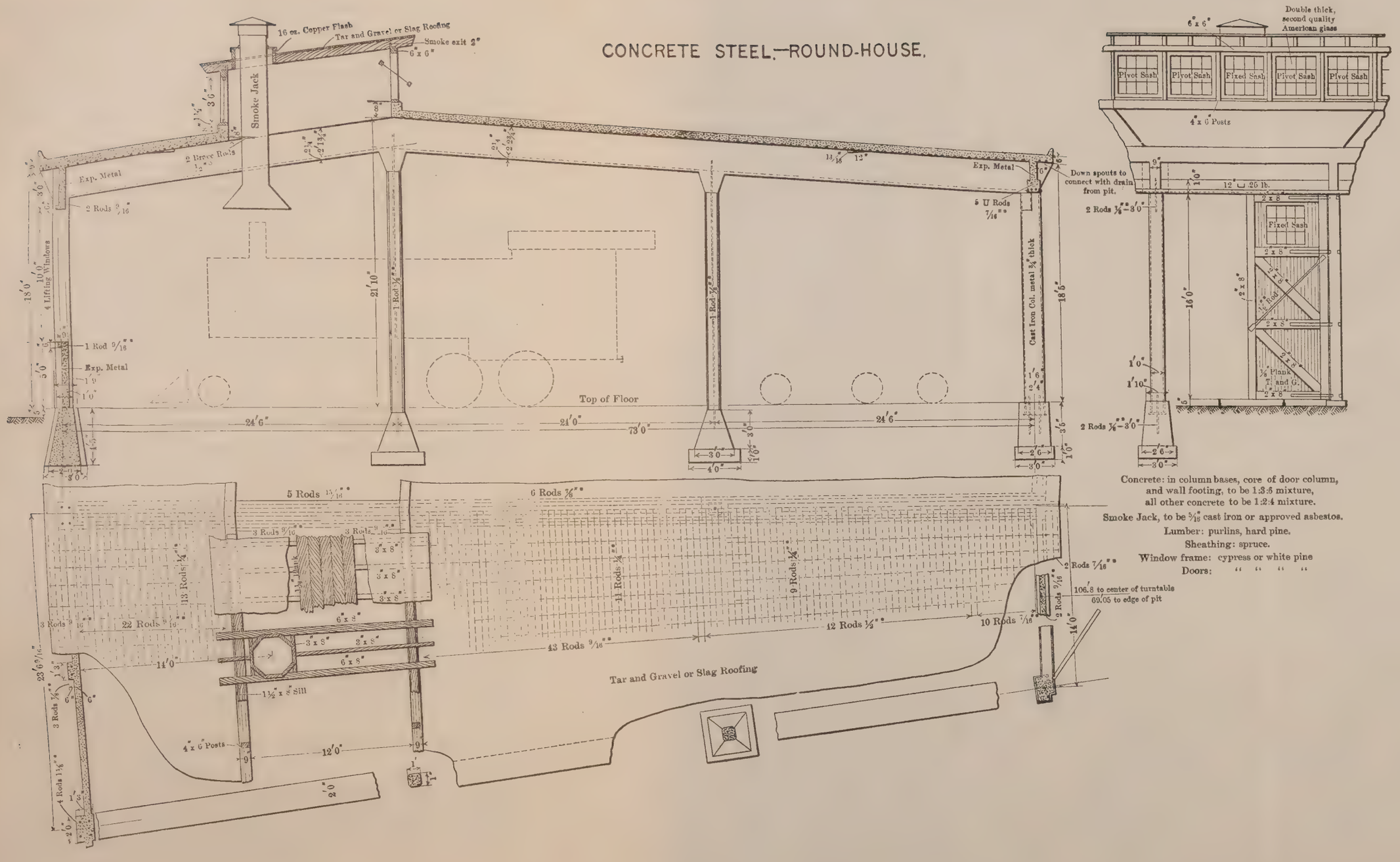
The use of cast-iron door posts is justifiable. They can be made economically of greater thickness, and thus be less affected by corrosion. The brittleness of cast iron occasionally seems to be an actual advantage. An engine recently jumped the track on leaving the turntable, and carried away the cast-iron door post. Although the brick arches fell, the roof merely sagged somewhat, cracking

* Presented at the meeting of May 3d, 1905.

along the lines of the adjacent bays. Had the post been of structural material, particularly if connected with the truss, the two bays of the roof would undoubtedly have been pulled down. Wooden doors seem to the writer to be preferable, both on account of cost and resistance to corrosion, when compared with steel doors of either the rolling or ordinary type. Lifting wooden doors, in themselves, are neater looking contrivances than the old swinging type, and should be subject to fewer collisions. They are more likely to get out of order from minor accidents, however, and require a greater height of building at the interior wall. Within limits, the entrance doors should be of ample width, as they are likely to be injured either by a blow from a damaged cab, or on account of carelessness in fastening.

Apart from the question of fire, wooden round-houses would seem to have an advantage in cost and life over anything except a protected or concrete-steel design. The question of fire may be a very serious one, but appears to be over-rated. Proper pipe connections in cases where the house is constantly in use, should greatly reduce this danger. As will be noted by an examination of the different designs, light and ventilation have been made important factors. In these days a good working light is rightly regarded as of great importance in all shops. Better results from labor, ease of inspection—indeed, less need of inspection—and a superior condition of cleanliness and order, are directly traceable to proper lighting. As a round-house is one of the places peculiarly in need of ventilation, a generous supply of smoke exits is true economy. Any corners in the roof, or purlins running across the slope and thus acting as baffles to the smoke, are very undesirable. The designs shown have been made to eliminate this trouble as far as possible. In the old-fashioned round-houses it is frequently almost impossible to see the roof sheathing on account of the smoke. The condition of light riveted-iron trusses in such an atmosphere should not need demonstration. Under such circumstances, from ten to fifteen years is probably the maximum life of the roof. The writer knows of several cases where trusses under similar conditions were literally falling to pieces at the end of that period. If steel is to be used, beam sections having a greater thickness of material than the small angles frequently used, and which are more readily painted and in-

CONCRETE STEEL.—ROUND-HOUSE.



Concrete: in column bases, core of door column, and wall footing, to be 1:3:6 mixture, all other concrete to be 1:2:4 mixture.

Smoke Jack, to be 5/16" cast iron or approved asbestos.

Lumber: purlins, hard pine.

Sheathing: spruce.

Window frame: cypress or white pine

Doors: " " " "

spected, should be used in the design. In any case, periodical inspection and painting are absolutely necessary.

Smoke-jacks have been constructed of a variety of materials. Wood, cast iron, tile and asbestos have given satisfactory results. Smoke-jacks of thin, rolled plate have a very short life, and, in the writer's estimation, are not worth installing. Wood lasts rather better than might be expected, and, in connection with a fire-proof roof, should prove economical and safe. It is not necessary to sand the interior, though the exterior should be well painted.

Cast iron, if heavy, has a fair length of service. Tile is more expensive, and its weight and liability to break, if detachable, are objectionable features. Asbestos is light in weight and is fire-proof, but is more expensive in first cost.

The smoke-jacks shown on the drawings are not intended to represent any particular type. A telescoping jack, provided with a bell having a diameter of about 4 ft., would be the writer's preference.

Gutters and down-spouts, while necessary in some localities, are often omitted. In any case, thin metal should not be used, as corrosion marks them for an early grave. In cold climates the down-spouts may advantageously be carried down inside the house to connect with the pit drainage system.

In designing a reinforced concrete roof, the question arises, will the gases penetrate hair cracks, and attack the metal? Such cracks, though invisible, will probably be present in any design, unless it is abnormally heavy. The writer is of the opinion that no failure need be apprehended from this cause.

Certain of the large roads are now building round-houses, arranged to serve partly as shops, by installing overhead cranes and hoists. It is suggested that a portable gantry crane might be used in houses where it is not desirable to incur the expense of a regular overhead installation.

The writer submits Fig. 3 as a suggestion for a steel, expanded metal, and concrete design.

The unit prices given in the following tables will vary, of course, in different localities and at different times. Such differences in cost, combined with local conditions, might render one of the more expensive designs in reality economical.

The loading and stress limits used are given in Table 1.

TABLE 1.—Loads and Units.

Dead load = Weight of material.
Wind and snow = 25 lb. per sq. ft.
Wind and snow = 30 lb. per sq. ft. (concrete-steel design).

Unit stresses.	Wood design	Purlins	900 lb.	
		Girders	1 200 "	
	Steel design	Purlins	10 000 "	
		Girder	9 000 "	(longer).
		Girder	9 700 "	(shorter)
	Concrete-steel design	Steel	16 000 "	
		Concrete	800 "	(1 month).

TABLE 2.—Quantities for Round-House of Wooden Design. One Bay. Fig. 1.

Items.	Quantities.	Prices.	Amounts.	Totals.
Roof and Center Columns.				
Monitor sheathing (spruce)	380 ft., B. M.	$35.00	$13.30	
Monitor purlins (pine)	320 " "	40.00	12.80	
Monitor framing (cypress)	345 " "	60.00	20.70	
Roof sheathing (spruce)	1 512 " "	35.00	52.92	
Roof purlins (pine)	2 238 " "	40.00	89.52	
Girders (pine)	675 " "	40.00	27.00	
Columns and caps (pine)	601 " "	40.00	24.04	
Framing, bridging, etc. (spruce)	65 " "	40.00	2.60	
Bolts	70 lb.	0.03	2.10	
Pivot windows (3 ft. 6 in. × 3 ft. 7 in.)	4	4.00	16.00	
Pivot windows (3 ft. 6 in. × 3 ft. 3 in.)	4	4.00	16.00	
Fixed windows (3 ft. 6 in. × 3 ft. 3 in. and 3 ft. 6 in. × 3 ft. 7 in.)	2	2.50	5.00	
Column foundations (concrete)	2.92 cu. yd.	6.00	17.52	
Roofing	1 513 sq. ft.	0.04	60.52	
Smoke-jack			30.00	
Painting	4 200 sq. ft.	0.0225	94.50	
Cast-iron column base	700 lb.	0.0275	19.25	
				$504.77
Walls.				
Brick wall	12.5 cu. yd.	$6.50	$81.25	
Brick arch	1.8 " "	8.00	14.40	
Cast-iron column	3 200 lb.	0.0275	88.00	
Wall foundation	7.2 cu. yd.	6.00	43.20	
Post foundation	1.46 " "	6.00	8.76	
Lifting windows (4 ft. 6 in. × 9 ft. 6 in.)	2	10.00	20.00	
Window framing (cypress)	200 ft., B. M.	60.00	12.00	
Double door			50.00	
				317.61
Total				$822.38

Painting of sash and door included in cost of same.

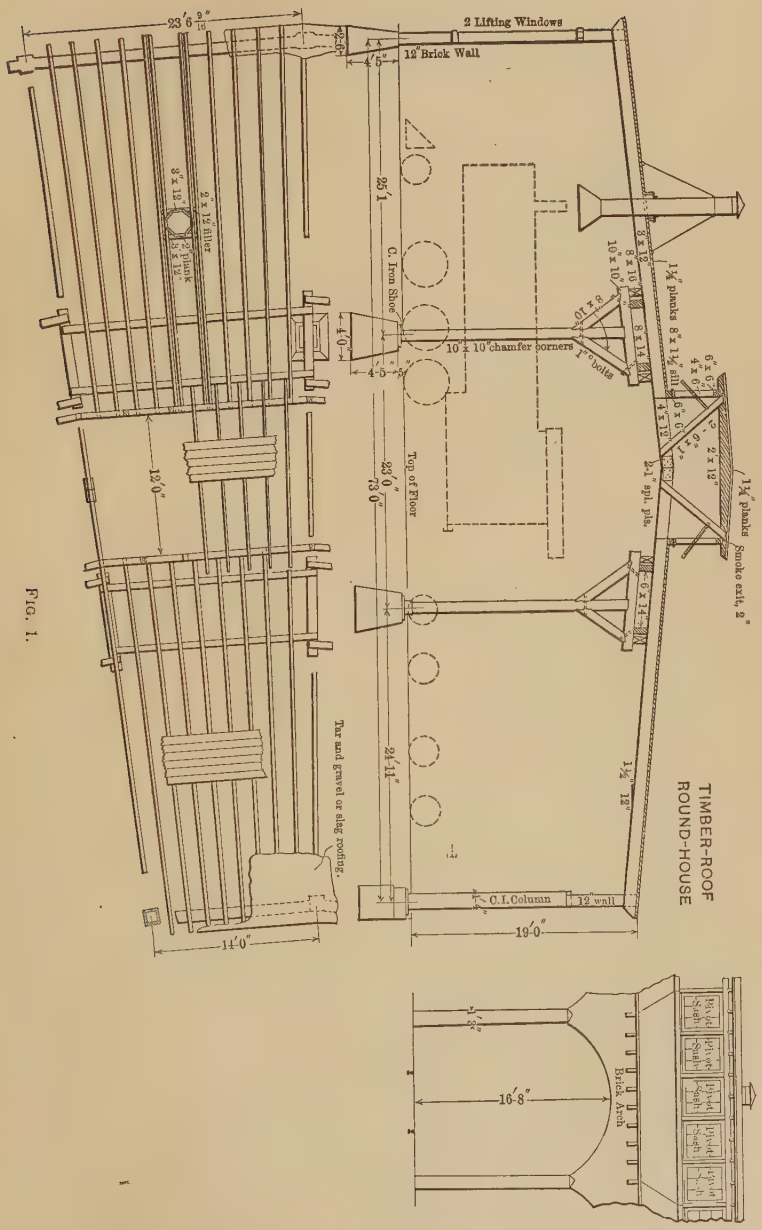

2 Lifting Windows
12" Brick Wall
C. Iron Shoe
10" x 10" chamfer corners
1" bolts
Top of Floor
Smoke exit, 2"
Tar and gravel or slag roofing.
C.I. Column
12" wall
Brick Arch
TIMBER-ROOF ROUND-HOUSE

FIG. 1.

TABLE 3.—Quantities for Round-House of Steel Design. One Bay. Fig. 2.

Items.	Quantities.	Prices.	Amounts.	Totals.
Roof and Center Columns.				
Monitor sheathing (spruce)	380 ft., B. M.	$35.00	$13.30	
Monitor purlins (pine)	320 " "	40.00	12.80	
Monitor framing (cypress)	345 " "	60.00	20.70	
Roof sheathing (spruce)	2 330 " "	35.00	81.55	
Nailing strips (spruce)	135 " "	40.00	5.40	
Steel columns 1 550 lb.				
Purlins 7 600 "				
Girders 1 900 "				
Knees, etc 450 "				
Bolts and fillers 100 "				
	11 600 lb.	0.03	348.00	
Pivot windows (3 ft. 6 in. × 3 ft. 7 in.)	4	4.00	16.00	
Pivot windows (3 ft. 6 in. × 3 ft. 3 in.)	4	4.00	16.00	
Fixed windows (3 ft. 6 in. × 3 ft. 3 in. and 3 ft. 6 in. × 3 ft. 7 in.)	2	2.50	5.00	
Column foundations. Concrete	2.26 cu. yd.	6.00	13.56	
Cap	0.14 " "	10.00	1.40	
Roofing	1 470 sq. ft.	0.04	58.80	
Smoke-jack			30.00	
Painting. Steel	1 250 sq. ft.	0.01	12.50	
Wood	1 900 " "	0.0225	42.75	
				$677.76
Walls.				
Brick wall	12.5 cu. yd.	$6.50	$81.25	
Brick arch	1.8 " "	8.00	14.40	
Cast-iron columns	3 100 lb.	0.0275	85.25	
Wall foundation	7.2 cu. yd.	6.00	43.20	
Post. Concrete	1.0 " "	6.00	6.00	
Cap	0.25 " "	10.00	2.50	
Lifting windows (4 ft. 6 in. × 9 ft. 6 in.)	2	10.00	20.00	
Window framing (cypress)	200 ft., B. M.	60.00	12.00	
Double door			50.00	
				314.60
Total				$992.36

Specifications for Round-House of Wooden Design.

Foundation.—The foundation shall be of concrete, masonry, or brick, to suit local conditions.

Brick Walls.—The brick shall be good, sound, hard-burned brick, well laid and bonded. The mortar shall be of Portland cement and sand in the proportions of 1 to 2.

Lumber.—All lumber shall be of the best quality, thoroughly seasoned and surfaced on all sides.

Sheathing .....Matched spruce.
Cross-bridging .....Spruce.
Window framing.....Cypress or white pine.
Sash ..... " " " "
Doors ..... " " " "
All other lumber .....Long-leaf hard pine.

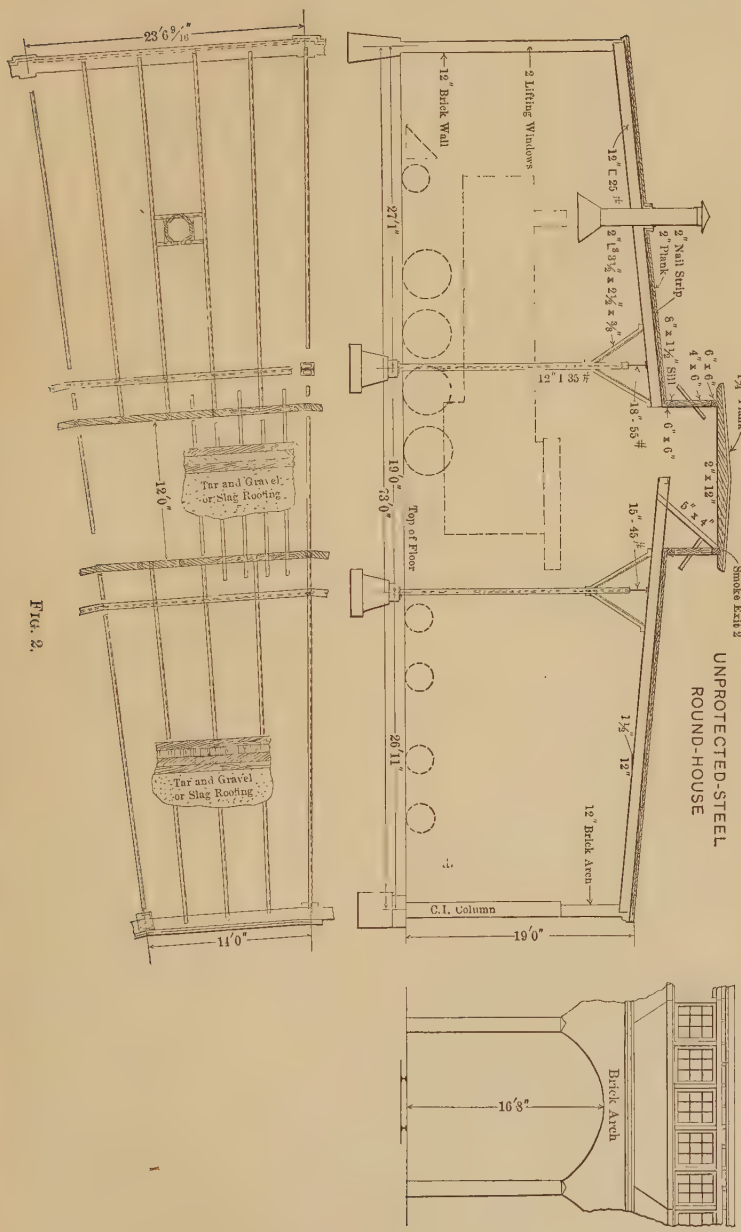

UNPROTECTED-STEEL ROUND-HOUSE
2 Lifting Windows
12" Brick Wall
12" C 25 #
2" Nail Strip
2" Plank
2 L 3½" x 2½" x ⅜"
8" x 1½" Sill
6" x 6"
4" x 6"
12" I 35 #
18" 55 #
6" x 6"
1¼" Plank
2 x 12"
Smoke Exit 2"
15" 45 #
Top of Floor
27'1"
19'0"
23'0"
12'0"
Tar and Gravel or Slag Roofing
Tar and Gravel or Slag Roofing
23'6 9/10"
11'0"
26'11"
1½"
12"
12" Brick Arch
C.I. Column
19'0"
Brick Arch
16'8"

FIG. 2.

TABLE 4.—Quantities for Round-House of Concrete-Steel Design. One Bay. Plate XV.

Items.	Quantities.	Prices.	Amounts.	Totals.
Roof and Center Columns.				
Rods	3 770 lb.	$0.03	$113.10	
Concrete (superstructure)	42.59 cu. yd.	15.00	638.88	
Concrete (column bases)	2.3 " "	6.00	13.80	
Monitor purlins (pine)	410 ft., B. M.	40.00	16.40	
Monitor sheathing (spruce)	420 " "	35.00	14.70	
Monitor frame (cypress)	280 " "	60.00	16.80	
Pivot windows (3 ft. 6 in. × 3 ft. 8 in.)	4	4.00	16.00	
Pivot windows (3 ft. 6 in. × 4 ft. 0 in.)	4	4.00	16.00	
Fixed windows (3 ft. 6 in. × 4 ft. 0 in. and 3 ft. 6 in. × 3 ft. 8 in.)	2	2.50	5.00	
Roofing	1 440 sq. ft.	0.04	57.60	
Gutter	38 lin. ft.	0.16	6.08	
Down-spout	18 " "	0.30	5.40	
Smoke-jack			30.00	
Painting	700 sq. ft.	0.0225	15.75	$965.51
Walls.				
Rods	640 lb.	$0.03	$19.20	
Channels	350 "	0.03	10.50	
Cast-iron column	2 330 "	0.0275	64.07	
Expanded metal (No. 10—6-in. mesh)	215 sq. ft.	0.027	5.80	
Concrete (superstructure)	642 cu. yd.	15.00	96.30	
Concrete (foundations)	7.09 " "	6.00	46.98	
Concrete (core of door post)	0.74 " "			
Lifting windows (4 ft. 6 in. × 9 ft. 6 in.)	4	10.00	40.00	
Window frame (cypress)	400 ft., B. M.	60.00	24.00	
Double door (12 ft. 8 in × 16 ft.)			40.00	
				346.85
Total				$1 312.36

Lintels and Caps.—Lintels and sills for wall windows, and cap stones for columns and door posts shall be of granite (or other suitable stone found in the locality), dressed even, and all exposed surfaces shall be six-cut.

Windows.—The windows shall have box frames. Both sashes shall be hung with best flax sash-cord, steel axle pulleys and iron counterweights; and shall be provided with strong sash locks. The glass shall be of second-quality, double-thick, American, thoroughly bedded, bradded and back-puttied. The pivot windows shall be provided with suitable stops, steel axles and approved operating device, and shall operate from the floor of the house.

Doors.—Each half door shall have four wrought-iron hinges, and one bolt and chain to the pair; door-stop and drop-bolt at the bottom properly fastened to the floor; and strong thumb-latch. A latching post shall be set in front of each door post, and suitable

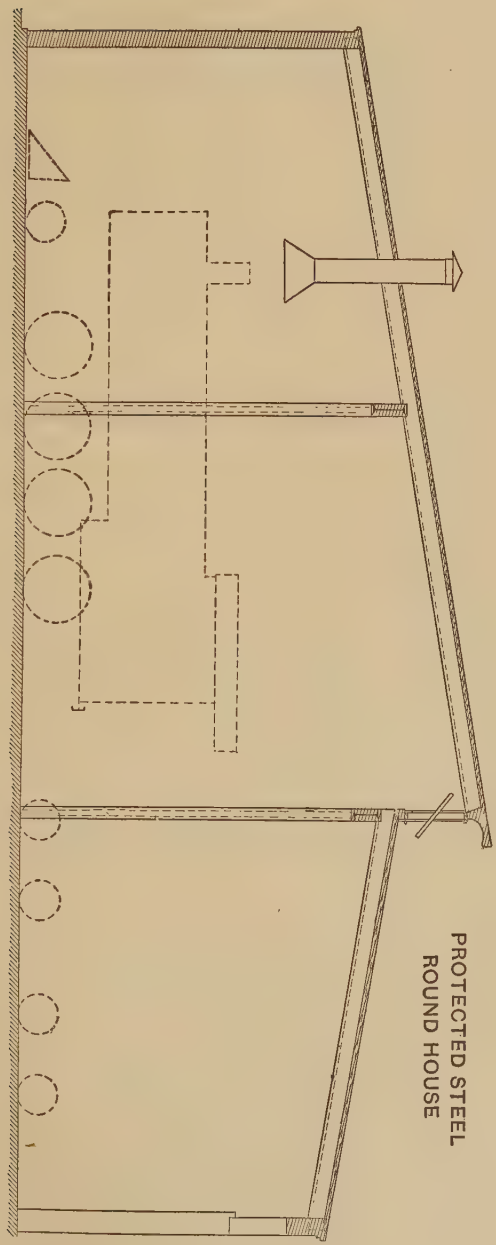


FIG. 3.

heavy hooks provided to hold the doors open. Certain specified doors shall have small entrance doors opening outward.

Paint.—All nail and screw heads shall be punched and puttied before painting, and all bolt-head countersinks shall be swabbed with tar. All woodwork shall receive two good coats of white lead and oil.

Roofing.—The roofing shall be of three thicknesses of tarred felt, laid over resin-sized sheathing paper, and covered with a uniform thickness of pitch and slag.

SPECIFICATIONS FOR ROUND-HOUSE OF CONCRETE-STEEL DESIGN.

Steel.—The steel shall be open-hearth.

Ultimate tension	62 000 to 68 000 lb.
Elastic limit	55% of ultimate.
Elongation	22% in 8 in.
Sulphur	0.05 per cent.
Phosphorus	0.06 " " (Acid.)
Phosphorus	0.04 " " (Basic.)

The steel shall be free from oil, dirt and rust. Care shall be taken to obtain a good contact between the steel and the concrete; and the position of the rods, as shown on the drawings, shall be carefully maintained. The coarser material in the concrete shall be excluded from under the rods.

Concrete.—The concrete shall be mixed in the following proportions: 1 part of Portland cement, 2 parts of sand and 4 parts of stone.

Cement.—******, or other approved brand of Portland cement shall be used.

Fineness: Residue by weight on No. 100 sieve, not more than 5 per cent.

Residue by weight on No. 200 sieve, not more than 25 per cent.

Initial set: Not less than 30 minutes.

Tensile strength of 1 sq. in., neat:

1 day	200 to 300 lb.
7 days	500 to 600 "
28 "	600 to 700 "

The cement, otherwise, shall be in conformity with the specifications, for Portland cement, of the American Society for Testing Materials. The tests will be made in conformity with the standards recommended by the Special Committee on Uniform Tests of Cement, of this Society.

Sand.—The sand shall be clean, coarse and sharp, and free from silt or loam.

Stone.—All stone shall be hard, sound limestone, free from dirt or dust. In the superstructure no stone shall be more than ¾ in. in any dimension; and, in the foundation, no stone shall be more than 2½ in. in any dimension.

Mixing.—The concrete shall be thoroughly mixed in the specified proportions immediately before using, and any which has begun to set shall not be used. The stone shall be thoroughly wetted before mixing. The sand and cement shall be well mixed before adding the stone and water, and the whole shall be mixed again until the stone is thoroughly coated. The consistency of the concrete shall be such that water rises to the surface without ramming, and the mass "quakes" when additional quantities are thrown in.

Forms.—The supporting forms shall be practically unyielding under loads or ramming. Facing planks shall be dressed on one side and two edges, and closely jointed. The forms shall be left in place for four weeks, and after removal the exposed surfaces of the structure shall be rubbed down or trowelled with cement mortar to remove irregularities. The lagging shall be sprinkled at intervals to prevent shrinkage.

Laying Concrete.—The concrete shall be laid in 6-in. layers (except as hereinafter specified), and rammed. The coarser material shall be worked back from the forms so as to bring an excess of mortar to the face. Each column, girder, sill and lintel shall be laid continuously, *i. e.*, on each of these, work shall not be stopped until it is completed. The roof slabs shall be laid in one layer and continuously from girder to girder. On laying new work adjoining that which has already set, the latter shall be cleaned and wetted and cement-mortar placed between. To join girders or walls already set with roof slabs, they shall be left rough to form a bond with the slabs. No concrete in the superstructure shall be laid in freezing weather. Finished surfaces shall be sprinkled daily for a

week, and kept covered with straw or canvas for three weeks. Completed portions shall not be loaded with materials or used as walks.

Expansion.—Over every fourth girder there shall be a halved joint in the roof slab, separated by two thicknesses of heavy roofing felt.

Roofing.—The surface of the roof shall be well swabbed with coal-tar pitch, and covered with three layers of roofing felt laid with overlapping joints. Each layer shall be swabbed with pitch, and over the top surface thus made there shall be spread a uniform coating of pitch and slag (or gravel).

Specification for Concrete Floor.

After excavating to the required depth, the subsoil shall be thoroughly compacted with heavy rammers. On this surface 4 in. of 1:3:6 concrete shall be laid and also an additional 2 in. of wearing surface composed of 1:2:4 concrete, finished with a 1 to 1 Portland cement and sand surface. These layers shall be laid as one layer and given a 2-in. crown between the pits.

Specification for Brick Floor.

After excavating to the required depth, the subsoil shall be thoroughly compacted with heavy rammers to a practically level surface. On this shall be laid a 4-in. bed of good "dead end" cinders, rolled with a roller weighing not less than one ton. Above this shall be placed 2 in. of sand thoroughly rolled. On this surface, which shall be given a 2-in. crown between the pits, shall be laid the brick floor. The brick shall be the best quality of hard-burned, machine-pressed, paving brick, laid closely together, on edge, breaking joints.

The floor shall be given a final rolling to an even surface, and the joints shall be filled with 1 to 1 Portland cement mortar.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1003.

A FEW REMARKS ON FOUNDATIONS.*

By L. L. Buck, M. Am. Soc. C. E.

In studying foundations for heavy concentrated loads, the writer has observed the effects of such loads upon existing structures. In such cases, where defects soon appeared, one could easily trace their causes. Where such defects arise from the use of poor stone or mortar, they are not worth considering; but there are many cases, where such defects appear, in which both stone and mortar seem to be excellent. In such cases it is evident that the question of distribution must be examined.

Sometimes a heavy and heavily loaded column is found resting on a foot-box made up of plates riveted to the bottom of the columns with angles riveted to the lower edges of these plates, and then a horizontal plate having an area of seven or eight times the area of the cross-section of the column, and supposed to distribute the pressure over the area covered by the plate. This is an extreme case, and does not occur as often of late years as it did earlier; but there have been cases, within a few years, where a heavily loaded column rested on a casting so shallow that it could not

* Presented at the meeting of May 3d, 1905.

nearly distribute its load over the whole area of the base of the casting.

The writer has in mind one case in which a heavy column rested on a casting, and the casting rested on a large nest of rollers. The outer ends of all the rollers and the outer rollers at each end of the bed were not borne upon at all. It is not easy to distribute enormous concentration of weight over a sufficiently large area to give the proper unit pressure over the whole area; but this should be done, and it requires greater height in proportion to the base of the bed than is in many cases given.

Having properly proportioned the foot of the column, attention must next be given to the masonry pedestal on which it is to rest. Supposing the bed-plate to distribute practically its whole load over its entire area equally. The question will arise as to how far outside of the bed-plate the stone of the pedestal can project (not only advantageously, but with safety), in the top course, below the coping, and how far it can go without absolute detriment to the work. If it is extended more than about 2 in. per foot of thickness of the course, no perceptible advantage will be gained. Of course, it must project sufficiently to keep the pressure per square foot within proper limits, including the weight of the masonry, and the wind pressure, and, if it is to rest on concrete, to keep the pressure on the concrete within proper limits.

The difficulty, with trying to extend the masonry too far beyond the bed-plate, is that the stone may crack, and, if so, it will start under the edge of the bed-plate, and the crack may incline still further under the bed-plate at the bottom of the course. Moreover, with too large a mass of masonry, there is a tendency to put the best masonry outside, and a poorer quality within. In this case the poorest masonry sustains the load. This is similar to making the pedestal of a bag filled with sand. In this case the bag will be pretty sure to burst, whether made of cloth or masonry.

Fig. 1 represents two courses made of cut stone, with $\frac{1}{2}$-in. joints. This form secures excellent bond, and from it the writer has obtained excellent results.

Where four or more heavily laden columns rest on one foundation, the writer thinks it is best to have the center of pressure on

each column so nearly placed that the distance between each pair of columns, each way, shall be about equal to the sum of the distances of the extension of the masonry outside of the columns, and then build the pier in such a manner that there shall be a pedestal of first-class masonry under each column, similar to that of the single-column foundation. But the pedestals must be bonded to whatever masonry is outside of or between these pedestals.

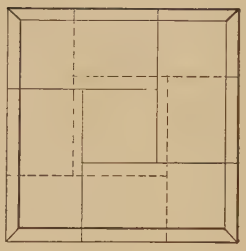


FIG. 1.

In some cases, where the load on one column is very great, a casting, in order to distribute its load properly over the whole area of its base, must either be as high as the length of the base, or the

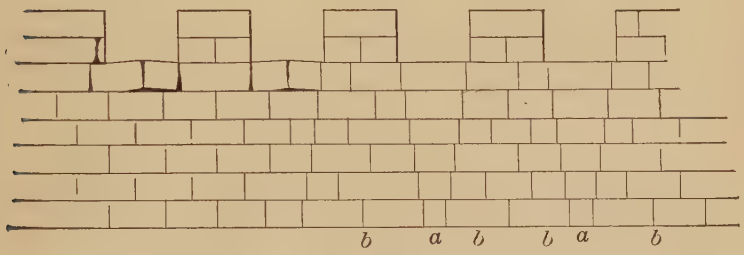


FIG. 2.

casting must be in several pieces bolted together for convenience in handling. In this case the bottom length of the column can be given such a taper that its lower end will cover a space equal to half the area of the base of the casting, when the casting will require only about half the height required in the other case. Of course, the casting, in both cases, must be thoroughly ribbed. There are other ways of accomplishing this result, but it is doubtful if, on the score of economy, there are any better.

In the case of heavy walls having frequent window openings, one will sometimes find them showing disagreeable cracks, similar to those indicated in Fig. 2.

This trouble might have been avoided by a form of construction similar to the right-hand portion of the diagram. Construct the

area of *b* so that the area of its base, in proportion to that of *a* shall be equal to that of the weights they respectively sustain.

The whole idea is to use the pedestal form, and not to project the stones under the heavily loaded portions more than the transverse strength of the stone will safely sustain.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
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TRANSACTIONS.

Paper No. 1004.

THE IRRIGATION SYSTEM OF ONTARIO, CALIFORNIA—ITS DEVELOPMENT AND COST.*

By F. E. Trask, M. Am. Soc. C. E.

With Discussion by Messrs. Arthur S. Hobby and F. E. Trask.

This paper treats of the development and present status of an irrigation plant in the "Orange Belt" of Southern California, and gives the details and cost of a system which has been designed to meet exacting demands for "high duty and maximum utility." The results accomplished have been so satisfactory and pronounced that the United State Geological Survey projected this system in detail upon a relief map, for the purpose of exhibition at the St. Louis Fair; and the writer presents a photograph (Plate XVI) of the model, in lieu of a map of the tract described in this paper, believing that it better portrays the controlling conditions in the development of this system.

The highest point shown on the relief map is Mount San Antonio, with an elevation of 10 080 ft., the south side of which is in the catchment area of the San Antonio Cañon. This cañon has an area of about 23 sq. miles, with an elevation of 2 200 ft. at the mouth, where it debouches upon the plain. From the mountains the plain slopes to the south, and, at the lower limit of the colony, 8 miles from

* Presented at the meeting of May 17th, 1905.

the cañon, has an elevation of 800 ft. The precipitous character of the San Antonio Cañon, together with its water supply, have made the development and use of water power an important adjunct and factor in the design and consummation of this irrigation system.

On Plate XVI the points, *A*, *B* and *C*, show the locations of Power Plants Nos. 1, 3 and 2, respectively; *D* and *E* indicate the locations of underground water supplies which are pumped from gravel beds; *F* marks the position of the lighting sub-station; and *x* and *y* represent the locations of the San Antonio and Cucamonga tunnel developments, respectively.

Location.—The Town of Ontario is located in San Bernardino County, California, 39 miles east of the City of Los Angeles, and 19 miles west of the county seat. It extends from the Sierra Madre Mountains on the north, from 7 to 8 miles southward, and has an average width of about 3 miles, with a population of 4 000. The colony was surveyed and opened to settlement in 1883, and its principal industry, to which it owes its development and present success, is the production of citrus fruits. Just as good soils are to be found both east and west, but, without the water supply, which originally consisted of one source, the San Antonio Cañon, the place would not now exist.

Control and Management.—The ownership of the system, from its inception, has been in the San Antonio Water Company, a corporation; and, excepting the initial work, it has been constructed, extended and developed by this company to meet the present exacting requirements of the stockholders. The San Antonio Water Company is a mutual company, one of the first organized in California, and, as such, has been an object lesson and pattern to which more recent communities have looked, and from the experience of which they have profited. Its stock is owned by the owners of the realty. It has no right to supply other than its stockholders, and its business is conducted solely for them, there being no dividends or other profits accruing to the stockholders except such profit as they realize from the use of the water on their lands.

In 1902 it became necessary to acquire additional lands and water rights and to construct an electric plant. For this purpose a subsidiary company was formed, *viz.*, the Ontario Power Company, the stock of which is held by the San Antonio Water Company. In

RELIEF MODEL OF ONTARIO, CAL.

this paper, however, the writer will speak of the entire enterprise as the property of the latter company.

Water Supply.—The original supply was by gravity, being one-half the discharge from San Antonio Cañon. This was first increased by tunnel developments, and lastly by pumping from artesian sources and saturated gravel beds. A twenty-year July average gives about 700 miners' in. as the mean run-off from this cañon. The Ontario miners' inch represents $\frac{1}{50}$ cu. ft. per second—continuous flow—and the average duty of this water is 1 in. to 7 acres.

In the early days of the colony, the promoters discovered that there would be insufficient surface water from the San Antonio Cañon to supply all the requirements of their enterprise, and began the construction of what they claimed would be a bed-rock tunnel under the wash of the San Antonio Cañon at or near its mouth. This tunnel has a cross-section of about $3\frac{1}{2}$ by $6\frac{1}{2}$ ft. and a length of about 3 000 ft. The upper 600 ft. penetrated bed-rock below the wash material of the creek, the bottom of the tunnel being about 110 ft. below the creek bed. The cost was about $50 000, and records for the past fifteen years show an average July output of 116 miners' in.

Fig. 1, Plate XVII, shows the mouth of the tunnel and the measuring weir. At the time the picture was taken there were about 150 miners' in. discharging from the tunnel. Later, the company acquired another tunnel, east of the colony, known as the "Cucamonga" tunnel, which has a discharge record of about 200 miners' in. While ample for all demands in years of average rainfall, this gravity supply was insufficient for years of drought. To meet the needs of dry years large tracts of water-bearing lands were purchased, both east and west of the colony, at Claremont and Cucamonga. Fifteen wells have been sunk, and pumping plants installed, with the result that the company is able, at all times. to supply the demands of the stockholders.

Pipe System.—The whole colony is piped throughout, the water company delivering water to domestic users through iron pipes; or, in the case of subsidiary companies, to some point sufficiently high to give domestic pressure; and to irrigators, at the highest corner of each 10-acre tract. The whole system consists principally of cement concrete pipe and vitrified clay pipe, ranging in diameter

from 8 to 40 in. The system is complete and economic in that the loss of water in distribution is practically nothing. It is fitted with measuring weirs, division boxes and turn-out boxes built of concrete and so designed and arranged as to permit of rapid and complete control and distribution of the waters from the several sources from which the company draws its total supply.

Four large mains deliver to the colony the water from the four sources, *viz.*, San Antonio Cañon, Claremont, Cucamonga tunnel and Cucamonga gravel beds. Supplemental distributing and diagonal mains have been laid throughout the colony at different points, and in such a manner that those in control can readily shift the supply from different sources to different sections of the colony, thereby insuring an equal subdivision of the water to the various stockholders. Lateral pipe lines, mostly 8 in. in diameter, run north and south on lot lines, ⅛ mile apart, throughout the colony, delivering to irrigators a "30-in. head" every 30 days. The mileage and cost of the irrigation system, in detail, are shown in Table 1.

TABLE 1.—Mileage and Cost of the Irrigation System in Detail.

San Antonio tunnel	1.00 mile.	$50 000
Cañon ditch	0.60 "	6 000
" tunnel	0.10 "	1 000
16-in. cement conduit	0.62 "	1 980
3 x 4-ft. concrete conduit	0.61 "	6 480
30-in. cement pipe	0.57 "	4 500
24 " " "	0.80 "	4 725
22 " " "	1.00 "	4 752
20 " vitrified pipe	1.13 "	4 800
18 " " "	4.09 "	16 200
18 " cement "	0.40 "	1 470
16 " vitrified "	3.40 "	12 600
14 " " "	5.32 "	16 124
12 " " "	2.85 "	6 900
12 " cement "	0.91 "	2 208
10 " " "	9.52 "	18 105
10 " vitrified "	5.55 "	11 134
10 " sheet-steel "	1.00 "	2 112
8 " vitrified "	16.20 "	25 680
8 " cement "	30.07 "	44 464
7 " " "	0.62 "	660
Weir and division boxes, gates, etc		25 000
Totals	86.36 miles.	$266 894

Hydro-Electric Plant.—Several years of expensive steam pumping influenced the San Antonio Water Company in a determination to develop its own water power in the cañon, not only to supply its own needs, but for sale and profit. This plant, completed in 1902,

FIG. 1. MOUTH OF SAN ANTONIO TUNNEL.

FIG. 2.—POWER PLANT NO. 1, TAIL-RACE AND INTAKE TO PLANT NO. 2.

is now supplying all lighting demands of the colony, pumping from the company's wells, and contributing a considerable surplus to adjoining sections for various purposes. This is the third plant to utilize the San Antonio Cañon water for power, and it has proved a good investment.

Fig. 1, Plate XVIII, and Figs. 1 and 2, Plate XIX, show this plant; Figs. 1 and 2, Plate XX, show the sub-stations; Fig. 2, Plate XVIII, shows the laying of the pressure pipe; Fig. 2, Plate XVII, shows Power Plant No. 1, with its tail-race and the intake to Plant No. 3, or the San Antonio Water Company plant; and Figs. 1 and 2, Plate XXI, show Plant No. 2. Plants Nos. 1 and 2 are the property of the Pacific Light and Power Company, of Los Angeles, and are mentioned here solely because they are factors in developing the maximum utility of the water supply of this system.

Plant No. 3, as the property of this water company, became the important factor in the perfection of this irrigation system. It rounded out the completed design to such a degree that the management is able—at a nominal cost—to maintain a constant and ample supply of water for all consumers, notwithstanding the great fluctuations in the annual rainfall. It is located at *B*, on Plate XVI, and diverts the water from the creek at *A*, into and through a 30-in. gravity conduit. The principal part of this conduit is cement concrete pipe. A portion of it is laid through tunnels, the remainder in a trench around the mountain side, with a part of the line, composed of steel inverted siphons, crossing side cañons. The wheels at the power-house have a working head of 700 ft., the water being delivered from the end of the gravity line, northwest of the power-house, through a steel pressure line, part of which is composed of riveted steel, the remainder or lower end being of 20-in. lap-welded tubing. The pressure lines were designed with a factor of safety of 5, and, after two years' service, are in perfect condition.

The equipment in the station consists of three direct-connected units, each composed of a 5-ft. Doble wheel and a 250- k. w. General Electric generator. The units are similar, and each is controlled by a Lombard governor. Electricity is generated at 11 500 volts and is transmitted direct to the sub-stations at this voltage. The station building is of stone, with a steel roof, and is supplied with a traveling crane. The equipment is the latest and best put out by

the General Electric Company and the Abner Doble Company, who, together, supplied the entire installation. The normal output of this plant is 750 k. w., which, at times, is increased by over-loading.

The writer made tests of these units, and found the three to have an average efficiency of 77.7% at the switch-board, the efficiency of the Doble wheel being 83.6 per cent.

Near the station the company built two stone cottages for employees, and in the rear there is a shop for repairs and experimental work.

The sub-stations are substantially built of wash boulders and neatly and thoroughly equipped with switch-boards, transformers and accessories. Pumping operations are directed from the upper station, one man being sufficient to operate as many as eight wells within a radius of a mile. The well equipment consists of an induction motor, belted to a centrifugal pump, and, when properly adjusted, it requires very little attention from the operator. The cost of the electric plant was as follows:

Power-house and two cottages	$ 5 647
Gravity conduit, 30-in	63 000
Pressure pipe line	14 863
Three Doble 5-ft. wheels and accessories	8 218
Power-house wiring, etc	1 873
Three alternating, 3-phase, revolving-field, 16-pole, 250-k. w. generators, with switch-boards, transformers and all accessories, complete in place	25 686
Lighting sub-station, building and equipment	10 814
Pumping " " " " "	6 275
Distributing system, motors, meters, lines, etc	55 713
Total	$192 093

Conclusion.—The natural conditions at Ontario made it possible to develop this irrigation system with a high degree of economy and a maximum utility. The small surface stream of the mountains first develops electrical energy to pump additional water from the gravel beds below, to light and to heat the homes of the people, and to propel the street car lines of the colony; second, it irrigates the

FIG. 1.—GENERAL VIEW OF LOCATION OF POWER-HOUSE.

FIG. 2.—LAYING PRESSURE PIPE.

orchards lying on the foothills and above the gravel beds of the pumped water supply; and, third, that portion of the water not consumed by evaporation and plant life sinks into the gravel beds underlying these orchards—replenishing the storage there—to be pumped to the surface and used again for irrigating orchards at a lower level. The conduit system, being closed and adequately provided with division weirs, measuring boxes and gates, delivers the supply to consumers without loss.

The initial work on this system was under Messrs. Chaffee and Dunlap, engineers, who were interested with the promoters of the enterprise. In 1887 the writer took charge of the works, designing, remodeling and extending them from time to time, as the increasing needs of consumers dictated.

W. H. Sanders, M. Am. Soc. C. E., was engineer for Electric Plants Nos. 1 and 3, the writer acting in the same capacity for Electric Plant No. 2.

DISCUSSION.

Mr. Hobby. ARTHUR S. HOBBY, ASSOC. M. AM. SOC. C. E. (by letter).—In the general layout of the Ontario Colony, the promoters divided the land into farms, villa plots and a town site, the latter comprising about 160 acres of land lying in a square on the north side of the Southern Pacific Railroad. Each acre of the original colony lands was allotted one share of stock in the San Antonio Water Company, so that the water supply for the town was derived from its 160 shares of water company stock, which, under the original allotment, was supposed to represent 16 miners' in. of water.

As a general rule, improved conditions result from strife, and Ontario was no exception. The Ontario Land Company, successors to the original promoters, laid out a town site on the south side of the railroad, and opposite the original town, and proceeded to boom it, with a view to swinging the business center to that side, where the land company still owned practically all the land, one of the arguments used being that there was a more abundant water supply on the "South Side," as it was locally called (and which designation will be used here to distinguish it from the "Town" to the north of the railroad).

These two sections received their water through a common pipe line running to a point about 2 miles north of the "Town" where it was separated and delivered into their respective private lines.

In the spring of 1894 the Town Trustees engaged the writer to design a more equitable method of securing for the "Town" its share of the water, as they considered that it was not then getting its full share. Upon investigation, it was found that at the point of separation the combined water was run into a concrete chamber which was arranged at one end with two submerged weirs or apertures conforming in area to the number of miners' inches each was supposed to deliver (under a 4-in. head above the center of the aperture) to its respective pipe line. The weir for the "Town" was 2 by 8 in. and, if the writer remembers, the "South Side" weir was 2 by 4 in.; however, that is immaterial. To maintain this 4-in. head, an open weir was built all along one side of the main chamber and having a length of 3 ft. or more, while its crest was 4 in. above the center of the above-mentioned apertures, so that any increase in flow tending to raise the head in the chamber would waste over this long weir without materially affecting the head and consequent pressure over the submerged apertures. The flow over this long waste weir tailed into the "South Side" pipe line, so that, whenever there was an increase of flow, the "South Side" received all the benefit, while if there was a decrease, so that the head in the chamber fell below the normal 4 in., the "Town" stood its share of the shrink-

FIG. 1.—INTERIOR OF POWER-HOUSE,
SHOWING TWO OF THE THREE GENERATOR SETS OF PLANT NO. 3.

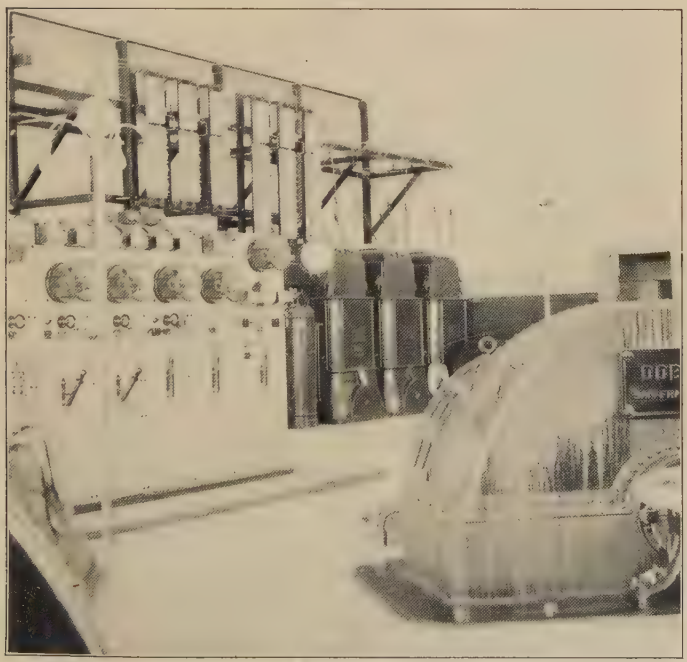

FIG. 2.—INTERIOR OF POWER-HOUSE.
SHOWING SWITCH-BOARD AND TRANSFORMERS OF PLANT NO. 3.

age. The writer reconstructed the box of concrete, making it about 5 by 10 ft., and provided effective baffles to bring the water to a state of rest before passing it over two open weirs with their crests at the same elevation, while, on the up-stream side of the weirs, sliding gates of zinc were placed, having the edges that formed the movable ends of the weirs brought up and turned over the rail at the top of the weirs as an index finger. A table of discharges for varying lengths and heads was tacked up inside one of the lids of the box. The variable-length weir was adopted for the reason that it was evident that either party might desire to rent or lease additional water at times, thereby altering the ratios of division, while any fluctuation in flow, either increase or decrease, would be shared by each in just proportion. As far as the writer is aware, that was the first division device comprising those features attempted anywhere in that section of the country. The zinc sliding gates leaked to a certain extent, as was to be expected, but, as the leakage was practically the same into each pipe line, it caused no trouble, and the device was so satisfactory on the whole that Mr. Trask constructed one on about the same lines for the water company, at a point farther up the colony. Mr. Hobby.

Shortly afterward, the writer was engaged by the water company to design and construct a division and measuring device at a point where various mains diverged from a trunk line, and where it was necessary to run varying quantities of water through any of the lines according to the requirements of that day's service. This was made practically as the box described, only with more weirs, and these were improved upon by using tightly fitted wooden gates, each with a zinc edge forming the end of the weir, thus practically eliminating leakage, while on the rail at the top of the weirs was a zinc scale, computed and marked to show the number of miners' inches passing for given heads for which the weir might be set. Another scale was fixed in the main chamber, indicating the head. These devices proved so satisfactory that a large number of them were constructed throughout the colony by Mr. Trask during the year. Until their adoption, the water had been apportioned to the various mains and consumers by the man in charge of that work pulling up a gate here and there (which gates were not visible, as only the stem projected above the ground) until he guessed that about the right quantity of water was passing; but that first little division box, which grew out of the strife between the "Town" and the "South Side," demonstrated to the consumers that their water could be accurately measured to them at a not unreasonable cost, and they were not slow to develop the system accordingly.

F. E. Trask, M. Am. Soc. C. E. (by letter).—In mentioning the utility of this system, the writer overlooked one novel feature which is worthy of note. Electric Plant No. 2, which supplies the power Mr. Trask.

Mr. Trask. for the electric railway system, is at times operated solely on the developed water from the San Antonio tunnel. This was true for some months of the season, 1904-05, during which time the discharge from this tunnel was in excess of 200 miners' in. At all times the output of this tunnel is utilized for power purposes, and, for years of minimum surface, or creek, supply, the product of the tunnel has been a valuable adjunct to the power interests, as well as to the water consumers.

The subject of an adjustable weir crest is not a new one, and Mr. Hobby was not the first to introduce it in the Ontario system. The writer used a slide gate, similar to that used by Mr. Carpenter and others, back in the Eighties, in a division box of the old domestic water system of Ontario, which box was destroyed in the early Nineties, during a period of changes in and re-adjustment of that system. Some years later Mr. Hobby utilized this idea in one of the city measuring boxes, and also in a box of the water company. The weir installed by Mr. Hobby consisted of a rectangular notch in a plank with a wooden slide gate, both the weir crests and the crest of the end of the slide gate being covered with thin metal, making a sharp metal edge over which the water contracted. This slide gate was carried in a wooden groove, built under the crest and upon the up-stream face of the weir, in such a way as to interfere seriously with the crest contractions. There were two serious objections to this weir: First, sand and gravel lodged in the groove, interfering at times with the adjustments of the slide; second, the wooden frame of the weir did not make a suitable material to build into masonry, the leakage along the lines of contact being considerable. The experience of the writer with an adjustable-crest weir led to the conclusion that it should be entirely of metal, and the following was adopted: From a sheet of metal of the necessary weight and dimensions, a rectangular orifice of ample size is cut—leaving a 2-in. margin for the top and a 6 to 8-in. margin at the ends and bottom—to this, angle-irons and anchor bolts are attached, and the weir is built into the wall, flush with the up-stream face. Two U-irons are riveted to the slide, and, when hooked over the top of the weir, the slide gate is in place, and can be moved to any position the water superintendent may choose. In the adjustment of the weir and the division of the water, the writer obtains the best results by supplying the party in charge with a standard weir table, leaving the regulation to his judgment and experience. The introduction of acceptable methods of dividing and measuring the water of this company dates from the time sufficient moneys were realized from the sale of bonds to meet the expense of such improvements, and not from any other circumstances or incidents.

FIG. 1.—FRONT ELEVATION OF LIGHTING SUB-STATION.

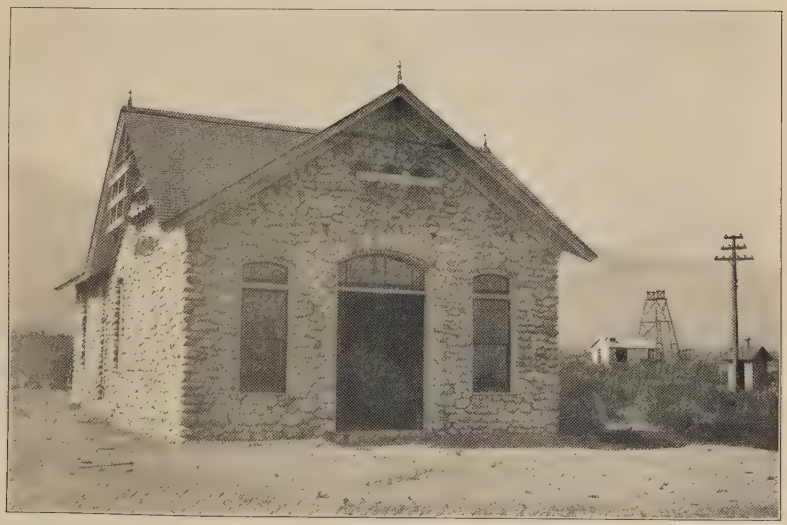

FIG. 2.—SUB-STATION FROM WHICH THE COMPANY'S WELLS ARE PUMPED.

FIG. 1.—POWER-HOUSE NO. 2.

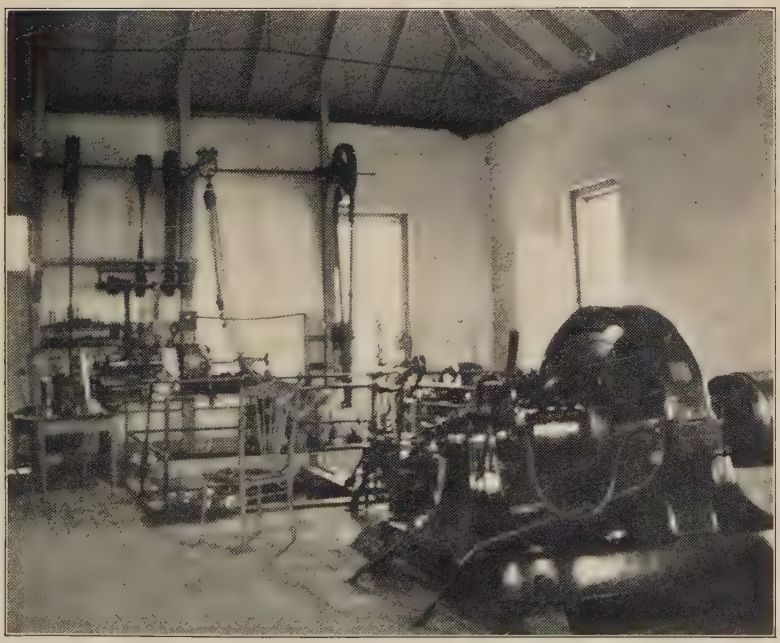

FIG. 2.—INTERIOR OF PLANT NO. 2.

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TRANSACTIONS.

Paper No. 1005.

A FEW POINTS IN THE DESIGN OF REINFORCED CONCRETE ARCHES.*

By B. R. Leffler, Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. William Cain, Wilbur J. Watson, Almon H. Fuller and B. R. Leffler.

The object of this paper is to set forth some short processes in the graphical analysis of the elastic arch. The method given by William Cain, M. Am. Soc. C. E., in his book on Steel-Concrete Arches will be the basis. The reader is supposed to be familiar with this method.

The design of the arch section for a given thrust and moment, as determined by the line of resistance, will be commented upon. The effect of a direct thrust and moment acting simultaneously on a section is not clearly understood by some designers.

Let t be the vertical distance from the gravity axis to the line of resistance. Then, if the arch ring is properly divided into small lengths, the following are the fundamental conditions for an arch without hinges: $\Sigma\ t = 0$, $\Sigma\ t\ x = 0$, and $\Sigma\ t\ y = 0$. In which x and y are the co-ordinates of any point. Briefly, the preliminary work is this: The external loads having been determined, a load

* Presented at the meeting of June 7th, 1905.

line is laid off, a trial pole assumed, and a trial equilibrium polygon is drawn. The true closing line of this polygon is then determined.

It is a common practice to find four lines of resistance, corresponding to the arch fully loaded, three-quarters loaded, one-half loaded, and one-quarter loaded. Any method of lessening the labor of determining the closing line for the trial equilibrium polygon is desirable, since there must be a trial polygon for each loading. One object of this paper is to set forth a short method.

Let Fig. 1 represent an outline of an arch ring with a trial equilibrium polygon located. Let $m\ m'$ be the true closing line. By Cain's method, a trial closing line, $n\ n'$, would be first assumed. Now, no restrictions being placed upon the position of $n\ n'$, it will be assumed entirely out of the trial polygon. In this same figure, let $A\ B\ C$ be any right triangle in which $A\ B$ covers all the ordinates covered by the trial polygon. Treat all the ordinates included in the triangle as forces, finding their resultant in amount and position. Similarly, as per Cain's method, this resultant is a trial T', and will be located to the right of the center of the arch. A like trial T is found at the same distance to the left of the center. By this method $T' = T$ always.

The following proposition will now be enunciated. *These trial T's, in position and amount, can be used for locating the true closing line for any trial equilibrium polygon that may be drawn.*

The demonstration of this proposition is so simple that it will not be given.

Let R be the resultant, in position and amount, of the ordinates included in the trial equilibrium polygon. Having R and the position of the trial T's, which is the same as for the true T's, we can find the true T's as per Cain's method. The ordinates for the true closing line can then be computed.

$$V_2\, m = \frac{C\,B \times \text{true } T}{\text{Trial } T}, \text{ and } V_1\, m' = \frac{C\,B \times \text{true } T'}{\text{Trial } T}.$$

In the above expressions, $\frac{C\,B}{\text{Trial } T}$ is a constant. Let n = the number of divisions in the span. Trial $T = \frac{C\,B\,(n+1)}{2}$. Hence $\frac{C\,B}{\text{Trial } T} = \frac{2}{n+1}$. Substituting in the above expressions, $V_2\ m = \frac{2}{+1}$ (true T), $v_1\ m' = \frac{2}{n+1}$ (true T').

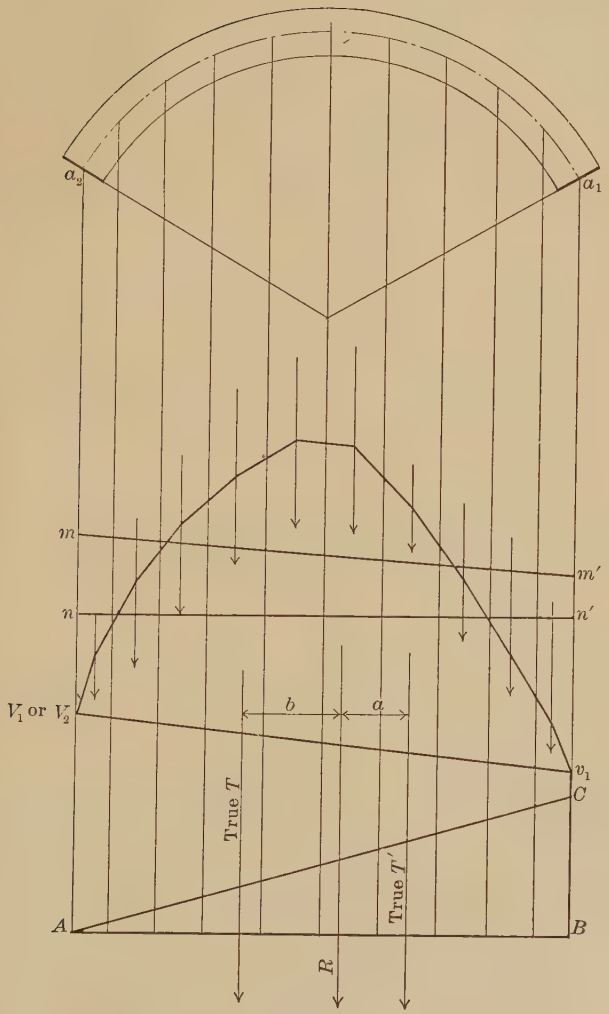


FIG. 1.

These expressions are still further simplified by dividing the arch span into equal parts.

Assume the arch span divided into equal parts. As to the requirements for this condition, the reader is referred to Cain's Solid and Braced Elastic Arches.* If these requirements are ignored, the error is probably small. A somewhat similar condition is met in computing the reactions of draw bridges. The writer refers to the assumption of a constant moment of inertia.

A method of calculating $V_1 m$ and $v_1 m'$, the ordinates of the true closing line, will now be deduced. As before, assume a right

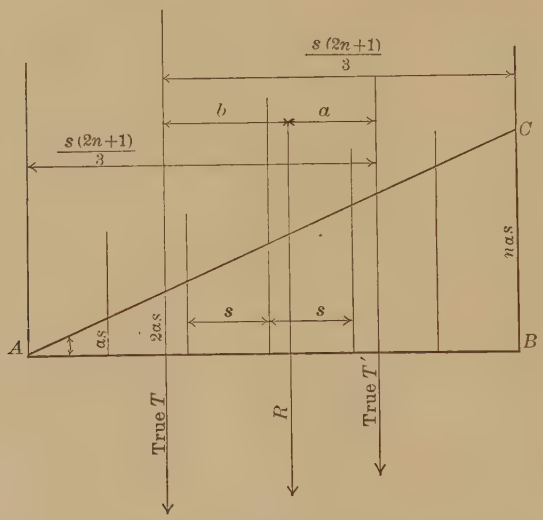


FIG. 2.

triangle, *A B C*, Fig. 2, but having its base divided into equal parts. Let α be the tangent of the angle at *A*. Let n equal the number of equal parts in the span. The remaining symbols are evident from the figure. As before, treat the ordinates as forces. Taking moments about *A*:

$$M = \alpha s^2 + 4 \alpha s^2 + 9 \alpha s^2 + \ldots\ldots n^2 \alpha s^2.$$

$$\text{Distance to trial } T' = \frac{\alpha s^2 (1 + 4 + 9 + \ldots\ldots n^2)}{n \alpha \frac{s}{2} (n+1)} =$$

$$\frac{2 s}{n (n+1)} \left[\frac{n (n+1) (2n+1)}{6}\right] = s \frac{(2n+1)}{3}.$$

*Page 76.

Trial $T' = \alpha\, s\, n\, \frac{(n+1)}{2}$, evidently.

True T' and T will now be calculated. Let a be the distance from T' to R, and b of T to R. The distance between T' and T is $\frac{2\,s}{3}\,(2\,n+1) - n\,s$, which, of course, equals $a + b$. Take origin of moments on T'. Then $R\,a = \left[\frac{2\,s\,(2\,n+1)}{3} - n\,s\right]$ true T,

$$\text{True } T = \frac{3\,R\,a}{2\,s\,(2\,n+1) - 3\,n\,s},$$

$$\text{and True } T' = \frac{3\,R\,b}{2\,s\,(2\,n+1) - 3\,n\,s}.$$

Using these values for the true T's we have, as before,

$$V_1\,m = \frac{2}{n+1}\,\frac{3\,R\,a}{2\,s\,(2\,n+1) - 3\,n\,s} = \frac{6\,R\,a}{s\,(n+2)\,(n+1)}.$$

Similarly, for $v_1 m'$.

It is impossible to present the full theory of the elastic arch in a single paper. For this reason the reader is supposed to be familiar with Cain's method.

The figures shown are not actual examples of an arch, but are simply intended to illustrate the location of the true closing line of the equilibrium polygon. Hence, no force diagram, or line of resistance is shown.

After the true closing line and true pole are located, the line of resistance can be drawn in its correct position in the arch ring. It can then be seen where the line passes out of the middle third.

Wherever the line of resistance passes out of the middle third of the plain concrete section steel is required.

The steel is usually placed symmetrically about the gravity axis of the concrete. The center of gravity of the compound section is thus the same as for the plain concrete section. The moment is measured by the product of the direct thrust into the distance of the line of resistance from the center of gravity. *This moment cannot produce tension in the concrete unless the line of resistance is out of the middle third of the plain concrete section.*

There is only one set of formulas, as far as the writer knows, which seems to show correctly the effect of the direct thrust and moment on an arch section of reinforced concrete. These formulas

are given by Cain, and were first published by Thacher. The formulas are:

$$s_1 = \frac{T}{A_1 + n A_2} \pm \frac{M v_1}{I_1 + n I_2};$$

$$\text{and } s_2 = \left[\frac{T}{A_1 + n A_2} \pm \frac{M v_2}{I_1 + n I_2}\right] n.$$

If these formulas are applied to any section in which the line of resistance passes within the middle third, it will be found that the steel takes no tension.

This middle-third theory cannot be too strongly emphasized, for the impression seems somewhat prevalent that steel is required wherever there is moment; whereas it is only required where tension in the concrete is produced.

The Railway Age of January to April, 1904, contained a series of articles, afterward issued in pamphlet form, in which the theory of putting in steel to take moment is exemplified.* A reinforced concrete beam formula similar to Johnson's was used. The writer fails to see how a formula like Johnson's or Hatt's can be rationally applied to a case of combined thrust and moment.

Theory shows that the voussoir arch with good joints behaves as a curved elastic beam as long as the line of resistance does not pass out of the middle third. Railroad engineers build reinforced concrete arches with considerable depth of ring, almost the same as for a voussoir arch. This being the case, the line of resistance does not deviate very much from the middle third, except in the case of the dead load being a small part of the total load. Hence, for a deep concrete ring very little steel will be required for flexure. But if steel is put in for moment, erroneously, much steel may be used. The use of steel to prevent shrinkage cracks is not now under consideration.

It is probable that railroad engineers are too conservative in the use of an arch ring of shallow depth. It is only by the use of such a ring that the steel can be made to resist tension.

The main objects of this paper are to introduce some improvements in the graphical analysis of the arch, and to emphasize the middle-third theory.

* See pages 38 and 39 of pamphlet.

DISCUSSION.

Mr. Cain. WILLIAM CAIN, M. AM. SOC. C. E. (by letter).—The author's formulas, relative to locating the true closing line of the trial equilibrium polygon for an arch, to satisfy the conditions, $\Sigma\ t = 0$, $\Sigma\ t\ x = 0$, are of interest and value. The resulting work is slightly briefer, however, if, taking a hint from the method used for fixing the line, $k\ k_1$, we put,

$$v\ n = v_1\ n_1 = \frac{R}{n+1},$$

which method is used in Buel and Hill's "Reinforced Concrete."

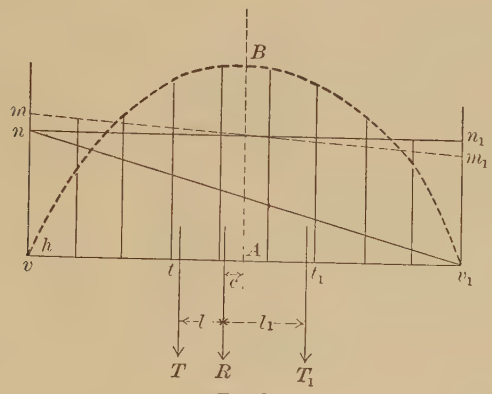


FIG. 3.

In Fig. 3, let $v\ B\ v_1$ be the trial equilibrium polygon, the ordinates being supposed to be placed at equal or unequal distances apart. Then, treating the ordinates from $v\ v_1$ to the equilibrium polygon as forces, their resultant, R, equal to the sum of the ordinates, is found, in magnitude and position. Suppose R acts c feet to the left of the center of $v\ v_1$.

If there are n divisions of $v\ v_1$ (nine in the figure), lay off on the verticals at v and v_1, $v\ n = v_1\ n_1 = R \div (n + 1)$, and draw the lines, $n\ n_1$ and $n\ v_1$. It follows that the sum of the ordinates in the triangle, $v\ n\ v_1$, or trial $T = R \div 2 =$ the sum of the ordinates in the triangle, $v_1\ n\ n_1$, or trial T_1, since $R = (n + 1)\ v\ n =$ the sum of the ordinates in the parallelogram, $v\ n\ n_1\ v_1$. The position of trial T is found by taking moments about A. (The work is nearly doubled when moments are taken about any other point.) Suppose that trial T is found to act $(l + c)$ feet to the left of A, then trial T_1 acts an equal distance $(l_1 - c)$ feet to the right of A, or through

Mr. Cain. t and t_1, respectively. Therefore, true T and true T_1 act also through t and t_1.

Now, since R is the resultant in magnitude and position of true T and true T_1, we have, on taking moments in turn about t_1 and t,

$$\text{true } T\ (l + l_1) = R\ l_1 = 2 \text{ trial } T\ l_1$$
$$\text{true } T_1\ (l + l_1) = R\ l\ = 2 \text{ trial } T_1\ l,$$

$$\text{therefore, } \frac{\text{true } T}{\text{trial } T} = \frac{2\ l_1}{l + l_1}; \quad \frac{\text{true } T_1}{\text{trial } T_1} = \frac{2\ l}{l + l_1}.$$

$$\text{Therefore, } v\ m = \frac{\text{true } T}{\text{trial } T} v\ n = \frac{2\ l_1}{l + l_1} \frac{R}{n + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

$$v_1\ m_1 = \frac{\text{true } T_1}{\text{trial } T_1} v_1 n_1 = \frac{2\ l}{l + l_1} \frac{R}{n + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

The points, m and m_1, having thus been located, the true closing line, $m\ m_1$, is drawn.

It will be observed that by this method, true T and true T_1 do not have to be computed, which is a slight gain.

For another equilibrium polygon, corresponding to a different loading, R and c must be found as before, but t and t_1 remain the same, so that at once are found, $l = A\ t - c$, $l_1 = A\ t_1 + c$, and Formulas (1) and (2) give the new values of $v\ m$ and $v_1 m_1$.

The foregoing formulas are true, whether the ordinates are spaced at equal or unequal distances apart.

Suppose, next, that the ordinates are h feet apart, everywhere; then, from the interesting investigation of the author on pages 186–187,

$$v_1\ t = \frac{2\ n + 1}{3} h; \quad l + l_1 = \frac{n + 2}{3} h$$

$$A\ t = A\ t_1 = v_1\ t - v_1\ A = \frac{2\ n + 1}{3} h - \frac{n\ h}{2} = \frac{(n + 2)\ h}{6}.$$

$$\text{Therefore, } l_1 = A\ t_1 + c = \frac{(n + 2)\ h + 6\ c}{6},$$

$$\text{and } l = A\ t - c = \frac{(n + 2)\ h - 6\ c}{6}.$$

On substituting these values of l and l_1 in Formulas (1) and (2), we derive,

$$v\ m = \frac{(n + 2)\ h + 6\ c}{(n + 2)\ h} \frac{R}{n + 1} = \left(1 + \frac{6\ c}{(n + 2)\ h}\right) \frac{R}{n + 1},$$

$$v_1\ m_1 = \frac{(n + 2)\ h - 6\ c}{(n + 2)\ h} \frac{R}{n + 1} = \left(1 - \frac{6\ c}{(n + 2)\ h}\right) \frac{R}{n + 1}.$$

Since R and c are known, the values of $v\ m$ and $v_1 m_1$ can be at once computed and laid off, and the true closing line drawn. Note, here, that $(n + 2)\ h = n\ h + 2\ h = v\ v_1 + 2\ h$. Unfortunately, this brief method will rarely be applicable, as it only holds for

"($s \div E I$) constant," or, approximately, for very flat arches having everywhere a constant cross-section. Mr. Cain.

The author states, relative to the assumed positions of the live load (for maximum stresses), that the load is usually made to cover, successively, the whole span, the half span, one-fourth, and three-fourths of the span. Perhaps the last two loadings can be improved upon, but the writer has no conclusive figures to offer. However, some insight into the matter can be gained by a consideration of the loadings giving maximum t and M for certain bridges which have been examined, in connection with the formula for unit stress in the concrete at the upper and lower edges of a joint,

$$s_1 = \frac{T}{A_1 + n A_2} \pm \frac{H t v_1}{I_1 + n I_2},$$

where H is the horizontal thrust, t the vertical distance from the center of any joint to the side of the equilibrium polygon pertaining to it, and T the normal component of the thrust on the joint.

Since the remaining quantities are constant for the same joint, we can write for any joint,

$$s_1 = c\,T \pm c'\,H\,t,$$

where c and c' are constants for any one joint. By using subscripts to c and c' pertaining to the joint, the formula can be supposed to

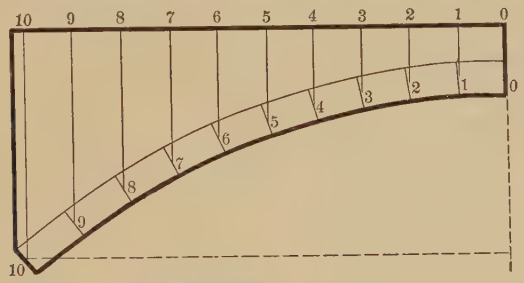


FIG. 4.

apply to any joint. Thus, if the crown is Joint 0, since, then, $T = H$, we can write,

$$s_1 = (c_0 \pm c'_0\,t)\,H.$$

The object is to find that position of the load which makes s_1 a positive or negative maximum, the former referring to compression, the latter to tension. The variables in the general formula are T, H and t, where any particular joint is under consideration.

The loadings for a series of segmental arches of constant section, varying from 12.5 to 150-ft. spans, the rise being one-fifth of the span, are given in the writer's "Theory of Voussoir Arches,"* which make t a maximum at certain joints.

Thus, in Fig. 4, representing a span of 100 ft., rise 20 ft., the half

* Pages 129-139.

Mr. Cain. span of the center line of the arch ring was divided into ten equal parts, and verticals were drawn through the points of division, meeting the roadway at 0, 1, 2, .., 10, and the joints on these verticals are similarly marked. Then it was found, for the joint stated below, that the maximum departure of the equilibrium polygon from the center of the joint (maximum t) corresponds to the position of the live load as given.

Joint.	Position of live load.
Right springing	0 to 10 (inclusive)
Crown	4 " 10 "
5	4 " 10 "
6	4 " 10 "
Left springing	5 " 10 "

Also at Joint 3, to the right of the crown, the live load covered the left half of the span (0 to 10).

It is evident from these results, that, for the voussoir arch, at least two positions of the uniform live load should be tried, one covering half the span, the other, say, three-tenths of the span from the springing. It will be seen, then, whether, at the critical joints, the centers of pressure lie within proper limits, say, the middle third.

But, for such loadings, although t is made a maximum, it does not follow that s_1 is a maximum, since, at any joint, s_1 varies, not only with t, but also with H and T. Besides, these results may not hold for segmental arches of varying section, or for many-centered arches of any kind.

Some additional light is thrown on the subject by Mr. W. S. Edge, of Cornell University, in a thesis on "Steel-Concrete Arches," a brief abstract* of which has been made by E. J. McCaustland, Assoc. M. Am. Soc. C. E.

The arch bridge considered by Mr. Edge was of 80 ft. span and 10 ft. rise., and Sections 1 to 48 are at the springs. The arch was divided up and the computation made after "Cain's Method."

It was found that:

"For the crown section the bending moment is the greatest when the load is about three-fifths on the bridge. The greatest positive moment, however, occurs with the arch practically one-half loaded and takes place in Section 1. The greatest negative moment occurs when the arch is three-fifths loaded, and is found at Section 48."

These conclusions refer to the positions of the load giving maximum $M = H\,t$, which may differ from those positions giving maximum

$$s_1 = c\,T \pm c'\,H\,t,$$

* *Engineering News*, April 21st, 1904.

since T varies in this formula as well as $M = H\,t$. Any increase of the load on the bridge increases both H and T, but it may diminish t, and *vice versa*. Mr. Cain.

Thus, at the spring, when the uniform live load covers half the span, T, H and t have certain values; when the load covers the whole span, H and T are doubled, but t is diminished, so that perhaps s_1 is less than before.

It is usual, with some constructors, to test the arch for two loadings, one covering the half span, the other the whole span. It seems, from what has preceded, that two other positions should likewise be tried, one extending from a springing three-tenths, and the other, six-tenths, of the span of the center line of the arch ring; for, although nothing conclusive has as yet been established, even with regard to the arch of constant section and radius, yet the indications point to these loadings.

It is to be hoped that some one who has the time will ere long investigate this important subject, for certain usual forms of the arch, and will publish the results.

The writer agrees with the author that reinforced concrete arches should not be given the dimensions of stone arches of the same span and rise. The theory of design in the two cases is different. If the radial depth of the reinforced concrete arch ring is made greater than is necessary for the maximum unit stresses on the concrete to remain within certain safe limits, then, not only is the metal not utilized to a proper extent, but the temperature stresses increase very much. In fact, if $I_1 + n\,I_2$, or the moment of inertia of the "revised section," is everywhere doubled, the horizontal thrust, H, is doubled; also, T is doubled, as well as $M = H\,t$. Consequently, s_1 is increased, for the term, $\frac{H\,t\,v_1}{I_1 + n\,I_2}$, is greater because v_1 is greater; also, since $A_1 + n\,A_2$ is not as much as twice the first value, the term, $\frac{T}{A_1 + n\,A_2}$, is increased; hence,

$$s_1 = \frac{T}{A_1 + n\,A_2} + \frac{H\,t\,v_1}{I_1 + n\,I_2}$$

is increased.

To diminish temperature stresses, it is thus necessary to diminish the cross-sections of the arch.

It seems, also, absurd to require that there shall be no tension anywhere in the concrete. It leads to abnormal sections at the springings. If the concrete cracks anywhere, it may admit moisture and air, which is detrimental to the steel; but, otherwise, it can do no harm, as the steel is generally ample in section to take up all tensile stresses.

It seems advisable, then, to design the sections so that there shall

Mr. Cain. be no cracks in the concrete, but this can be done perhaps by specifying for the maximum tensile stress in the concrete, say, one-half its breaking strength, or perhaps more. This will diminish the cost considerably, and make a more sightly structure.

Mr. Watson. WILBUR J. WATSON, M. AM. SOC. C. E. (by letter).—The average bridge engineer does not have occasion to compute the stresses in an arch ring often enough to keep thoroughly in touch with any good method for their computation, and when the necessity does arise, the information is usually desired quickly, and there is not time for an extended theoretical investigation of the subject.

Some years ago the writer made, for his own satisfaction, a study of some of the more common methods used in the solution of the arch, and at that time decided upon the use of Cain's method, as being, in his judgment, the best practicable method for his purpose. He found, however, that whenever the occasion arose to use the method, it was necessary to make a complete restudy of the theory, and even then mistakes in method were almost sure to be made, involving much waste of time.

In order to expedite the work, and also to serve as a check upon the same, a standard stress sheet, Plate XXII, was gotten up, showing in detail the various steps.

It will be noted that the writer has departed from Professor Cain's method in two important particulars:

First.—The loads are assumed at points on the arch ring independent of the points used in the division of the neutral axis, which latter are used only in the solution of the three conditions of equilibrium.

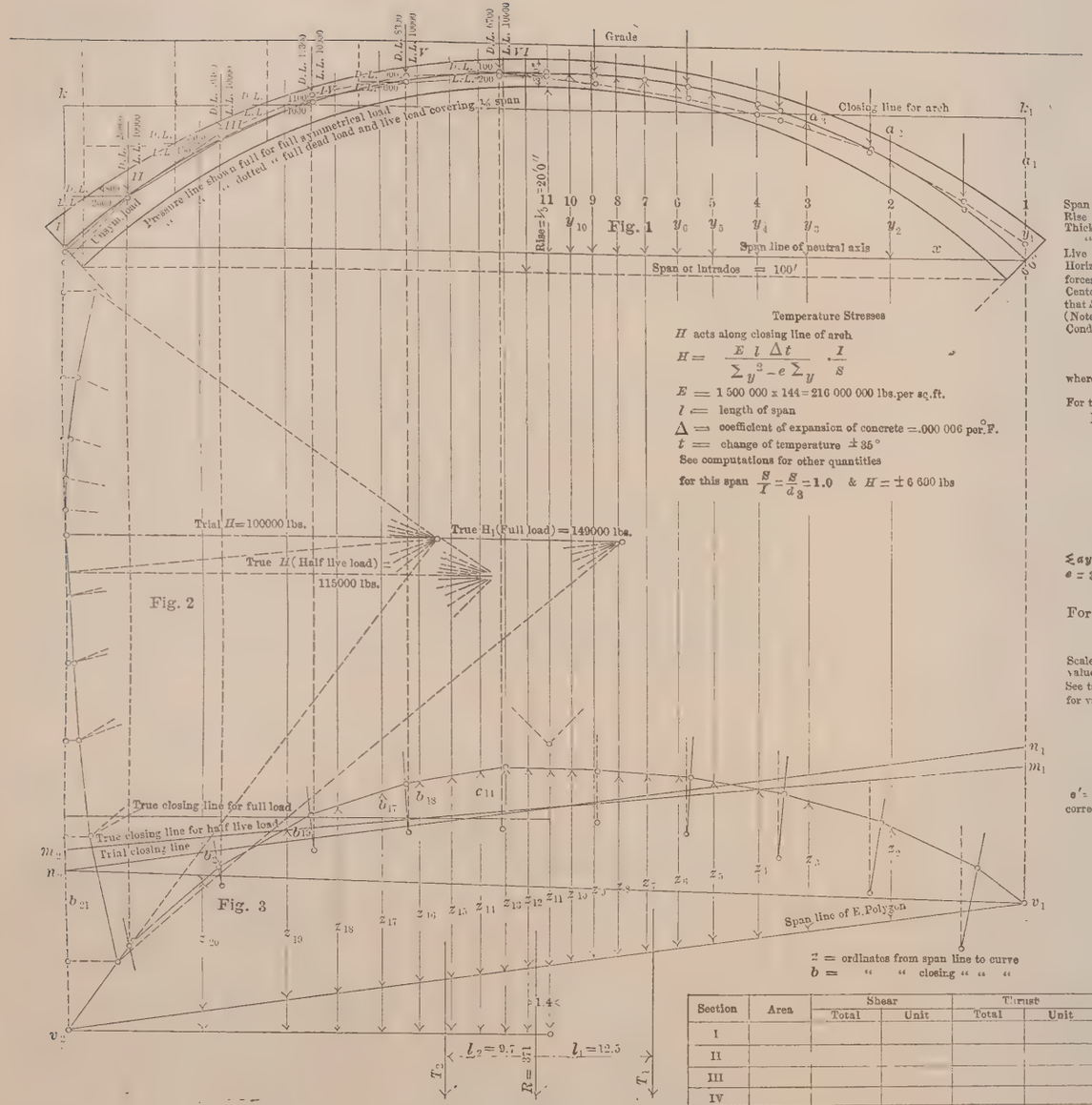
This is done in order that the method may be used for concentrated loads, and especially for cases in which the loads are brought upon the arch by means of spandrel arches.

The writer is aware that this method introduces an element of error in scaling the lengths of the ordinates to the trial equilibrium polygon, but believes that this error is so small as to be a negligible quantity.

Second.—The writer takes into account the horizontal forces acting on the extrados of the arch. He has not yet had time to make an investigation to determine the amount of error due to their omission, but he can see no good reason why they should be omitted.

It will also be noted that in the division of the neutral axis no account is taken of the moment of inertia of the reinforcement, the divisions are, therefore, only exact for a concrete arch without reinforcement, but as this method is much less laborious than the more exact one used by Professor Cain, the writer has used it also for reinforced arches, under the impression that the error due thereto must be comparatively small.

ANALYSIS OF 100 FT. X 20 FT. REINFORCED CONCRETE ARCH


Grade
Closing line for arch
Pressure line shown full for full asymmetrical load
" dotted " full dead load and live load covering ½ span
Rise = ½ = 20.0'
Span line of neutral axis
Span or Intrados = 100'
Fig. 1
Trial H = 100000 lbs.
True H₁ (Full load) = 149000 lbs.
True H (Half live load) = 115000 lbs.
Fig. 2
True closing line for full load
True closing line for half live load
Trial closing line
Fig. 3
Span line of E. Polygon
l₂ = 9.7
l₁ = 12.5
T₂
R = 31.1
T₁

Temperature Stresses

H acts along closing line of arch

$$H = \frac{E\ I\ \Delta t}{\sum y^2 - e \sum y} \cdot \frac{I}{S}$$

E = 1 500 000 x 144 = 216 000 000 lbs. per sq. ft.
l = length of span
Δ = coefficient of expansion of concrete = .000 006 per °F.
t = change of temperature ± 35°
See computations for other quantities
for this span $\frac{S}{I} = \frac{S}{a_3} = 1.0$ & $H = \pm 6\,600$ lbs

Span of intrados = L = 100'
Rise " " = R = 20'
Thickness at crown = t_1 = 3'
" at spring = t_2 = 6'
Live Load = Cooper's E 50, assumed to be equivalent to 1 000 lbs. per sq. ft.
Horizontal Forces assumed to be equal to one-third Vertical forces x tan. angle of inclination of extrados.
Center line of arch ring divided by trial in divisions S such that $S \div d$ = constant, where d = depth of arch ring.
(Note: begin at crown with division of of arch ring.)
Conditions for equilibrium:
1. = $\Sigma\, l = 0$
2. = $\Sigma\, dx = 0$
3. = $\Sigma\, dy = 0$
where d = ordinate from curve to equilibrium polygon

For the curve Fig. 1	y_1 = 0	y_1^2 = 0	
	y_2 = 10.20	y_2^2 = 104.0	
	y_3 = 14.35	206.0	
	y_4 = 16.30	266.0	
	y_5 = 17.50	306.3	
	y_6 = 18.30	334.9	
	y_7 = 18.90	357.2	
	y_8 = 19.20	368.6	
	y_9 = 19.40	376.4	
	y_{10} = 19.50	380.2	2698.6 x 2 = 5397.2
½ of	y_{11} = 9.80		334.2
	½ Σy = 163.45	Σy = 326.9	Σy^2 = 5731.4

$\Sigma a y = \Sigma (y\ e)\ y = \Sigma y^2 - e \Sigma y$
e = 326.9 ÷ 21 (= number of divisions + 1) = 15.6 — 326.9 x 15.6 = 5099.6
Diff. = 681.8

For the trial polygon, symmetrical loads

Fig. 3	z_1 = 0	b_1 = −23.8 x y_1 = 0		corrected b_1 = −16.0
	z_2 = 16.5	b_2 = −7.2 x y_2 = −73.5		−93.6
Scale for	z_3 = 22.4	b_3 = −1.4 x y_3 = −20.1		
values of z & b	z_4 = 25.0	b_4 = +1.2 x y_4 = +19.6		
See table above	z_5 = 26.7	b_5 = +2.9 x y_5 = +50.8		
for values of y	z_6 = 27.8	b_6 = +4.1 x y_6 = +75.0		+601.7
	z_7 = 28.5	b_7 = +4.7 x y_7 = +88.8		
	z_8 = 29.0	b_8 = +5.2 x y_8 = +100.0		
	z_9 = 29.4	b_9 = +5.5 x y_9 = +107.0		
	z_{10} = 29.4	b_{10} = +5.5 x y_{10} = +107.0		
½ of	z_{11} = 14.7	b_{11} = +2.7 x y_{11} = + 53.6		corrected b_{11} = 3.69
	½ Σz = 249.4	Σz = 498.8 = R		Diff. = 508.1 x 2 = 1016.2

e' = 498.8 ÷ 21 = 23.8 = dist. of true closing line above v
correct pole distance = true H = 100 000 x $\frac{1016}{682}$ = 149 000 lbs.

For the trial polygon, unsymmetrical loads Fig. 3

z_1 = 0	x dist. from ctr.			trial c_1 = (17.7-0) x 352 = 920.0	true	c_1 = −15.50 x y_1 =	± 0	
z_2 = 10.4	x 37.2 =	387		$c_2 - c_{20}$ = (15.1-2.6) x 37.2 = 465.0		c_2 = −5.60 x y_2 =	−56.1	
z_3 = 14.6	x 28.2 =	411		(13.6-4.1) x 28.2 = 268.0		c_3 = −1.80 x y_3 =	−25.8	
z_4 = 16.6	x 22.7 =	377		(12.6-5.1) x 22.7 = 170.0		c_4 = ∓ 00 x y_4 =	± 0	
z_5 = 18.3	x 17.9 =	326	2272	(11.9-5.8) x 17.9 = 109.0		c_5 = +1.20 x y_5 =	+21.0	
z_6 = 19.3	x 13.9 =	268		(11.2-6.5) x 13.9 = 65.3		c_6 = +2.10 x y_6 =	+38.4	
z_7 = 19.9	x 10.4 =	207		(10.6-7.1) x 10.4 = 36.4		c_7 = +2.70 x y_7 =	+51.0	
z_8 = 20.6	x 7.4 =	152		(10.0-7.7) x 7.4 = 17.0		c_8 = +3.30 x y_8 =	+63.3	
z_9 = 20.2	x 4.7 =	95		(9.6-8.1) x 4.7 = 7.1		c_9 = +3.70 x y_9 =	+71.8	
z_{10} = 21.5	x 2.3 =	49		(9.2-8.5) x 2.3 = 1.6		c_{10} = +3.90 x y_{10} =	+76.0	
z_{11} = 21.9	x 0 =	0		8.8	2059.4	c_{11} = +4.10 x y_{11} =	+80.3	
z_{12} = 22.3	x 2.3 =	51		8.5		c_{12} = +4.40 x y_{12} =	+85.8	
z_{13} = 22.6	x 4.7 =	106		8.1 ÷185.7 (= R ÷ 2) = 11.1'		c_{13} = +4.70 x y_{13} =	+91.0	
z_{14} = 22.6	x 7.4 =	167		7.7 from center line		c_{14} = +4.60 x y_{14} =	+88.2	
z_{15} = 22.5	x 10.4 =	234	2797	7.1 l_1 = 12.5' l_2 = 9.7'		c_{15} = +4.30 x y_{15} =	+81.2	
z_{16} = 22.4	x 13.9 =	311	525	6.5 true $v_1 m_1$ = $\frac{2 \times 9.7}{22.2}$		c_{16} = +4.10 x y_{16} =	+75.0	
z_{17} = 21.7	x 17.9 =	388		5.8		c_{17} = +3.30 x y_{17} =	+57.8	
z_{18} = 20.8	x 22.7 =	472		5.1 x 17.7 = 15.5		c_{18} = +2.00 x y_{18} =	+32.6	
z_{19} = 19.0	x 28.2 =	536		4.1 true $v_2 m_2$ = $\frac{2 \times 12.5}{22.5}$		c_{19} = -0.00 x y_{19} =	± 0	
z_{20} = 14.3	x 37.2 =	532		2.6		c_{20} = -4.90 x y_{20} =	- 50.0	
z_{21} = 0				0 x 17.7 = 19.9'		c_{21} = -19.90 x y_{21} =	± 0	
						Σc = +48.30	+913.4	
						-47.60	-131.9	
						Diff. =	781.5	

Σz = 371.4 ÷ R = 525 ÷ 371.4 = 1.4'
for trial distance, $v_1 m_1$ = $v_2 m_2$ = 371.4 ÷ 21 = 17.7'
True H = 100 000 x 781.5 ÷ 682 = 115 000 lbs.
True c_1 & c_{21} = 13.55' & 17.45'

z = ordinates from span line to curve
b = " " " closing " " "

Section	Area	Shear		Thrust		Moment	Lever Arm.	Flange Stress	Flange Area	Remarks
		Total	Unit	Total	Unit					
I										
II										
III										
IV										
V										
VI										

Note: Steel ribs must take all bending stress due to dead + live load @ 16000 lbs. unit stress, and dead + live + temp. @ 24000 lbs unit stress.

Mr. Watson.

The notation varies somewhat from that of Professor Cain, in the use of single letters to denote ordinates.

It has been found that, in order to secure accurate results with this method, a large scale should be used, and that all arithmetical work should be carefully checked as the work progresses, as any error in multiplying or adding will affect all subsequent work.

Data regarding the range of temperature to which arch structures are subjected are much needed. As far as the writer can judge from a few observations of the movement of expansion joints in existing structures, these joints do not show over $\frac{1}{4}$ in. motion in a length of 100 ft., in the most exposed cases. Assuming a coefficient of expansion of 0.000006, this would correspond to a maximum change of temperature of about 35° fahr.

The method of locating the true closing line of the equilibrium polygon for unsymmetrical loads, given by Professor Cain, has been used in Plate XXII, in place of the original method as given in his book on Steel-Concrete Arches.

Mr. Fuller.

ALMON H. FULLER, ASSOC. M. AM. SOC. C. E. (by letter).—During the summer of 1905, the writer worked, to a certain extent, on the theory and design of masonry and reinforced concrete arches and is, therefore, in a position to appreciate any suggestions that will tend to simplify and shorten the application of the elastic theory. In determining the closing line he has found the suggestions of the author, particularly as modified in Professor Cain's discussion, to be a decided improvement upon the method as first given by Professor Cain.

In the computations for the divisions, s, of the arch ring to make $\frac{s}{I_c + n I_s}$ constant, the writer found that much time could be saved by platting not only $I_c + n I_s$ in terms of d, but also the values of d in terms of the distance from the abutment on the same sheet, as shown on Fig. 5. Then with the distance from abutment (l at middle of s) as argument, the values of $I_c + n I_s$ may be read as indicated by the dotted line on Fig. 5, and, with the slide rule, with the constant setting, $\frac{s_1}{I_{1c} + n I_{1s}}$, the values of s may be quite readily obtained. There has been some discussion on the possibility of making $\frac{s}{d_3}$ instead of $\frac{s}{I_c + n I_s}$ a constant, without appreciably affecting the results. The apparent saving, however, is only in platting the curve. An additional advantage would be that, in case of a revision of section, the new depths could be chosen inversely as the cube roots of the lengths (s), and the revised loads, with their line of action, could be obtained with a small fraction of the work necessary when the values of s must be changed. In the

Mr. Fuller. one problem considered by the writer, the values of s resulting from the two conditions were so nearly the same that, for similar conditions, results would be sufficiently precise by determining the values of s for plain concrete sections and considering the effect of the reinforcement only in computing the stresses.

The writer would like to emphasize the distinction made by the author between maximum moments and maximum stresses, and to express his appreciation of the data furnished by Professor Cain in regard to the positions of loading which will produce maximum deviation of the line of resistance from the center line. As pointed out by the author, these will not necessarily indicate maximum stresses, but are a decided aid in that direction.

The writer recently made a design for a 100-ft. span and has investigated it from a number of different standpoints. The assumed conditions were:

Span	100	ft.
Rise of center line	12	"
Surcharge at crown	2.3	"
Weight of earth	100	lb. per cu. ft.
Weight of concrete	140	" " "
Live load	140	" " sq. ft.
Modulus of elasticity of concrete	2 900 000	" " sq. in.
Modulus of elasticity of steel	29 000 000	" " "
Coefficient of expansion of concrete and steel	0.0000065	
Range of temperature	52° (26° each way from normal).	

The results for four different points are submitted in Table 1: At the crown, and at the points, 6, 2 and 1, obtained by dividing the half span into 14 divisions according to Cain's method. Point 1 is at the center of the end section; Point 6 is about midway between it and the crown. They are tabulated under three distinct heads, showing the effect of varying: First, the position of the live load; second, the material, *i. e.*, omitting the reinforcement; and, third, the effect of hinges. The stresses are excessive at the end section, but as they are within allowable limits at the next section, provision may be made to reduce them by a local deepening of the arch ring near the abutment.

A careful perusal of the table may reveal apparent discrepancies in the maximum and minimum stresses. It is believed, however, that the proper combinations have been made. The maximum crown stress for three-fifths load, for instance, seems to be too low, but for this loading the line of resistance is above the center line at the

Mr. Fuller.

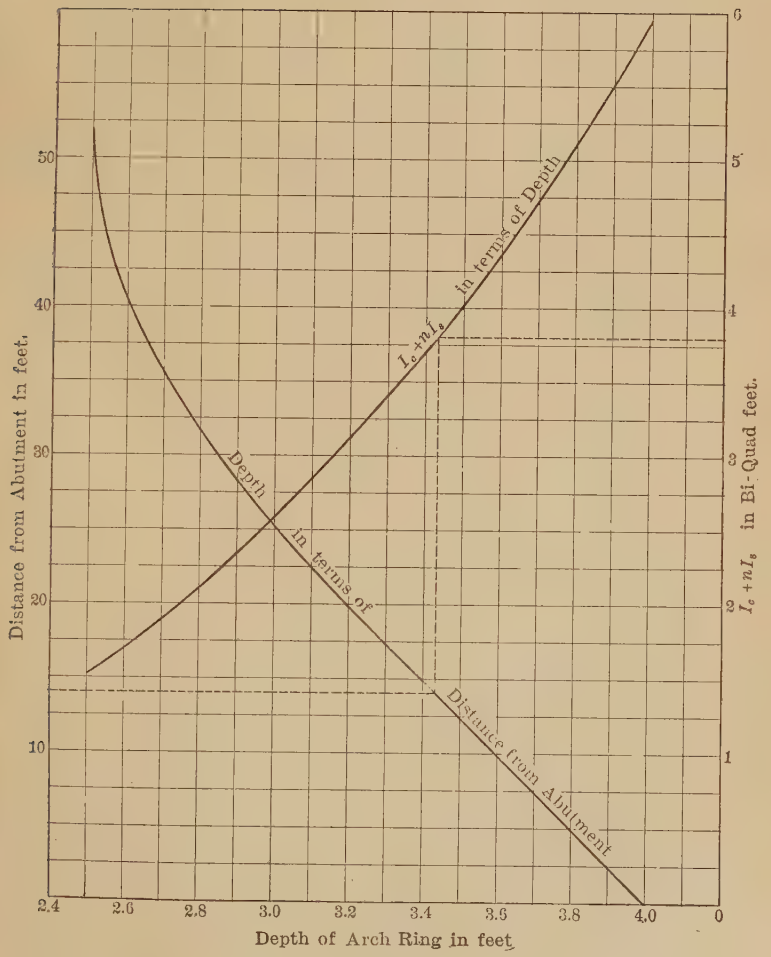


FIG. 5.

TABLE 1.—STRESSES IN REINFORCED CONCRETE ARCH BRIDGES.

Span, 100 ft. 0 in. Rise, 12 ft. 0 in.

Depth of Arch Ring: Crown, 2.0 ft.; Point 6, 2.29 ft.; Point 1, 3.05 ft.

Surcharge at Crown, 2.3 ft. Live Load, 140 lb. per sq. ft. Temperature, 26°.

	Hor. Thrust in Pounds.		Stresses in Concrete in pounds per square inch.															
			Crown.				Point 6.				Point 1.				Point 2.			
	D. & L. Loads.	Temperature.	D. & L.	T	Max.	Min.	D. & L.	T.	Max.	Min.	D. & L.	T.	Max.	Min.	D. & L.	T.	Max	Min.
Varying loads—Fixed ends—¾% Steel, at intrados and extrados.																		
Full live load	90 300	11 250	293	217	510	100	285	61	287	146	237	330	525	—145	220	239	451	28
Three-fifths live load	85 800	11 250	.....	217	449	15	313	61	315	94	272	330	560	—194	237	239	476	—13
Half live load	82 800	11 250	283	217	500	67	339	61	341	52	310	330	598	—248	243	239	482	—33
Two-fifths live load	79 700	11 250	290	217	507	40	332	61	334	43	308	330	591	—253	218	239	457	—69
Varying material—Fixed ends—Half live load.																		
¾% Steel, at intrados and extrados	82 800	11 250	283	217	500	67	339	61	341	52	310	330	598	—248	243	239	482	—33
Plain concrete	82 800	8 650	351	214	565	120	400	57	405	53	356	309	630	—233	274	239	513	—22
Varying conditions—Half live load—¾% Steel, at intrados and extrados.																		
Fixed ends	82 800	11 250	283	217	500	67	339	61	341	52	310	330	598	—248	243	239	482	—33
2-hinges	82 900	960	296	94	390	116	405	62	462	—15	197	9	206	188	256	20	272	124
3- "	82 800	.....	250	.....	250	250	370	.....	370	82	196	.....	196	196	263	.....	263	137

The minus sign denotes tension.

Mr. Fuller.

crown, and therefore the maximum stress, due to dead and live load, is at the upper edge of the section, while the maximum temperature stress is at the lower edge and cannot be combined with it. For all other loadings given, the line of resistance falls below the center line at the crown, and therefore the maximum stress from the loads and temperature occurs at the same point, on the intrados. Mr. Fuller.

Stresses are given for full, half, three-fifths and two-fifths live load. The computations for the last two loadings are apparently not required. At the present time there seems to be far greater need for careful consideration for the most advantageous curve for the center line, and a more definite knowledge of the possible temperature changes in the mass of concrete than for more refinement in determining the effect of the loads.

Table 1 shows the temperature stresses to be surprisingly large, even under the conservative assumption of a range of 26° each way from the normal, and indicates the necessity for reliable data concerning the relative changes in temperature of the atmosphere, and in a mass of concrete covered with earth, before accurate determination of stresses can be made.

As the temperature stresses vary directly with the change in degrees, Table 1 forms a basis for comparison of maximum and minimum stresses under any assumed range.

B. R. Leffler, Assoc. M. Am. Soc. C. E. (by letter).—Referring to Mr. Cain's formulas (1) and (2), we get by addition, Mr. Leffler.

$$\frac{v\,m + v_1\,m_1}{2} = \frac{R}{n+1},$$

and by subtraction and substitution,

$$v\,m - v_1\,m_1 = \frac{4\,c\,R}{(l_1 + l)\,(n+1)}.$$

The first gives the value of the middle ordinate of the true closing line; the second, the difference of the end ordinates. These locate the true closing line. Notice that for any particular arch, $l_1 + l$ is a constant; and, since $c\,R$ is the moment of the ordinates, c can remain unknown.

Let Fig. 6 represent a trial equilibrium polygon in which the span is divided into an even number of equal parts. Draw $v_1\,w$ parallel to $m\,m_1$. From the formulas on page 190, $v\,w = \dfrac{12\,c\,R}{(n+1)\,(n+2)\,h}$. This can be deduced also from the foregoing general formula.

The value of $v\,w$ can be put into a more convenient form. By reason of the spacing of the span, an ordinate occurs at the center. The ordinates are numbered in opposite directions from the center. Let d_1, d_2, d_n, etc., be the differences of symmetric ordinates. The subscript shows the pair of ordinates taken. Substituting the value of

Mr. Leffler.

$c\ R,\ v\ w = \frac{12\ (d_1 + 2\ d_2 + 3d_3 \ldots\ldots)}{(n+1)\ (n+2)}$. This expression is short because only small numbers are used. Notice that each difference is multiplied by its subscript.

Regarding the short method, the writer considers it too great a time-saver and too valuable to be ignored. Moreover, any new arch can be dimensioned so that the method is exact. The writer begs permission to call attention to Mr. Cain's "Solid and Braced Elastic Arches," for the correct way of dimensioning the arch ring. The short method is not exact in the investigation of arches already built, unless they were built to conform to the conditions required. Valuable insight would be obtained, if some one would locate the lines of resistance for a number of arches by the short method, and by the long method. A comparison of results would show the percentage of error.

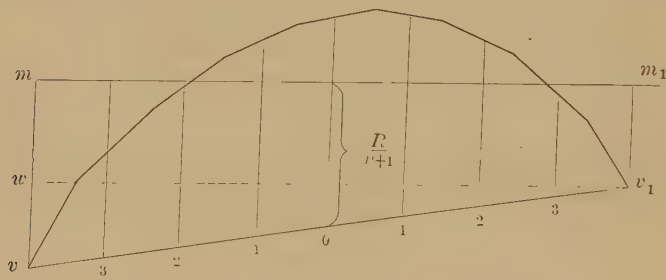


FIG. 6.

By the short method, two good draftsmen can locate the line of resistance and determine the proper section of an arch ring, for four different loadings, in about thirty hours of total labor. Much depends upon the proper selection of nomenclature, and tabulation of work.

Regarding the tension in concrete, the tendency seems to be to neglect this.* Experiments show that, if the steel is stressed up to 5 000 lb. per sq. in., the tensile strength of the concrete is exceeded. In short, it is impossible to make the steel do full duty and use the tensile strength of the concrete.

The writer is compelled to design for a live load of Cooper's E-60. The arch ring is often close to the track, and the load is applied soon after the arch is finished. Under these conditions, the tensile strength of concrete is too uncertain a factor.

The writer desires to thank Mr. Cain for his interesting discussion.

* "Concrete, Plain and Reinforced," by Taylor and Thompson, page 290.

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TRANSACTIONS.

Paper No. 1006.

THEORY OF THE SPHERICAL OR CONICAL DOME OF REINFORCED CONCRETE OR METAL.*

By William Cain, M. Am. Soc. C. E.

1.—The domes that will be considered in this paper are supposed to be solids of revolution, as to figure; and to be constructed of material such as metal or reinforced concrete, capable of sustaining either tensile or compressive stresses.

The masonry dome can only resist compressive stresses, and, as it has been considered somewhat in detail by the writer,† it will not receive special mention in what follows. It may be remarked, however, that the theory concerning the upper part of any dome, where compressive stresses only are experienced, is common to all domes, whether of metal, reinforced concrete or masonry, so that much that follows pertains to both classes of domes. Both analytical and graphical methods of solution will be given, though the former only applies to the spherical or conical dome of constant thickness, while, in the latter, no such restriction is imposed. Another advantage in the graphical method lies in its simplicity and clearness in interpreting results exactly. In this way it is an aid to the analytical method.

In Fig. 1 is shown a dome in which the soffit is supposed to be

*Presented at the meeting of June 7th, 1905.

†"Theory of Steel-Concrete Arches and Vaulted Structures." D. Van Nostrand Co., publisher.

generated by revolving some curve about the vertical axis. The extrados may be similarly generated, using either a curve or any other figure for the generating line. If we pass, through the axis, two meridian planes making an angle, ψ, with each other, we cut from the dome a portion of a wedge, $F\ C$. Next conceive conical joints perpendicular to the soffit, certain distances apart, and call the part of the dome, as $D\ E$, lying between any two successive conical joints a crown. The four curved edges of a crown evidently lie in horizontal planes. It is convenient to call the portion, V, of the crown, $D\ E$, lying between the two meridian planes, a voussoir, though, of course, there are no true voussoirs in the metal or concrete dome.

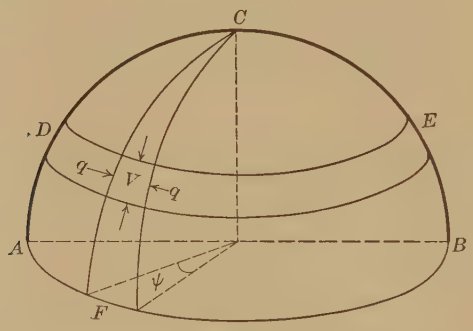

FIG. 1.

A voussoir is thus the portion of a dome included between two contiguous conical joints and between two meridian planes which make an angle, ψ, with each other

The upper part of the dome, under the influence of gravity, tends to fall, and thus induces stresses at all the supposed joints; the magnitude of these stresses it is the object to find. The voussoir, V, as part of the crown, $D\ E$, is subjected to horizontal pressures (or tensions), q, on its vertical faces, and it likewise sustains a meridional thrust acting downward on its upper conical face and a meridional thrust acting upward on its lower conical face. The forces acting on the voussoir, which must be in equilibrium, are thus, its own weight, the two forces, q, and the two meridional thrusts

mentioned. The latter are called meridional thrusts because they act in a meridian plane dividing the voussoir in half.

2.—In the investigation that follows, the resultant, Q, of the two forces, q, q, is first found, and from it and the given angle, ψ, q is computed from the formula below. Since q, q are both horizontal, it follows that Q is horizontal.

To find the relation between Q and q, consider the plan of the voussoir, $A\,B$, Fig. 2, where Q, of course, bisects the angle, ψ, and q, q act perpendicularly to the vertical faces, and hence to the radii drawn at A and B. The sum of the components of q, q, parallel to Q, equals Q.

$$\text{Therefore, } Q = 2\,q \text{ sin. } \frac{1}{2}\,\psi.$$

As eventually we will suppose ψ to be expressed in circular measure and equal to $\frac{1}{n}$ of the circumference of a unit circle $\left(i.\,e., \psi = \frac{2\,\pi}{n}\right)$, and will suppose n very large; then ψ will be very small, and can be made to differ from sin. ψ by as small a quantity as we choose, by sufficiently increasing n. Hence, on replacing sin. $\frac{1}{2}\,\psi$ by $\frac{1}{2}\,\psi$,

$$q = \frac{Q}{\psi} = \frac{Q\,n}{2\,\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Here, Q is the resultant of the forces, q, q. If Q acts to the left, as in the figure, q, q, cause compression in the "crown," $D\,E$, Fig. 1; whereas, if Q acts to the right, the directions of q, q, are reversed, and tension is exerted in the crown.

If a circular steel band or hoop is subjected to a uniform normal pressure, the relation between the force, F, representing the resultant of the pressures on a portion, $A\,B$, of the hoop, subtending the very small angle, ψ, and the resulting tension, T, in the hoop, Fig. 3, is similarly found to be:

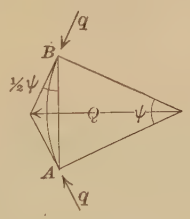

Fig. 2.

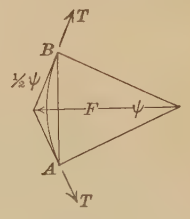

Fig. 3.

$$T = \frac{F}{\psi} = \frac{F\,n}{2\,\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

on noting that T, T and F are in equilibrium. F is here opposed to the resultant of the forces, T, T.

3.—Complete Spherical Dome of Constant Thickness, t feet. Graphical Method.—Let r be the radius, in feet, of the center line of the portion of the dome, $F\,C$, Fig. 1, included between the two meridional planes.

In Fig. 4(a), this center line is represented by the quadrant, $A\,B$, described with C as a center and r as radius $= B\,C = A\,C$. The axis, $C\,A$, is vertical. Divide $A\,C$ into any number of equal parts (10 in the figure), each of length a, and draw horizontals

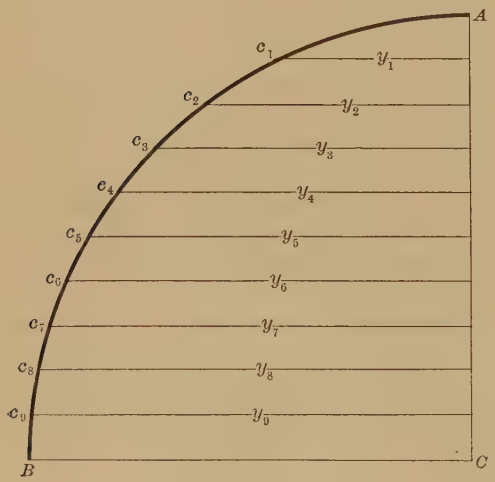

Fig. 4(a).

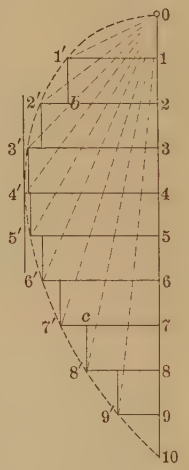

Fig. 4(b).

through the points of division, meeting the curve at c_1, c_2, ... The area of the zone generated by revolving any of the arcs, Ac_1, c_1c_2, c_2c_3, ..., about $A\,C$, each having the altitude, a, is, by geometry, $2\,\pi\,r\,a$, and the weight of the corresponding shell (or "crown" of Fig. 1) is, if the material weighs w pounds per cubic foot, $2\,\pi\,r\,a\,w\,t$. The weight of the corresponding voussoir, V (Fig. 1), is $\frac{1}{n}$ of this.

Therefore, weight of voussoir $= \frac{1}{n}\,2\,\pi\,r\,a\,w\,t = \psi\,r\,a\,w\,t$.

This represents the weight of the successive voussoirs, Ac_1, c_1c_2, c_2c_3, ..., of Fig. 4(a), as the succcessive arcs will now be supposed to refer only to the corresponding voussoirs.

Compute $r\ a\ w\ t$, and, to any scale, lay off on the vertical, 0-10, Fig. 4(b), 0 1 = 1 2 = 2 3, etc., each equal to $r\ a\ w\ t$. The weight of any voussoir is thus represented by ψ times the lengths, 0 1, 1 2, etc., so that the measure of each line in the force diagrams must be multiplied by ψ to give the true result. Of course, the scales used in Figs. 4(a) and 4(b) should be as large as convenient. 0-10 is not generally of the length, $A\ C$. It was drawn so in the figure the better to suit the conveniences of the page. The meridional thrusts at c_1, c_2, ..., are assumed, as in the analytical theory, to be tangent to the center line ($A\ c_1c_2 \ldots B$). Hence, in Figs. 4(a) and 4(b), draw 0 1′, 0 2′, ..., perpendicular to the radii at c_1, c_2, ..., to give the direction of the meridional thrusts at c_1, c_2, ... There is no thrust at the summit, A, since the surface of the conical joint there reduces to a line and there can be no pressure on a line. The forces acting on the first voussoir, $A\ c_1$, are its weight (represented by 0 1 in Fig. 4(b)), the upward meridional thrust at c_1, and the two forces, q, q, the resultant of which replaces them in the triangle of forces, 0 1 1′, formed by drawing a horizontal through 1 to meet 0 1′ at 1′. On following around the triangle in the order 0 1 1′ 0, we see that Q_1, represented by 1 1′, acts to the left; hence the forces, q, q, cause compression. Q_1, of course, acts at the intersection of the tangent at c_1 with the vertical through the center of gravity of the voussoir, $A\ c_1$. It will not be necessary, though, to find the position of any of the Q's.

Draw, in Fig. 4(b), horizontals through 2, 3, ..., to meet 0 2′, 0 3′, ..., at 2′, 3′, ..., respectively. Also, verticals are drawn at 1′, 2′, 3′, ..., as shown. The force diagram for the second voussoir, c_1c_2, is 0 1′ b 2′ 0; 0 1′ representing the downward meridional thrust at c_1; 1′ b the weight of the voussoir; and b 2′, Q_2 and 2′ 0 the upward meridional thrust exerted by the third voussoir upon the second at c_2. Since Q_2 acts to the left, the crown in which the second voussoir is placed is everywhere in compression. The construction is continued as before for all the voussoirs, as shown in Fig. 4(b). About c_4 the forces, Q, change sign, being directed to the left above c_4 and below it, to the right.

Thus the polygon of forces for the voussoir, c_7c_8, is 0 7′ c 8′ 0; 0 7′ representing the downward meridional thrust at c_7, 7′ c the resultant of the corresponding q's $= Q_8$; c 8′ the weight of the voussoir, c_7c_8, and 8′ 0 the upward meridional thrust at c_8, exerted by the ninth voussoir upon the eighth. Since 7′ c acts to the right, there is tension exerted on its vertical faces throughout the crown, of which voussoir, c_7c_8, is a part. Q is greatest near the summit and near the base, and least about c_4.

In a concrete dome, the tension exerted around the successive crowns can be resisted mainly by an enclosed steel hoop, preferably shaped so as to give a mechanical bond between the steel and the concrete.

4.—Meridional Thrust per Unit of Length of Horizontal Circumference.—Since the weights of successive voussoirs were assumed to be ψ 0 1 $= \psi$ 0 2, etc., the numerical measures of the lines in Fig. 4(b), representing meridional thrusts, values of Q, etc., must each be multiplied by ψ to get the true thrust, etc., in pounds. Thus the meridional thrust at c_2 is ψ 0 2′. This thrust acts on a horizontal width of voussoir $\frac{2\,\pi\,y_2}{n} = \psi\, y_2$, if we call y_2 the radius of the horizontal circle at c_2. Hence, the meridional thrust per unit of length of horizontal circumference is ψ 0 2′ divided by ψ y_2, or $\frac{0\ 2'}{y_2}$.

In this way we find the meridional thrusts per unit of length of horizontal circumference

$$\text{at}\ldots\ldots\ldots\ldots c_1,\quad c_2,\quad c_3,\ldots\ldots\ldots\ldots,$$
$$\text{to be},\ldots\ldots\ldots\ldots \frac{0\ 1'}{y_1},\ \frac{0\ 2'}{y_2},\ \frac{0\ 3'}{y_3},\ldots\ldots\ldots\ldots;$$

where 0 1′, 0 2′, 0 3′, mean the numerical measures of the lines, 0 1′, 0 2′, 0 3′, to the scale of force used in Fig. 4(b). The thrusts are thus given in pounds. They each act on an area, $1 \times t = t$ square feet, nearly, so that, on dividing any thrust by t we obtain the unit stress, in pounds per square foot, on the conical joint at that depth. The thrusts increase in going from A to B, Fig. 4(a).

5.—Average Horizontal Side Thrust (or Pull) on the Vertical Face of a Voussoir, per Unit of Length of Meridian.—Let us con-

sider the second voussoir. For it, $Q_2 = \psi\, b\, 2'$; whence, by Equation 1, the thrust around the "crown," including voussoir, c_1c_2, is,

$$q_2 = \frac{Q_2}{\psi} = b\,2'.$$

Let the length, in feet, of the arc, c_1c_2, be designated by s_2; then the average side thrust on the voussoir, per unit of length of meridian, is $\frac{b\,2'}{s_2}$, and the average stress exerted, per unit of area, is $\frac{b\,2'}{s_2\,t}$.

As before, $b\ 2'$ means the numerical measure of the line, $b\ 2'$, Fig. 4(b), to the scale of force.

Similarly, if the length of the arc, c_7c_8, in feet $= s_8$, the pull around the crown in which the voussoir, c_7c_8, lies, per unit of length of meridian, is $\frac{7'\,2}{s_8}$; or generally, if we designate the horizontal projection lengths (measured to scale of force) of $0\,1'$, $1'\,2'$, $2'\,3'$,...., by b_1, b_2, b_3,...., and the corresponding lengths of arc $A\,c_1$, c_1c_2, c_2c_3,...., in feet, by s_1, s_2, s_3,...., then the average side thrust (or pull) on the vertical faces of voussoirs,

$$Ac_1,\ c_1c_2,\ c_2c_3,\ \ldots.,$$

per unit of meridian length will be, $\frac{b_1}{s_1}, \frac{b_2}{s_2}, \frac{b_3}{s_3}\ldots.$, respectively.

These ratios are greatest for the highest and lowest voussoirs.

6.—Hoop Tension.—Let us conceive that the dome only extends from A to the horizontal circle through c_4, where it is supported by a cylindrical wall. The part of this wall, or abutment, contained between the two meridional planes making an angle, ψ, with each other, must be capable of resisting the downward thrust, $\psi\ 0\,4'$ pounds. If it is too narrow to withstand the horizontal component of this thrust, $\psi\ 4\ 4'$ pounds, a steel hoop may be made to encircle the dome at the level of c_4, and, by Equation 2, the tension, T, in it will be,

$$T = \frac{\psi\,4\,4'}{\psi} = 4\,4' \text{ pounds},$$

where $4\,4'$ means the numerical measure of the length, $4\,4'$, Fig. 4(b), measured to the scale of force.*

*This gives results agreeing with Rankine's formula for hoop tension (where it is a maximum), $0.3\ r^2\ w\ t$, on substituting $r = w = t = 1$. Here, $a = \frac{1}{10}\ r = \frac{1}{10} = 0\,1 = 0\,2$, etc., in Fig. 4($b$), since $0\,1 = r\ a\ w\ t$, and $4\,4'$ = hoop tension, measures 0.3. The point of maximum hoop tension is really at 0.618 r above C, Fig. 4(a); but the tension there differs from $4\,4'$ inappreciably, as is evident from Fig. 4(b).

Similarly, we can suppose the abutment successively at c_1, c_2, c_3,...., and the consequent hoop tensions will be in pounds, the lengths of 1 1′, 2 2′, 3 3′,...., in turn, all measured to the scale of force.

A sharp distinction must be drawn between the total horizontal thrust at a joint, other than c_1, and the value of Q pertaining to the voussoir in the crown just above it. Thus, for the second voussoir, $Q = \psi\ b\ 2'$, but the total horizontal thrusts exerted at c_1 and c_2 are ψ 1 1′ and ψ 2 2′, respectively. We have 2 2′ = 1 1′ + b 2′, also 7 7′ — c 7′ = 8 8′, etc. Overlooking this point has sometimes caused some confusion.

The stresses in a masonry dome extending from A to the horizontal circle through c_4 (about) are ascertained exactly as just described. If the dome extends below the level of c_4, the supposed wedges below c_4 contained between meridian planes, act as simple arches, and the thickness must be materially increased as we approach B, Fig. 4(a), which is expensive. The alternative is the use of a metal band or bands, unless the stones can be cut and joined so as to resist tension.

7.—*Open Spherical Dome.*—Suppose the portion of the dome, Fig. 4(a), from A to the level of c_1, to be removed. We will then have an open dome, and, of course, no pressure at c_1. The weight of the voussoir, Ac_1, is now zero; therefore 0 moves to 1 in Fig. 4(b) and the radiating meridional thrusts are drawn from 0 = 1.

The subsequent constructions and conclusions are exactly as above, save that the point of maximum hoop tension is lowered.

8.—*Open Spherical Dome with a Lantern.*—Let Fig. 5(a) represent a half meridian section of a spherical dome, the center line of the shell being c_0-c_{37}, the portion above the level of c_0 being removed, and a lantern, of total weight, W, resting on the crown of the shell, the center line of which is c_0-c_4. Divide the vertical projection of the arc, c_0c_4, into (say) four equal parts, each of altitude a' and draw the horizontal radii, y_0, y_1, y_2, y_3, y_4, from the vertical axis, through the points of division to c_0, c_1, c_2, c_3, c_4.

Let p = weight, in pounds per square foot, of the horizontal section of the lantern.

$$\text{Therefore, } p = \frac{W}{\pi\ (y_4^2 - y_0^2)}$$

As before, consider the portion of the dome and load between two meridian planes making an angle, $\psi = \frac{2\pi}{n}$, with each other. The weight of the lantern resting on the voussoir $c_0 c_1$ is thus,

$$\frac{1}{n} p \pi (y_1^2 - y_0^2) = \frac{2\pi}{n} \frac{p}{2} (y_1^2 - y_0^2) = \psi \frac{p}{2} (y_1^2 - y_0^2).$$

Similarly, the lantern loads resting on the voussoirs, c_1c_2, c_2c_3, c_3c_4, are, respectively,

$$\psi \frac{p}{2} (y_2^2 - y_1^2), \ \psi \frac{p}{2} (y_3^2 - y_2^2), \ \psi \frac{p}{2} (y_4^2 - y_3^2).$$

Add to each of these the weight of the voussoir on which it rests, $\psi\ r\ a'\ w\ t$ (Art. 4) and lay off to scale the successive sums divided by ψ from 0 on the vertical load line, Fig. 5(b), downward.

Thus the weight of the voussoir, $c_0 c_1$, and the lantern load = ψ 0 1; the weight of the voussoir, $c_1 c_2$, with load = ψ 1 2; of voussoir, $c_2 c_3$, and load, ψ 2 3, and for voussoir $c_3 c_4$, ψ 3 4.

Next, divide the vertical projection of the remaining arc, $c_4 c_{37}$ (Fig. 5(a)), into a number of equal parts, each of length a. Then, as in Art. 4, the weights of the corresponding voussoirs, c_4c_5, c_5c_6, etc., are each $\psi\ r\ a\ w\ t$. Hence, in Fig. 5(b), lay off 4 5 = 5 6, etc., each equal to $r\ a\ w\ t$, completing the load line 0-37.

As before, from 0, Fig. 5(b), draw 0 1′, 0 2′, 0 3′, ..., perpendicular to the radii of the sphere at c_1, c_2, c_3, ..., giving the directions of the tangents to the arc, c_0c_{37}, at c_1, c_2, c_3, ..., respectively. The horizontals through 1, 2, 3, ..., cut these rays at 1′, 2′, 3′, ..., respectively. The force diagram is then completed by marking off the horizontal projections of 1′ 2′, 2′ 3′, etc.

The meridional, crown and hoop stresses are found now exactly as detailed in Arts. 4, 5 and 6. Thus the meridional thrusts per unit of length of horizontal circumference at c_0, c_1, c_2, ..., are, respectively,

$$0, \frac{0\ 1'}{y_1}, \frac{0\ 2'}{y_2}, \ldots.$$

The tension in a hoop encircling the dome at the level of (say) c_4 is 4 4′ pounds; at c_9, 9 9′ pounds, etc.

Finally, if we designate the horizontal projections of 0 1′, 1′ 2′, 2′ 3′, 3′ 4′, ..., (measured to scale of force) by b_1, b_2, b_3, b_4, ..., and the lengths of arcs, c_0c_1, c_1c_2, c_2c_3, c_3c_4, ..., (to scale of

distance) in feet, by s_1, s_2, s_3, s_4, ...; then the average side stresses (compression or tension) on the vertical faces of voussoirs,

$$c_0c_1,\ c_1c_2,\ c_2c_3,\ c_3c_4,\ \ldots.,$$

per unit of meridian length (the "crown stresses"), are,

$$\frac{b_1}{s_1},\ \frac{b_2}{s_2},\ \frac{b_3}{s_3},\ \frac{b_4}{s_4},\ \ldots,\ \text{pounds, respectively.}$$

The greatest crown stress is that on the voussoir, c_0c_1, and is

$$\frac{b_1}{s_1} = \frac{1\ 1'\ \text{(to scale of force)}}{\text{length of } c_0\ c_1,\ \text{in feet}}\ \text{pounds compression.}$$

This is evidently greater than the average crown stress on the voussoir, c_0c_4, which is $4\ 4' \div$ length of the arc, c_0c_4.

In practice, the portion of the arc, c_0c_4, on which the lantern rests is of small length. Thus, in the dome of St. Peter's, at Rome, the thickness of the lantern at the base is only 2.1 ft. In Fig. 5(a), the lantern was given an abnormal thickness to make a plainer diagram and illustrate an important point hitherto unnoticed, *viz.*, that in a dome with a lantern, the slope of the curve, $0\ 1'\ 2'\ 3'\ 4'\ 5'\ldots.$, is discontinuous at 4′. The corresponding curve for a closed dome with no lantern is continuous as to slope. The rate of increase of the crown stress is thus discontinuous at the outside edge of the lantern. With a light lantern, even, although the crown stresses may be both compressive on either side of the outer edge of lantern, yet they make a sudden jump in passing this edge. With the very heavy lantern of Fig. 5(a), the crown stress in the voussoir, c_3c_4, is compressive, but for the voussoir, c_4c_5, and all below it, the crown stress is tensile. The sudden change in the slope of the curve occurs at 4′, and the curve below that has a point of inflection.

We are now prepared to find, in the analytical solution that follows, different formulas for the crown stresses on either side of c_4, however near to c_4 the points on either side of c_4 are taken.

This abnormality results from omitting the elastic yielding of the dome under stress. This yielding, due to crown stress, is greatest on the voussoir, c_0c_1, which brings the one below it more in action, as more of the weight of the lantern will then be transferred to it, and so on, down; besides, the direction of the meridional thrust just below c_4 is doubtless nearer the horizontal than drawn, throwing 5′ to the left. If the resultant meridional or crown stress passes

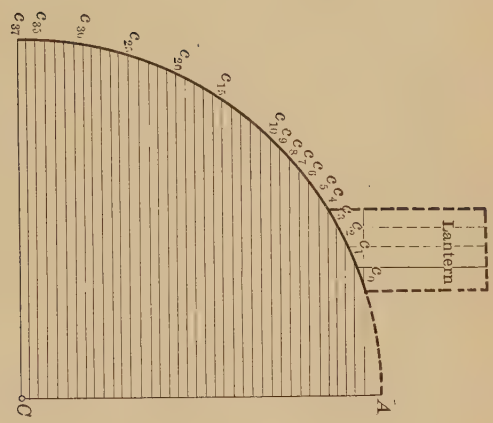

Fig. 5(a).

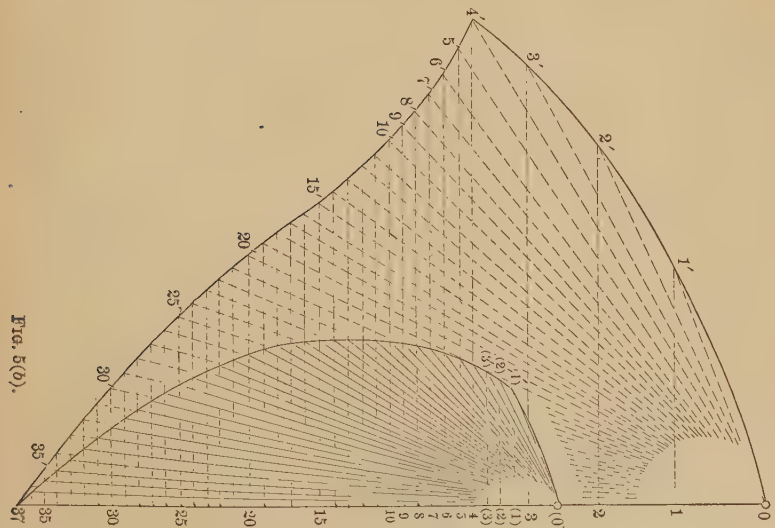

Fig. 5(b).

through the middle-third limit, the maximum unit stress on the joint is double the average. The elastic yielding of the dome will undoubtedly cause a non-uniform distribution of the stress over most of the (conceived) joints, and, to be on the safe side, it is wise to take the maximum unit stress as double the average, unless the dome is thicker than usual, when it may be assumed as less.

The lantern, for the given weight, p, per horizontal square foot, may be assumed to bear only upon c_0c_3, c_0c_2 and c_0c_1, in turn; but each curve, corresponding to $0\,1'\,2'\,3'\,\ldots$., will have the same characteristics. As p diminishes, when c_0c_1 alone supports a lantern load, the curve gradually approaches the usual form for a closed dome with no lantern. Thus, on the load line, lay off $4(3) = (3)(2) = (2)(1)$ to represent the weight of the voussoirs, c_4c_3, c_3c_2, c_2c_1, divided by ψ and $(1)(0) = \frac{1}{2}\,0\,1$, for the weight of the voussoir, c_0c_1, with its lantern load, also divided by ψ. Taking (0) as a pole, the curve corresponding to $0\,1'\,2'\,3'\,\ldots$., is drawn and it will be noticed that the crown stresses are not only compressive on either side of c_1 (the outer edge of the lantern), but continue so for some distance down. There is, apparently, still discontinuity of the slope at (1').

9.—Analytical Theory of Spherical Domes.—Let Fig. 6 represent half of a vertical section through the axis, $O\,C$, of the dome. To take the most general case, let the dome be open above, with a circular opening, of radius $E\,O = y_0$, having a lantern resting on the arc, $D\,E$, of total weight $= W$ pounds. The small constant thickness of the dome is t feet and its weight per cubic foot is w pounds.

Take the origin at O, x vertical, y horizontal, and call the coordinates of P, $x = O\,N$, $y = N\,P$. The mean radius of the dome, $C\,P = C\,A = r$; the angle, α (in circular measure) $=$ the angle, $A\,C\,P$, and $C\,N = h$. Let $s =$ the length of the arc, $A\,P$; whence $s = r\,\alpha$, and, differentiating, $\frac{d\,\alpha}{d\,s} = \frac{1}{r}$. The differential triangle is drawn at P, whence

$$\frac{d\,x}{d\,s} = \sin.\,\alpha = \frac{y}{r};\quad \frac{d\,y}{d\,s} = \cos.\,\alpha = \frac{h}{r}.$$

The whole weight of the lantern and the part of the dome with altitude x, as before, is $(2\,\pi\,r\,w\,t\,x + W)$. As in previous articles,

the part considered will be $\frac{1}{n}$ of this, contained between two meridian planes making an angle, $\psi = \frac{2\pi}{n}$, with each other. Thus

$$\frac{1}{n}(2\pi r w t x + W) = \psi r w t x + \frac{W}{n}$$

is the vertical load sustained at P, when P is below D, and since the meridional thrust there is supposed to act in the direction of the tangent at P, its value is,

$$\left(\psi r w t x + \frac{W}{n}\right) \text{ cosec. } \alpha = \left(\psi r w t x + \frac{W}{n}\right)\frac{r}{y}.$$

The meridional thrust per unit of horizontal circle at P, call it M, is found from this, by dividing by $\frac{2\pi y}{n} = \psi y$, to be,

$$M = \frac{r^2 w t}{y^2} x + \frac{W r}{2\pi y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This formula is true when P is anywhere between D and B.

The force triangle to the right in Fig. 6 indicates, by the vertical line, the load above $\left(\psi r w t x + \frac{W}{n}\right)$, supported at P, the hypothenuse is the meridional thrust at P and the base is the total horizontal thrust at $P = H$. This triangle is similar in meaning to any one of the triangles, as 0 3 3′ in Fig. 4(*b*) or 0 9 9′ in Fig. 5(*b*).

With the notation just preceding Equation (5), further on, the meridional thrust at a point (x, y) under the lantern load is readily found, as follows:

$$\text{The vertical load at } (x, y) = \psi r w t x + \frac{p\psi}{2}(y^2 - y_0^2).$$

On multiplying this by cosec. $\alpha = \frac{r}{y}$ and dividing by ψy, we find,

$$M = \left[r w t x + \frac{p}{2}(y^2 - y_0^2)\right]\frac{r}{y^2} \quad \ldots\ldots\ldots\ldots (3')$$

At E, $x = 0$, $y = y_0$; therefore $M = 0$; at D, (3′) = (3).

On changing x to $x + \Delta x$ in Fig. 6, y changes to $y + \Delta y$, s to $s + \Delta s$, α to $\alpha + \Delta \alpha$ and H to $H + \Delta H$.

$\frac{\Delta H}{\Delta s}$ = average rate of increase of H per unit of meridian length,

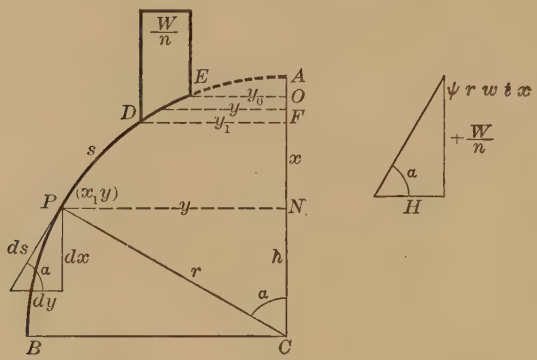

FIG. 6.

and the limit, as Δs approaches indefinitely 0, is,

$\frac{dH}{ds}$ = rate of increase at P per meridian unit

= intensity of Q (Art. 2) per meridian unit, at P.

From the force triangle, for points, P, below D,

$$H = \left(\psi\, r\, w\, t\, x + \frac{W}{n}\right) \text{cot. } \alpha.$$

Therefore

$$\frac{dH}{ds} = \left(\psi\, r\, w\, t\, x + \frac{W}{n}\right)\left(-\text{cosec.}^2\ \alpha\, \frac{d\alpha}{ds}\right) + \psi\, r\, w\, t\, \frac{dx}{ds} \text{cot. } \alpha.$$

$$= \left(\psi\, r\, w\, t\, x + \frac{W}{n}\right)\left(-\frac{r^2}{y^2}\,\frac{1}{r}\right) + \psi\, r\, w\, t\, \frac{y}{r}\,\frac{h}{y}.$$

Therefore $\frac{dH}{ds} = \psi\, r\, w\, t\, \left(-\frac{x\, r}{y^2} + \frac{h}{r}\right) - \frac{W\, r}{n\, y^2}$.

On dividing this by $\frac{2\,\pi}{n} = \psi$, Art. 2 (1), we get the side thrust on a voussoir per unit of meridian length, or the "crown" thrust at (x, y), when P is below D.

$$C = w\, t\, h - w\, t\, x\, \frac{r^2}{y^2} - \frac{W\, r}{2\,\pi\, y^2} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

This formula gives the intensity of the "crown" pressure (or tension), in pounds, at the point, (x, y).

If there is no lantern, $W = 0$, and the last term drops out, giving the crown pressure for the open dome.

If the dome is closed, or complete, the points, D, and O, move to A; and x must be reckoned from the summit, A.

By the use of Figs. 4(a), 4(b), the significance of the foregoing steps will be more apparent. Thus, if we conceive the point, P, of Fig. 6 to be at c_1 Fig. 4(a), whence $s = A\ c_1$ and $H = \psi\ 1\,1'$, Fig. 4(b), and that s is increased to $s + \Delta\ s = Ac_2$ (say); then $H + \Delta\ H = \psi\ 2\ 2'$; consequently, $\Delta\ H = \psi\ b\ 2' = Q_2$ (Art. 5) and $\Delta\ s = \text{arc}\ c_1\ c_2$. Hence, $\frac{\Delta\ H}{\Delta\ s} = \frac{Q_2}{\text{arc}\ c_1\ c_2}$, gives the average value of Q affecting the voussoir, c_1c_2, per unit of meridian length. This changes as $\Delta\ s$ becomes smaller and approaches a limit, as $\Delta\ s$ approaches indefinitely 0, $\frac{d\ H}{d\ s}$, which is defined to be the intensity of Q per unit meridian at the point, $(x, y) = c_1$, here.

If we suppose $P = (x, y)$ at c_7, Fig. 4(a), and that $\Delta\ s = \text{arc}\ c_7\ c_8$, then $H = \psi\ 7\ 7'$, $H + \Delta\ H = \psi\ 8\ 8'$; whence $\Delta\ H = -\psi\ c\ 7'$ and $\frac{\Delta\ H}{\Delta\ s} = \frac{Q_8}{\text{arc}\ c_7\ c_8}$ is negative, whence its limit is negative. We conclude that when Formula (4) gives a negative value for C, the crown thrust, tension is exerted around the crown.

The crown pressures, as given by Formula (4) and the diagram, are not identical, the one referring to intensity at a point, the other to average stress over an area. On assuming a large number of divisions of $A\ C$, and taking $P = (x, y)$, at the center of a corresponding voussoir, the two methods should give practically identical results.

Let us now find the crown stress at any point, (x, y), of the arc, $D\ E$, under the lantern load. As in Art. 8, let p = weight, in pounds per horizontal square foot, of the lantern, so that,

$$p = \frac{W}{\pi\,(y_1^2 - y_0^2)}$$

The weight of the lantern load resting on the voussoir extending from E, Fig. 6, to the point, (x, y), is thus,

$$\frac{1}{n}\,p\,\pi\,(y^2 - y_0^2) = \frac{p\ \psi}{2}\,(y^2 - y_0^2).$$

On adding the weight of the voussoir to this, and multiplying the sum by cot. α, as before, we find the total horizontal thrust exerted at (x, y).

$$H = [r\ w\ t\ x + \frac{p}{2}\,(y^2 - y_0^2)]\ \psi\ \text{cot.}\ \alpha.$$

Proceeding as above,

$$\frac{1}{\psi}\frac{d.H}{ds} = \left[r\ w\ t\ x + \frac{p}{2}(y^2 - y_0^2) \right]\left(- \text{cosec.}^2\ \alpha\ \frac{d\ \alpha}{d\ s} \right) + \left[r\ w\ t \frac{d\ x}{d\ s} + p\ y \frac{d\ y}{d\ s} \right]\frac{h}{y}.$$

$$\text{Therefore, } C = \left[r\ w\ t\ x + \frac{p}{2}(y^2 - y_0^2) \right]\left(- \frac{r}{y^2} \right) + \left[w\ t\ y + p\ y \frac{h}{r} \right]\frac{h}{y}.$$

$$\text{Therefore, } C = w\ t \left(h - r^2 \frac{x}{y^2} \right) + p \frac{h^2}{r} - \frac{p}{2}(y^2 - y_0^2)\ \frac{r}{y^2} \cdots\cdots (5)$$

This value of C is the crown pressure at a point, (x, y), on the arc lying between D and E.

At the inner edge of the lantern, E, $x = 0$, $y = y_0$, $h = C\ O$.

$$\text{Therefore, } C \text{ (at } E) = w\ t\ C\ O + p\ \frac{C\ O^2}{r}.$$

Just above D, the outer edge of the lantern, $x = O\ F = x_1$, $y = y_1$, $h = C\ F = h_1$; therefore, from Equation (5),

$$C = w\ t \left(h_1 - r^2 \frac{x_1}{{y_1}^2} \right) + \frac{p\ {h_1}^2}{r} - \frac{W\ r}{2\ \pi\ {y_1}^2}.$$

Just below D, from Equation (4),

$$C = w\ t \left(h_1 - r^2 \frac{x_1}{{y_1}^2} \right) - \frac{W\ r}{2\ \pi\ {y_1}^2}.$$

There are thus different values for C just above and just below D, exactly as was found by the graphical method, the difference being $\frac{p\ {h_1}^2}{r}$. If the first value of C is positive and the second is negative, the joint of rupture is at D. If both are positive, the joint of rupture is lower down. Compare results with Figs. 5(a), 5(b).

At the base, B, $x = O\ C$, $y = r$, $h = 0$; therefore, from Equation (4),

$$C = - w\ t\ C\ O - \frac{W}{2\ \pi\ r}\ \text{(tension)}.$$

When the origin is changed to F, the two formulas, (3) and (4), are identical in meaning with those given by E. Schmitt, Assoc. M. Am. Soc. C. E., in his paper on domes,* except for his unwarranted

* *Transactions*, Am. Soc. C. E., Vol. LII, p. 270.

use of the double sign in his Equation 27; but some results are given just below his Equation 27 which do not agree with those just found.

For a closed dome with no lantern, O is put at A, $x = A\ N = r - h$, and Equation (4) reduces to,

$$C = w\ t\ h - w\ t\ \frac{(r-h)\ r^2}{r^2 - h^2} = w\ t\ \left(h - \frac{r^2}{r+h}\right).$$

This is zero at $h = 0.618\ r$, corresponding to $\alpha = 51°\ 49'$. The joint at this point is called the "joint of rupture." Above this joint C is positive or in compression, below it is negative or in tension.

The hoop tension in a metal band encircling the dome at a height, h, above C, Fig. 6, is found as follows (see, also, Art. 6): Take the case of the open dome with the lantern. The total horizontal thrust at P (below D) exerted on the voussoir just below P is,

$$H = \left(\psi\ r\ w\ t\ x + \frac{W}{n}\right) \cot.\ \alpha = \left(\psi\ r\ w\ t\ x + \frac{W}{n}\right)\frac{h}{y}.$$

Therefore, by Equation (2), the tension, T, in the band is,

$$T = \frac{H}{\psi} = \left(r\ w\ t\ x + \frac{W}{2\pi}\right)\frac{h}{y} \quad \ldots\ldots\ldots\ldots (6)$$

For the closed dome with no lantern, $W = 0$, and with O at A, $x = r\ (1 - \cos.\ \alpha)$, $n = r \cos.\ \alpha$ and $y = r \sin.\ \alpha$,

$$\text{therefore } \frac{T}{w\ t} = r^2\ \frac{(1 - \cos.\ \alpha)\ \cos.\ \alpha}{\sin.\ \alpha} = r^2\ \frac{\sin.\ \alpha \cos.\ \alpha}{1 + \cos.\ \alpha},$$

which is Rankine's formula.* At the joint of rupture, where $\alpha = 51°\ 49'$,

$$T = 0.3\ w\ t\ r^2.$$

This is the maximum hoop tension for a spherical dome of constant (small) thickness, t.

10.—Let us discuss a little more fully the closed dome with no lantern. The origin, O, in Fig. 6, now moves to the summit, A;

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* "Applied Mechanics," p. 267.

NOTE.—To compare results with those given by Rankine for the closed dome with no lantern (the only case discussed by him), we observe that if P_x denotes the total weight of the dome to the level of (x, y), then, for $W = 0$, M (of Equation 3, above) $= \frac{P_x \text{ cosec. } \alpha}{2\pi y}$, C (of Equation 4, above) = Rankine's p_z and T (of Equation 6, above) = Rankine's $P_{y'}$.

Rankine's results, expressed in trigonometrical functions, can be easily expressed in terms of x, y and h. They are found to be identical with those given by Equations (3), (4), and (6), above. The results for a truncated conical dome with a lantern, given in Art. 11, are likewise identical with those given by Rankine, when reduced as before.

whence $x = A\ N = r - h$ and $y^2 = r^2 - h^2$. Also, $W = 0$. Formulas (3) and (4) reduce to,

$$M = w\ t\ \frac{r^2}{r + h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3')$$

$$C = w\ t\ \left(h - \frac{r^2}{r + h} \right) \quad \ldots\ldots\ldots\ldots\ldots (4')$$

M increases as h diminishes. Its least value, indefinitely near A, for which $h = r$, is $\frac{1}{2}\ w\ t\ r$ (compression). Its greatest value is at the base, B, for which $h = 0$, and is $w\ t\ r$, also compression. The derivative of C, with respect to h, is $\left(1 + \frac{r^2}{(r + h)^2} \right) w\ t$, a positive quantity, indicating that C increases algebraically as h increases. At the base, B, $h = 0$ and $C = -\ w\ t\ r$ (maximum tension); when $h = 0.618\ r$, $C = 0$; finally, indefinitely near the summit, A, $h = r$ and $C = \frac{1}{2}\ w\ t\ r$ (maximum compression).

The thrust, M, acts upon a conical joint the area of which is $t \times 1 = t$ square feet. Similarly, C acts upon a vertical joint of the same area, t. Whence, dividing the above values by t, the stresses, in pounds per square foot,

due to the meridional thrust at $A = \frac{1}{2}\ w\ r$, compression
" " " " " " $B = w\ r$, "
" " " crown " " $A = \frac{1}{2}\ w\ r$, "
" " " " " " $B = -w\ r$, tension.

These stresses represent the average stress, in pounds per square foot, over the corresponding joints. If the resultant force passes through the center of the joint, they are maximum stresses; if it passes $\frac{1}{3}\ t$ from the edge, the maximum unit stresses are double the above values. For safety, considering the elastic yielding of the dome, whatever the thickness, t, of the shell, double the average unit stresses above should be used, and perhaps more for very thin domes.

The remarkable fact concerning all unit stresses in a closed dome is that whatever the thickness, t, of the shell, the unit stresses are the same for the same radius. Increasing t does not increase the strength, unless the elastic yielding of the dome materially modifies results, by causing the resistance lines to depart so far from

the centers of the joints that the unit stresses are very much increased, and even considerable tension may be caused in some of the joints.

In the present state of our knowledge, it is impossible to locate accurately these lines connecting the centers of pressure on the joints of either a crown or a wedge. Hence, it is wise to be conservative in assuming the thickness of a dome.

Reinforced concrete seems to be admirably adapted to the construction of domes. The concrete can take all compressive stresses and the steel the tensile stresses developed in the lower crowns of the dome.

For a complete spherical concrete dome, let us assume $w = 150$ (lb. per cu. ft.), $r = 50$ (ft.); then, double the average meridional compressive stress at the base, is $2\ w\ r = 15\,000$ lb. per sq. ft. $=$ 104 lb. per sq. in. Double the average tension in the crown, 1 ft. high, resting on the base, is also 104 lb. per sq. in. A very small chain, or steel reinforcement, can take this tension without yielding appreciably under the stress. It would be risky to count on the concrete furnishing any tensile resistance.

The stresses at the summit, both crown and meridional, are compressive, and double the average is only 52 lb. per sq. in.

The stresses at the base are the maximum stresses, either crown or meridional, experienced anywhere in the closed dome, no matter what the thickness, if the assumptions are realized. Thus, if it were possible for concrete to stand a compressive stress of 104 lb. per sq. in., when of the thickness of an egg-shell, a spherical concrete-steel dome 100 ft. in diameter would be perfectly stable, if it were no thicker than an egg-shell. Of course, the assumptions are not exactly realized. The meridional and crown thrusts are rarely applied at the centers of the joints; in fact, for a very thin shell, not built perfectly true, they can easily, at some joints, not only lie outside of the middle third of the cross-section of the shell, but even outside the shell, causing tension on the opposite surface. With a thick shell, this can rarely, if ever, happen. As, in the theory of the simple (barrel) arch, the elastic yielding of the arch determines the line of the centers of pressure, so here, the elastic yielding of the dome must determine the corresponding centers of pressure, though

the theory has never been formulated. For a thin metal arch, the centers of pressure may lie outside the arch, and the same thing may happen for a thin concrete dome.

If a complete theory of the dome, considering its elasticity, were known, a proper thickness could be found to satisfy both strength and stability. As it is, our present imperfect theory gives no proper answer to this question, and the thickness has to be assumed; based, to some extent, on the successful practice of the past.

It is natural to think that, since the average stresses at the base of a complete spherical dome are double those at the summit, the thickness should be greater at the base to equalize the stresses more nearly. It is well to count the extra cost before doing so.

Thus, take the complete dome already examined, where $w = 150$, $r = 50$ (ft.). The average stress at the base is 52 lb. per sq. in., and at the summit 26 lb. per sq. in. for any thickness, t. Assume $t = 1$ (ft.). Then conceive a dome having the thickness, 1 ft. at the summit and 2 ft. at the base. The radius of the intrados is 50 ft. and that of the extrados is readily found to be 52.01 ft. The volume of the dome is the difference between the volume of the spherical segment with radius 52.01 and altitude 51 and that of the hemisphere of radius 50. It is found to be $7\,728\ \pi$ cu. ft.

The vertical component of the thrust at the base, $7\,728\ w\ \pi$, divided by the area of the base $= 204 \times 144\ \pi$ sq. in., gives the meridional stress at the base $= 39.6$ lb. per sq. in. Since the contour curves at the summit are parallel, the stress there will be the same as for a dome of constant thickness and mean radius 50.5, or equal to $\frac{1}{2}\ w\ r = \frac{1}{2}\ 150\ (50.05) \div 144 = 26$ lb. per sq. in., which still differs considerably from the stress at the base. Comparing results with the dome of constant thickness, 1 ft., and radius, 50 ft., the volume of which is $5\,101\ \pi$, we see that the unit stress at the base has been decreased from 52 to 39.6 lb. per sq. in., while the volume has been increased in the ratio 7 728 to 5 101.

If we reduce the thickness at the summit to 6 in. and retain the thickness of 2 ft. at the base, the stresses at the summit and base are 26.3 and 32.7 lb. per sq. in., respectively, and the volume $= 6\,410\ \pi$.

The unit stresses in the dome of radius 50 ft. and constant thickness $t = 0.5$ (ft.) are 26 and 52, as before, and the volume = 2 525 π.

The stresses at the base and summit have now been more nearly equalized, by varying the thickness from 0.5 ft. at the summit to 2 ft. at the base, but the volume has been increased over that of the dome with constant thickness, 0.5 ft., in the ratio 6 410 to 2 525. The reduction in stress seems hardly worth the extra cost in either case for a dome of this size unless the maximum unit stress was ten times the average maximum 52 lb. per sq. in., or 520 (the safe compressive strength) which could only happen when the thrust was near the intrados or extrados.

The limiting radius for a reinforced concrete dome of such thickness that the center of pressure on any joint is nowhere nearer the surface than one-third the thickness, and where the maximum stress allowed on concrete in compression is 500 lb. per sq. in., is found, by equating the maximum stress, 2 (w r), with 500 $\times$ 144, to be,

$$r = \frac{72\,000}{300} = 240 \text{ ft.}$$

In such a dome, it would certainly be advisable to increase the thickness from the summit down, for wind stresses have to be allowed for, in addition to other stresses due to fixed load. Concrete cannot always be depended on in tension, since shrinkage cracks have been known to appear. If its resistance in tension is ignored, we have another reason why the thickness of domes should not be too small; since, with a thin shell of, say, 6 in., the thrust need only move 1 in. from the center of the joint to double the stress and 2 in. from the center to quadruple the average stress, whereas, if the thickness is 12 in., the resultant on the joint would have to move double these distances to double and quadruple, respectively, the average stress. If the thrust, in either case, moved to the extrados or intrados (and still ignoring the concrete in tension), the stress would be infinite and the concrete would crush.

What would become of our "egg-shell" dome with a movement of the thrust only one-half the thickness of the egg-shell?

Such considerations go to show the theoretical necessity for having a dome of appreciable thickness.

If the thickness of the dome is varied, the graphical method,

hitherto given, affords a ready solution; but now the successive weights to be laid off along the vertical load line are not equal for equal altitudes, as before. The construction and final results are otherwise as given in Articles 3-8. It will suffice to describe the center line of the meridional section of the half dome with a radius, r, found by trial, having its center on the axis of the dome. Then, as in Art. 3, the weight of a voussoir will be $\psi\ r\ a\ w\ t$, approximately, where t = mean thickness of the voussoir. A large number of divisions should be used to cause the total volume to agree with that computed, as in the examples of this article. The meridional thrusts are, as before, drawn perpendicular to the radii of the center line at the points, c_1, c_2, etc.

11.—Truncated Conical Dome Supporting a Lantern of Weight, W.—Let Fig. 7 represent half a meridional section of the dome, c_0c_4, supporting a cylindrical lantern at the top, c_0. The intersection of the center line, c_0c_4, with the vertical axis of the dome is at O, the angle between these two lines is α, and O, being taken as the origin, x vertical and y horizontal, the co-ordinates of c_0 will be called (x_0, y_0) and of any point, c, (x, y) with the proper subscripts.

Divide the altitude of the dome into any number of equal parts (only four in the figure—a much larger number is desirable) each of height a feet, and pass horizontal planes through the points of division, cutting the dome at c_0, c_1, c_2.....Call the horizontal radius of the point midway between c_1 and c_2, r_2, the horizontal thickness of the shell t (both in feet), then, by the law of Pappus, the volume of the voussoir, c_1c_2, included between the two meridian planes making an angle, $\psi = \frac{2\pi}{n}$, with each other, is $a\ t\ r_2\ \psi$, and if w is the weight of the dome per cubic foot,

$$\text{the weight of the voussoir, } c_1c_2 = \psi\ a\ t\ r_2\ \psi$$

and similarly, for the weight of any voussoir. These weights vary with r.

The weight of the lantern contained between the two meridian planes is $\frac{W}{n} = \frac{2\pi}{n} : \frac{W}{2\pi} = \psi \frac{W}{2\pi}$.

The construction and interpretation of the force diagram is so similar to that given for the spherical dome (Arts. 8, 3, 4, 5, 6) that

reference to preceding articles will sufficiently explain the successive steps below.

In the force diagram, Fig. 7, lay off $A\ 0 = \frac{W}{2\pi}$ vertically; then $0\ 1 = w\ a\ t\ r_1$ for the voussoir, c_0c_1; $1\ 2 = w\ a\ t\ r_2$ for the voussoir, c_1c_2, etc.; whence the actual weights are, in turn, $\psi\ A\ 0$, $\psi\ 0\ 1$, $\psi\ 0\ 2, \ldots$, and the measures of all lines in the force diagram must be multiplied by ψ to give correct results.

Assuming, as for the spherical dome, that the meridional thrust at any point is tangent to the center line of part of the shell included between the two meridional planes making an angle, ψ, with each other, it is seen here that the meridional thrusts act along c_0c_4. Hence, draw $A\ 4'$ parallel to c_0c_4 and through the points, 0, 1, 2, ..., draw horizontals meeting at $A\ 4'$ at $0'\ 1'$, $2'$, ...

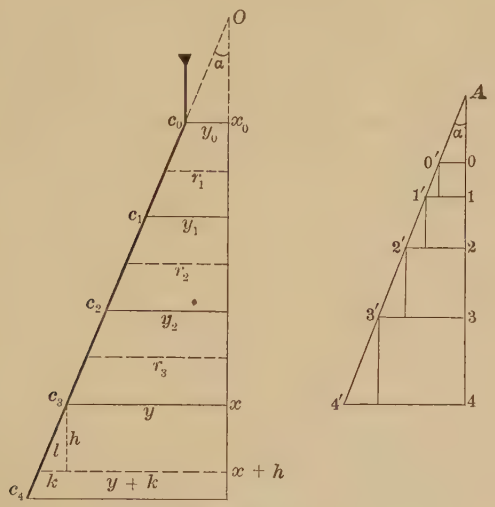


Fig. 7.

It follows that the meridional thrusts at c_0, c_1, c_2, ..., are exactly $\psi\ A\ 0'$, $\psi\ A\ 1'$, $\psi\ A\ 2'$, ...; whence the meridian thrusts per unit of length of horizontal circle through c_0, c_1, c_2, ..., are found by dividing these values in turn by $\frac{2\pi y}{n} = \psi y$, the horizontal width of the voussoir at the point, and are, respectively,

$$\frac{A\ 0'}{y_0} \text{ at } c_0, \frac{A\ 1'}{y_1} \text{ at } c_1, \frac{A\ 2'}{y_2} \text{ at } c_2, \ldots..$$

Also, as in Art. 6, the hoop tensions at c_0, c_1, c_2, ..., are 0 0′, 1 1′, 2 2′, ..., respectively.

The horizontal radial force exerted on the voussoir, $c_3\, c_4$, is evidently (see Art. 3) $Q_4 = \psi\ (4\,4' - 3\,3')$; and similarly for the others.

Call the lengths, $c_0c_1 = c_1c_2 = c_2c_3 = \text{etc.} = s$; then the average side thrust on the voussoir, c_4c_3, per foot of meridian (see Art. 2), $c_4 = \frac{q}{s} = \frac{Q}{\psi\, s} = \frac{4\,4' - 3\,3'}{s}$; or, generally, if we designate the horizontal projections of 0′ 1′, 1′ 2′, 2′ 3′, ..., by b_1, b_2, b_3, ..., respectively, the average side thrust on the voussoirs, c_0c_1, c_1c_2, c_2c_3, ..., per foot of meridian, will be, $\frac{b_1}{s}$, $\frac{b_2}{s}$, $\frac{b_3}{s}$, ..., respectively.

12.—Analytical Solution for the Truncated Dome with a Lantern of Weight, W.—The thickness of the shell, t, will now be taken at right angles to c_0c_4, and the weight of the dome will be regarded (for t small) as equal to the area of the convex surface of the frustum multiplied by $w\ t$. Let the co-ordinates of any point, c_3, be (x, y) and let the length, $O\ c_3 = s$, $O\ c_0 = s_0$, whence the weight of the entire dome above the point, (x, y), is,

$$\left(2\,\pi\, y\,\frac{s}{2} - 2\,\pi\, y_0\,\frac{s_0}{2}\right)$$

Therefore the total weight of $\frac{1}{n}$ the dome above (x, y) and $\frac{1}{n}$ the lantern is,

$$w\ t\,\frac{\psi}{2}\,(y\ s - y_0\, s_0) + \frac{W}{n}.$$

From the force triangle, $A\ 3\ 3'$, where $A\ 3$ may now be taken for this weight, the meridional thrust, $A\ 3'$, equals,

$$\left[\frac{w\ t\ \psi}{2}\ (y\ s - y_o\, s_o) + \frac{W}{n}\right] \text{sec. } \alpha\,;$$

whence the meridional thrust, per foot of horizontal circumference, is this expression divided by $\frac{2\,\pi\, y}{n} = \psi\, y$, or,

$$M = \left\{\frac{w\ t}{2}\ (y\ s - y_o\, s_o) + \frac{W}{2\,\pi}\right\}\frac{s}{x\, y} \qquad \ldots\ldots\ldots\ldots (7)$$

As in Art. 6, the hoop tension at (x, y) is $\frac{3\ 3'}{\psi}$, or,

$$T = \left\{ \frac{w\,t}{2}(y\,s - y_0\,s_0) + \frac{W}{2\,\pi} \right\} \text{tan. } \alpha =$$

$$\left\{ \frac{w\,t}{2}(y\,s - y_o\,s_o) + \frac{W}{2\,\pi} \right\} \frac{y}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

To find the thrust around the horizontal crown at (x, y), let us conceive x, y and s to take the simultaneous increments, h, k and l; the total weight now supported at the point, $(x + h, y + k)$, is,

$$\frac{w\,t\,\psi}{2}\left\{ (y + k)\,(s + l) - y_0\,s_0 \right\} + \frac{W}{n}.$$

On developing and subtracting from this the weight supported at (x, y) given above and multiplying by tan. α, to get Q = radial horizontal thrust acting on the voussoir of length l, we find,

$$Q = \frac{w\,t\,\psi}{2}\left\{ k\,(s + l) + y\,l \right\} \text{tan. } \alpha.$$

By Art. 2, the side thrust on the vertical face of the voussoir lying between (x, y) and $(x + h, y + k)$ is $q = \frac{Q}{\psi}$, and, dividing this by l, we get the average side thrust per foot of meridian length. As h, k and l simultaneously approach zero, this expression approaches a limit, C, which is defined to be the intensity of the pressure all around the crown at (x, y) per meridian unit. Therefore, as l and k tend indefinitely toward zero,

$$C = \text{lim. } \frac{Q}{\psi\,l} = \text{lim. } \frac{w\,t}{2}\left\{ \frac{k}{l}\,(s + l) + y \right\} \text{tan. } \alpha,$$

$$\text{or, } C = \text{lim. } \frac{w\,t}{2}\left\{ \frac{y}{s}\,s + y \right\} \text{tan. } \alpha = w\,t\,\frac{y^2}{x} \quad \ldots\ldots\ldots\ldots (9)$$

The formulas for C and M reduce to those given by Professor Church* for the complete dome with no lantern.

This is not equal to the average side thrust on the voussoirs just above or just below (x, y), but when the lengths of these voussoirs are very small, it approximates the mean of these two side thrusts. Or, put another way, if (x, y) is a point at the middle of any voussoir, the value of C above should give a close approximation to the side thrust on that voussoir as found graphically in Art. 11, provided the length of the voussoir is small enough.

The above values for M, T and C are correct for any point, (x, y), at or below c_0. It is to be remarked that Equation (9) is

* *Transactions*, Am. Soc. C. E., Vol. LII, p. 314, 70(*a*), 71(*a*).

always positive and independent of W; hence C is always in compression and has the same value for a dome with or without a lantern.

If the abutment cannot withstand the horizontal component of the meridional thrust there, a metal hoop must be placed around the base of the dome. The tension to be provided for in it is given by Equation (8), when x, y and s are given the values pertaining to c_4, Fig. 7.

It is seen from the foregoing that, in a conical dome, the base of which is kept from spreading, either by aid of friction on a firm abutment, or by means of a hoop, tension is nowhere exerted from the summit down. It is thus admirably adapted to masonry construction and to sustaining a heavy weight, as that of a lantern, at the summit. St. Paul's, London, is a novel illustration of the use of a conical dome to sustain the weight of a stone lantern, but having also "an inner dome, visible in the church, of brick 18 in. thick, except near the bottom, where it grows out of the cone of the same thickness, going up outside it." "Outside the cone is built a wooden dome covered with lead." This evidently gives a very strong construction.

A heavy lantern on a spherical dome, not only adds greatly to the stresses, but, in addition, it moves the joint of rupture much nearer the summit, thus causing more crowns to suffer tension, or, for the masonry dome which cannot take tension, the part below the joint will tend to separate into a series of simple arches. Assuming these simple arches to be each contained between two meridian planes, the line of resistance is drawn from the joint of rupture downward with the meridional thrust acting there and with its horizontal component as the constant horizontal thrust.

As stated in the beginning, it is not the intention of the writer to enter into a full discussion of the stone or brick dome. The reader will find in Professor H. T. Eddy's "New Constructions in Graphical Statics"* some interesting constructions and reflections concerning masonry domes, particularly as to the simple arch action referred to.

The writer's views are given in his "Theory of Steel-Concrete Arches and Vaulted Structures," where the dome of varying thick-

* D. Van Nostrand Co., publisher.

ness is discussed, and the dome with two shells, bearing a lantern load, receives a graphical treatment.

It is not the intention of the writer to go further in the subject of domes, but it may be added, in conclusion, that domes of revolution of any kind, as well as framed polygonal domes, are easily treated by the graphical method.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1007.

THE HYDRAULIC PLANT OF THE PUGET SOUND POWER COMPANY.*

By Edwin H. Warner, M. Am. Soc. C. E.

With Discussion by Messrs. William S. Post, Joseph H. Cunningham, Arthur L. Adams, Verne L. Havens and Edwin H. Warner.

The hydraulic plant of the Puget Sound Power Company is located in Pierce County, Washington, and is the development of the Puyallup River for the purpose of supplying power to the Seattle Electric Company, the Tacoma Railway and Power Company, and the Seattle-Tacoma Interurban Railway, associated companies of the Stone and Webster Syndicate, of Boston, Mass.

The Puyallup River is torrential in its upper reaches, having its source in the glaciers on the western slope of Mt. Rainier.

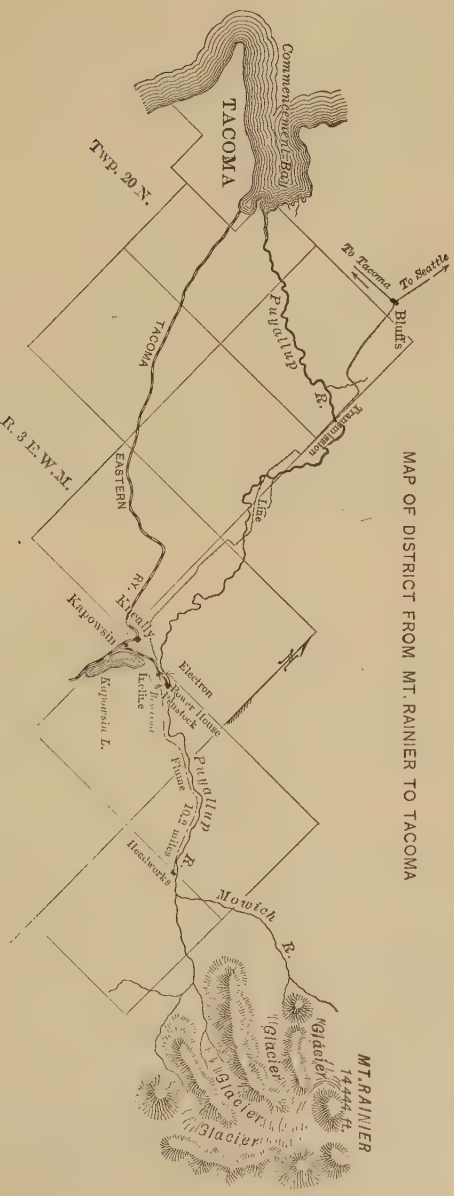
Fifteen miles below the glaciers are located the dam and headworks, from which a flume, 10.2 miles long, brings the water to the reservoir. Thence it is carried by four penstocks, each 2 100 ft. long, to the power station on the river bank. A transmission line, 22¼ miles long, delivers the current at Bluffs, on the Seattle-Tacoma Interurban Railway, where it is distributed to the two cities.

The power station is 26 miles from Tacoma, with which city

* Presented at the meeting of September 6th, 1905.

FIG. 1.—CLEARING AT SITE OF HEADWORKS.

FIG. 2.—HEADWORKS.



MAP OF DISTRICT FROM MT. RAINIER TO TACOMA

FIG. 1.

there is communication over the Tacoma Eastern Railway, a logging road doing business in the vicinity.

The work, preliminary to the construction of the plant, consisted of the improvement of 6 miles of county road, from Kneally; the building of 15 miles of wagon road past the reservoir site to the head-works; the construction of a cable incline to the reservoir site, rising 930 ft. in 4 100 ft.; the grading and laying of 3 miles of track to the commissary headquarters and saw-mill; the erection of one saw-mill near the head of the cable incline, and one within a mile of the head-works; the building of a machine shop, and the installation of a lighting plant.

In May, 1903, the railway company built a spur from Kapowsin Lake to the foot of the cable incline; this cut out the wagon haul from Kneally, and standard loads were hauled to Midway, at the foot of the steep grade on the cable incline, where they were reloaded on small cars.

At the foot of the incline, the power company built a depot, put in 1 500 ft. of storage track, and extended the railway spur 7 000 ft. to the site of the power station.

Dam and Head-Works.

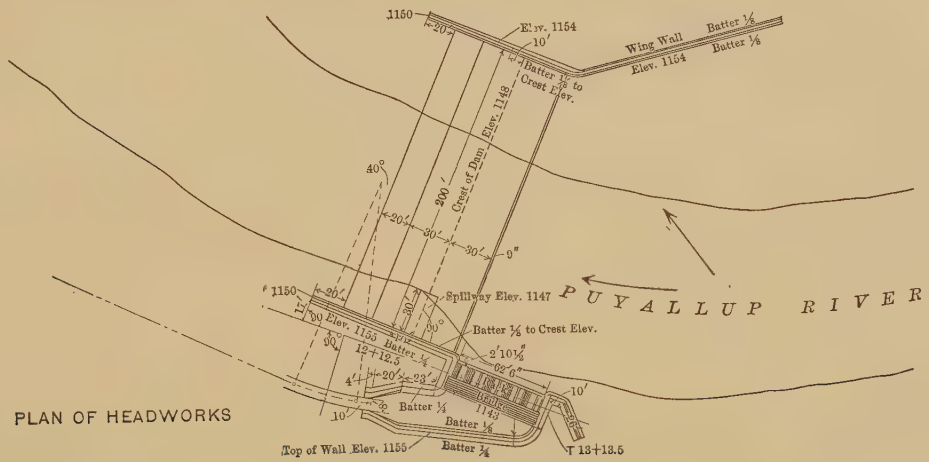
Dam.—The dam is a low timber-crib structure, triangular in section, and founded on gravel. It is 200 ft. long on the crest line, 60 ft. wide at the base, with an apron extending 20 ft. down stream. The crest line is 5 ft. above the bed of the river, and at the west end is dropped 1 ft., giving a spillway 30 ft. long and 12 ft. wide.

All the crib timbers are 12 by 12-in., and are fastened at the intersections with $\frac{3}{4}$-in. drift-bolts. The longitudinal timbers are built into the concrete abutments at each end.

The east half of the site was sheet-piled, and excavated to a depth of 4 ft., and foundation sills were set. Along the upper toe, under the center, and at the lower toe of the dam, 3 by 12-in. double-lap sheet-piling was driven to a further depth of 6 ft. In places where the driving was unsatisfactory, excavation was made, the piling was set and the excavation filled to the top of the foundation sills with concrete or clay puddle. By the time the east end was finished, work on the intake was so far advanced that the river was

FIG. 1.—CABLE INCLINE, FROM MIDWAY UP.

FIG. 2.—FLUME AT ROCKY POINT.



PLAN OF HEADWORKS

FIG. 2.

diverted, the west end of the dam site was similarly treated, and the dam completed. The upper face of the dam is sheathed with a triple lap of 3 by 12-in. plank spiked to the purlins. The lower face and apron are 8 by 12-in. timbers drift-bolted to the purlins with $\frac{5}{8}$-in. round iron. The crest of the dam has a protection of $\frac{1}{4}$ by 48-in. boiler plate, and the crib pockets are filled with rock and boulders. A lighting plant for night work was installed, and a 10-in. centrifugal pump was used to keep down the water inside the coffer-dam.

Abutments.—The east abutment is carried up stream 20 ft. from the toe of the dam, and then flares to the east for 140 ft. to prevent high water from passing through a low swale and thus around the structure. The east abutment is returned on itself and continued as the east wall of the intake.

Intake.—The intake has an entrance width of 60 ft., which narrows to 8 ft. at its connection with the flume. Flash-boards at the entrance permit of such regulation that surface water only may be taken, and the pasage of sand and gravel into the flume is prevented. Immediately back of the flash-boards are set $\frac{1}{4}$ by $3\frac{1}{2}$-in. iron rack-bars, spaced 2 in. apart. In the throat of the intake there

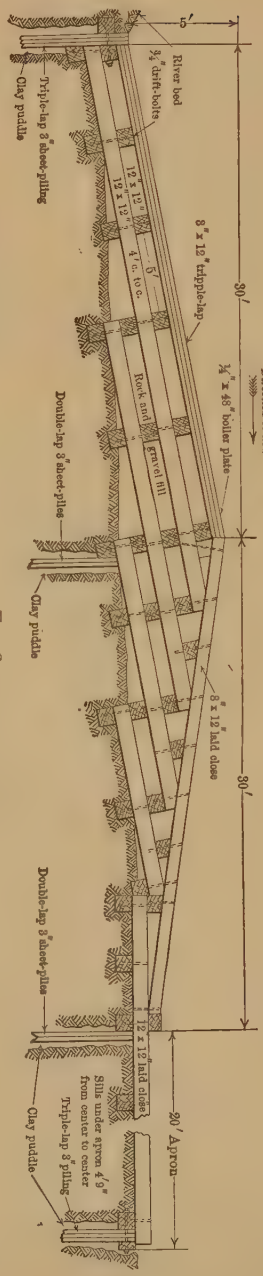
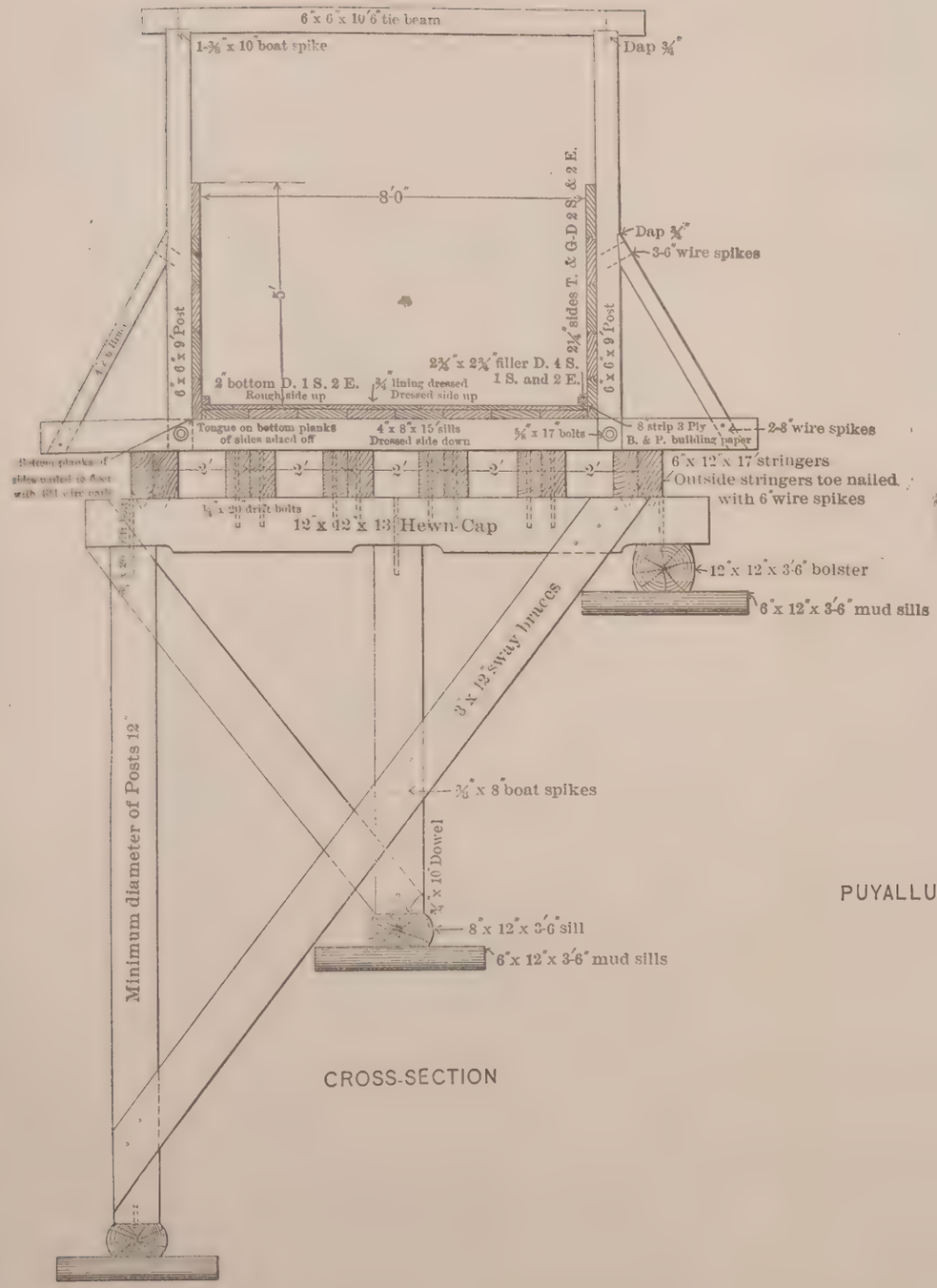
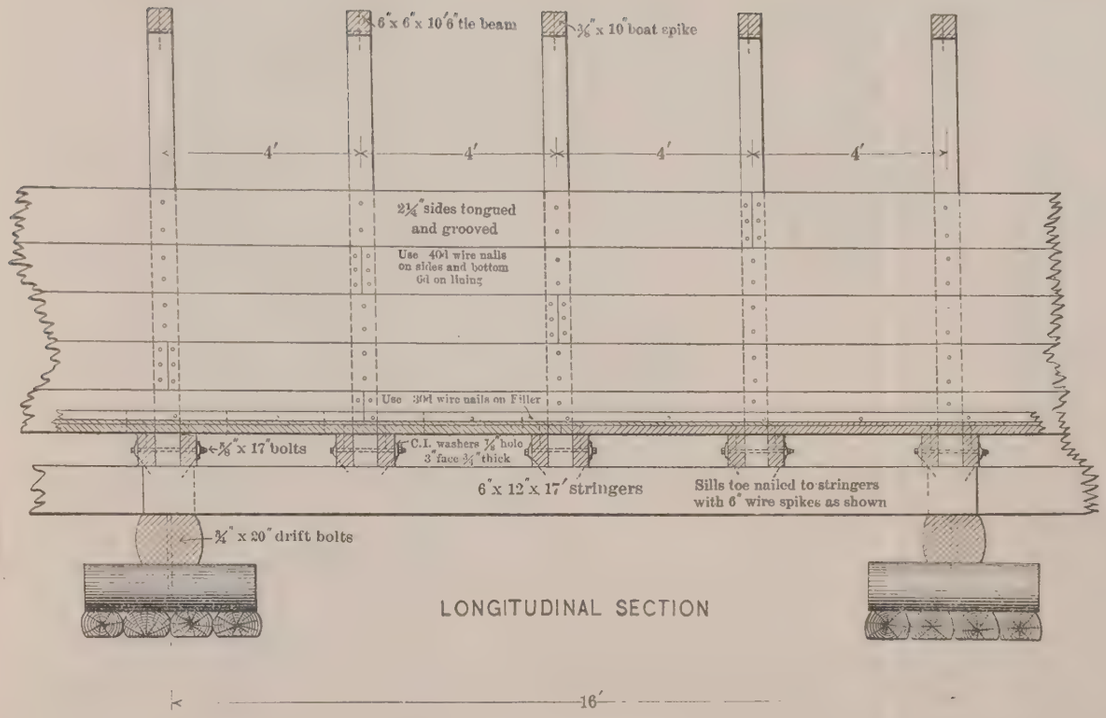


FIG. 3. SECTION OF DAM


6"x 6"x 10'6" tie beam
1-⅜" x 10" boat spike
Dap ¾"
8'-0"
Dap ¾"
3-6" wire spikes
2¼"x 2¼" sides T. & G. D 2 S. & 2 E.
6"x 6" x 9' Post
2¼"x 2¼" filler D. 4 S. 1 S. and 2 E.
2" bottom D. 1 S. 2 E. Rough side up
¾" lining dressed Dressed side up
Tongue on bottom planks of sides adzed off
4"x 8"x 15' sills Dressed side down
⅝" x 17" bolts
8 strip 3 Ply B. & P. building paper
2-8" wire spikes
6"x 12"x 17" stringers
Outside stringers toe nailed with 6" wire spikes
¾" x 20" drift bolts
12"x 12"x 13' Hewn Cap
12"x 12"x 3'6" bolster
6"x 12"x 3'6" mud sills
3"x 12" sway braces
Minimum diameter of Posts 12"
⅝" x 8" boat spikes
¾" x 16" Dowel
8"x 12"x 3'6" sill
6"x 12"x 3'6" mud sills

CROSS-SECTION


6"x 6"x 10'6" tie beam
⅜" x 10" boat spike
4'
4'
4'
4'
2¼" sides tongued and grooved
Use 40d wire nails on sides and bottom 6d on lining
Use 30d wire nails on Filler
⅝" x 17" bolts
C.I. washers ¼" hole 3" face ¼" thick
6"x 12"x 17' stringers
Sills toe nailed to stringers with 6" wire spikes as shown
¾" x 20" drift bolts
16'

LONGITUDINAL SECTION

FLUME

PUYALLUP RIVER DEVELOPMENT

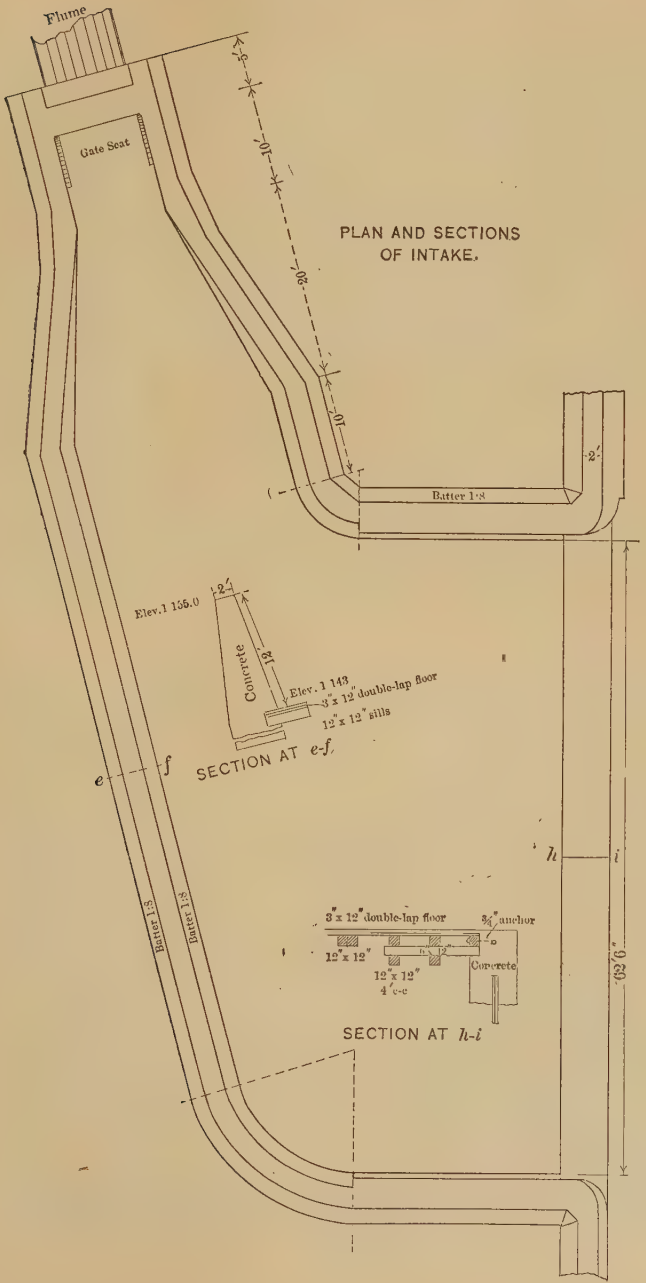

Flume
Gate Seat
5'
10'
30'
10'
1'
2'
Batter 1:8
PLAN AND SECTIONS
OF INTAKE.
Elev. 1 155.0
2'
12'
Concrete
Elev. 1 143
3" x 12" double-lap floor
12" x 12" sills
SECTION AT e-f
e
f
h
i
Batter 1:8
Batter 1:8
3" x 12" double-lap floor
¾" anchor
12" x 12"
12" x 12"
4 c-c
Concrete
SECTION AT h-i
62' 6"

FIG. 4.

is a radial gate with a tank counterbalance. The abutments and intake walls are of concrete, and the intake floor is a double lap of 3 by 12-in. planks spiked to 12 by 12-in. sills, the ends of which are built into the walls. A 3 by 24-in. box-drain relieves the floor from any upward pressure.

All concrete was hand-mixed in the proportions of 1 part of cement, 3 parts of sand and 6 parts of gravel. It was deposited from wheel-barrows, and tamped. The sand and gravel were found on the site. The gravel was washed before using, and a portion of the sand was similarly treated. The cement used was Alsen's German Portland, giving an average of 450 lb. for 7-day tests. A series of tests on 1 to 3 mortar gave an average of 160.5 lb. for the same time. Tests made immediately after breaking gave 12.5% moisture for the highest test briquettes; departure both ways from this percentage accompanied those below the average.

In this portion of the work, 1 426 cu. yd. of concrete 500 000 ft., B. M., of lumber and 13 tons of iron were used. The contract for the head-works was let to Messrs. Porter Brothers, of Spokane, Wash., for a lump sum of $45 000, with unit prices, to cover any increase or decrease of work, as follows: Concrete, $10; wet excavation, $8; dry excavation, $1, per cubic yard.

Flume.

The precipitous side-hill, upon which the flume line is located, made a trestle substructure necessary for practically its whole length. The calculated velocity in the flume was such that only rock would resist erosion, while the high angle of the hillside rendered any increase in flume section impossible, except at a prohibitive first cost and charge for maintenance.

The substructure consists of a sill laid on 6 by 12-in. mud-sills, 3 ft. 6 in. long, three plumb posts, at 5 ft. 6 in. centers, with dowels and pins to the sills, and caps, 13 ft. long, drift-bolted to the heads of the posts with $\frac{3}{4}$-in. round iron. On the caps, six 6 by 12-in. stringers, 17 ft. long, are laid and drift-bolted. With the exception of the stringers, round timber, cut near the site, was used. The bents were spaced at 16-ft. centers, and, where necessary, 3 by 12-in. sway-bracing was used. The height of the substructure ranges from

FIG. 1.—FLUME FRAME, WITH PART OF FLOOR AND SIDING IN PLACE.

FIG. 2.—FOREBAY, WITH FLUME DISCHARGING INTO POOL.

6 to 80 ft., reaching the latter figure, however, only in two gulch crossings.

After clearing the line, construction was begun at four different points, and, as fast as completed from the reservoir end, a broad-gauge track was laid with 35-lb. rails, and the distribution of the flume material was begun. Eight miles of track were thus laid and used, and material for the remaining 2.2 miles was cut at the upper saw-mill and delivered by wagon.

Approximately, 8 000 000 ft., B. M., of lumber and 60 tons of iron and nails were used in building the substructure. All clearing, excavation for the sills, the trestle bents to hardpan, and the erection of the substructure, including laying the stringers, ties and track, were done by the company's force.

The flume alignment, following closely a grade contour, shows of necessity a large amount of curvature. This aggregates 10 280°, a trifle more than 1 000° per mile, the maximum curve being 70 degrees. This curvature, while not remarkable in a flume, prohibited the use of standard equipment on the track, and the company built, at its shop at the reservoir site, forty small four-wheel flat cars, 7 by 16 ft. over all. The conditions precluded any super-elevation of the rail, and a loose box, giving large end play to the axle, was adopted. This served the purpose admirably, and speeds up to 12 miles an hour were attained without derailment. Three 16-ton four-wheel engines were used for work on the flume and in the yard.

A number of the cars were fitted up with strong backs, and were used on the steep portion of the cable incline from Midway up.

Superstructure.—To the stringers of the substructure are toe-nailed two 4 by 8-in. sills, 15 ft. long, and spaced 6 in. apart. A 6 by 6-in. post, 9 ft. long, is dropped between and held by a $\frac{3}{4}$-in. machine-bolt, 17 in. long. A 4 by 6-in. brace, 6 ft. long, is spiked to the sills and posts, and these, with a 6 by 6-in. tie-beam, 10 ft. 6 in. long, gained and spiked to the heads of the posts, form the flume framing, which is spaced at 4-ft. centers. The floor is of 2 by 12-in. plank with a 1 by 12-in. lining, and the sides are of $2\frac{1}{4}$-in. tongued and grooved stuff. In the corners there is a $2\frac{3}{4}$ by $2\frac{3}{4}$-in. fillet, and between the lining and the floor plank there is a 6-in. strip of tarred paper, turned up and held in place against the side by the fillet. The floor planks are laid straight, in lengths of from 16 to

32 ft., and on curves the segments are cut to a driving fit. The tongued and grooved siding, in similar lengths, was bent to place, held by clamps and driven down without difficulty.

With a view to its use independent of the reservoir, the flume is extended to the fore-bay, where it passes through the reservoir embankment. Two rows of sheet-piling, 20 ft. apart, were set in 4-ft. trenches which were filled with concrete. The sheet-piling was spiked to the flume, which was carried above the high-water line and extends 6 ft. into the embankment on each side. The trench was carefully puddled, and no seepage has occurred.

Within the reservoir, plank was laid on the trestle-sills, the flume was tied down with rods, and the plank loaded with boulders.

Five mud-boxes, at intervals along the flume, intercept any solid matter which passes the intake. The gates are of plank, slide in wooden grooves, and are raised with a bar for flushing.

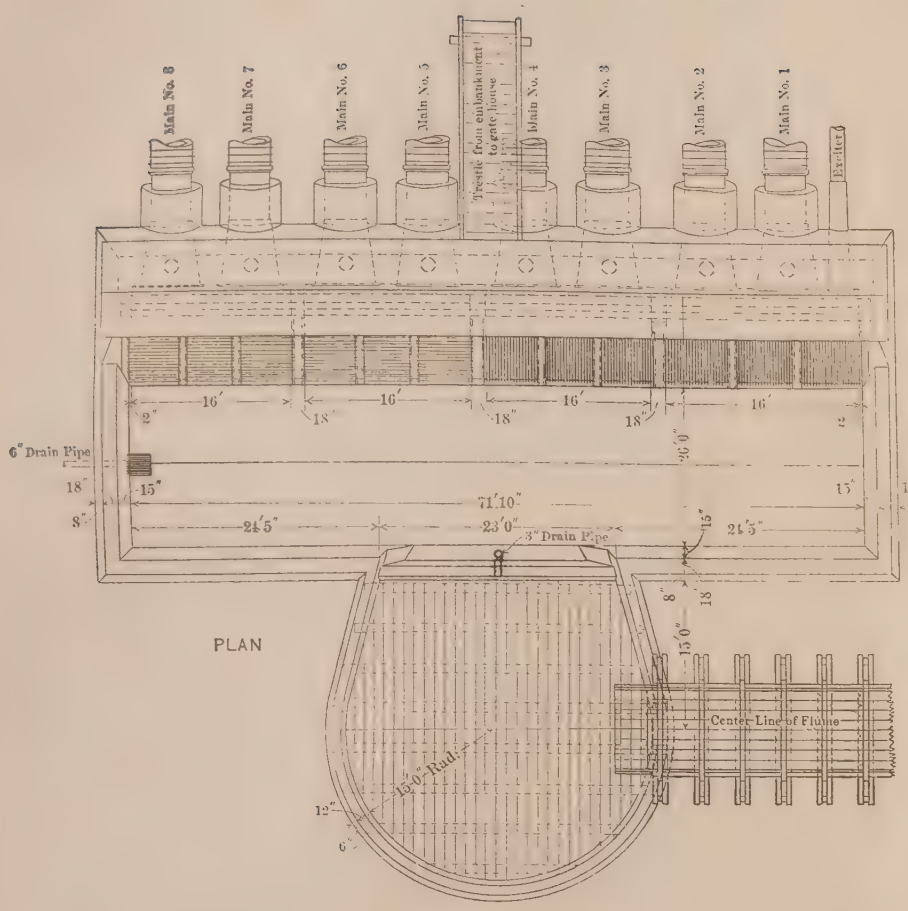
Seven spillways, with needle-gates below, permit the flow to be diverted when necessary. They are provided with an apron, and, where required, with a small flume to carry the water clear of the substructure.

There is one automatic regulating spillway, 1 800 ft. from the reservoir. It consists of an apron, 132 ft. long, the lip of which is level with the high-water line in the reservoir. A 4 by 6-ft. flume on a heavy grade carries the water beyond an adjoining rock bluff.

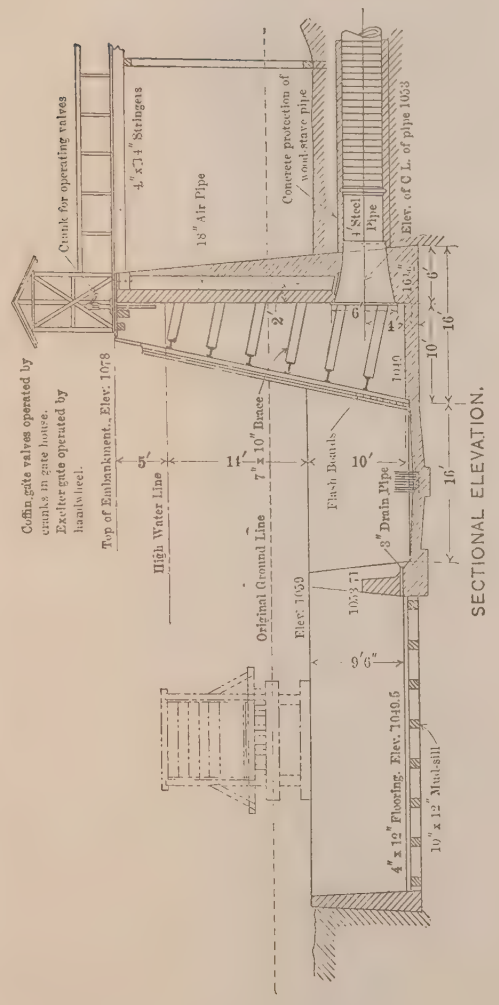
The flume is 8 ft. wide, in the clear. For the present installation of 20 000 h-p. the sides are built up 4 ft. 6 in., and will be raised to the full height of the framing when the plant is extended. The last 3 000 ft. of the flume is sided to the full height in order to make the regulating spillway operative. A gate of 3 by 3-in. needles, near the end of the flume, regulates the flow to the fore-bay. With the reservoir full, the gate will be retained as a safety device to prevent a back flow in case of a break in the flume close at hand. The grade of the flume is 0.00136, and, by Kutter's formula, gives a discharge of 280 cu. ft. per second, using 0.0105 as the value for "*n*," and with a depth of 4 ft.

As fast as the distribution of material by train was completed, at the end of the track, the rails were taken up and placed on the substructure caps, and work on the flume proper was begun. When completed, the track was relaid on top of the flume.

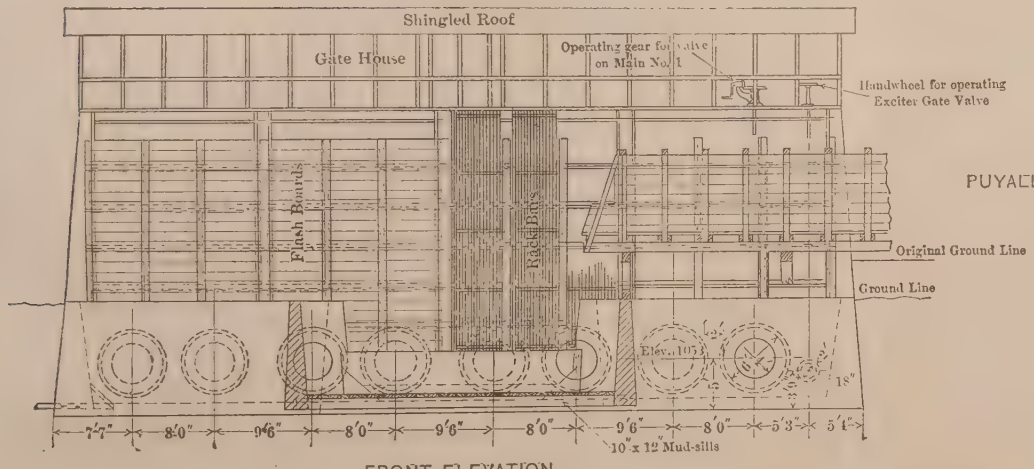
PLATE XXVII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1007.
WARNER ON
HYDRAULIC POWER PLANT.



PLAN


Crank for operating valves
4"x3½" Stringers
18" Air Pipe
Concrete protection of wood stave pipe
Steel Pipe
Elev. of C.L. of pipe 1053
Coffin gate valves operated by cranks in gate house. Exciter gate operated by hand wheel.
Top of Embankment, Elev. 1078
High Water Line
7"x10" Brace
Flash Boards
Original Ground Line
Elev. 1050
3" Drain Pipe
4"x12" Flooring, Elev. 1049.5
10"x12" Mud-sill

SECTIONAL ELEVATION.



FRONT ELEVATION
SECTION THROUGH POOL

FORE-BAY
PUYALLUP RIVER DEVELOPMENT.

Hand-cars are now used for patrol purposes and to deliver supplies for the use of the watchman at the head-works. The flume tie-beams are spaced at 4-ft. centers, and, as a derailment at any point would be serious, a safety device was originated, consisting of a 2 by 4-in. piece, 6 ft. long, suspended from wrought-iron hangers in front of the wheels and 1 in. above the track. When the wheels leave the track the car skids on this 2 by 4-in. piece until brought to rest. It works satisfactorily, and the patrolmen get up a speed of 10 miles per hour, seemingly confident of results.

In the superstructure, 6 000 000 ft., B. M., of lumber, and 75 tons of iron and nails were used. The company furnished all material, which cost $12 per thousand feet for lumber, and 4 cents per pound for iron. The contract for erection was let to Messrs. Porter Brothers, for a lump sum of $40 000, with a unit price of $8 per thousand feet for additional material.

Fore-bay and Gate-wall.—At the north end of the reservoir, and 20 ft. within the toe line, is located the fore-bay, into which the flume discharges. The 30-ft. horse-shoe pool is separated from the fore-bay proper by a wall 5 ft. high, which prevents a direct flow to the penstocks, and allows the escape of the air entrained in the fall from the flume. The fore-bay is 32 by 80 ft., with concrete walls and floor; three walls are built 10 ft. high, to the level of the bottom of the reservoir; the fourth is a gate-wall, 6 ft. wide at the base and 3 ft. at the top. It is carried 19 ft. higher and level with the top of the reservoir embankment; it has five buttresses on the fore-bay side which divide it into four panels. In each panel there are two 6-ft. penstock steel tapers; the east end panel contains, in addition, a 24-in. taper for the exciter pipe. Each taper has an air inlet running to the top of the gate-wall. The four now in use, and the exciter, are closed with Coffin valves, operated by hand, with provision for motor operation from the power station when desirable. A 6-in. blow-off pipe frees the fore-bay of water when necessary. The four tapers for future use are closed with wooden bulkheads, tightly fitted and caulked with oakum.

In the face of the buttresses are embedded horizontally six 7 to 12-in. I-beams which support the rack-bars; two sets of two 8-in. channels, with a 4-in. wooden filler, are bolted to the I-beams, subdividing each panel into three spaces of approximately 5 ft. each;

the top of the channels and a wooden cover piece form a groove in which the flash-boards slide, thus isolating for examination any two pipes from the fore-bay without emptying the latter or interrupting the operation of the plant.

The rack-bars are of wrought iron, ¼ by 3 in. in section, held together with ¾-in. rods, and packed with ⅞-in. cast-iron separators. This spacing gives a velocity of 0.87 ft. through the racks, with the water at the top of the fore-bay wall. This is reduced to 0.39 ft. with the water at the high-water line in the reservoir. In front of the exciter gate the spacing is reduced to ½ in., and an additional set of racks, similarly spaced, isolates the exciter from the other tapers within the panel.

The source of supply being high up in the mountains, very little foreign matter is brought down the flume, and no clogging of the racks has occurred.

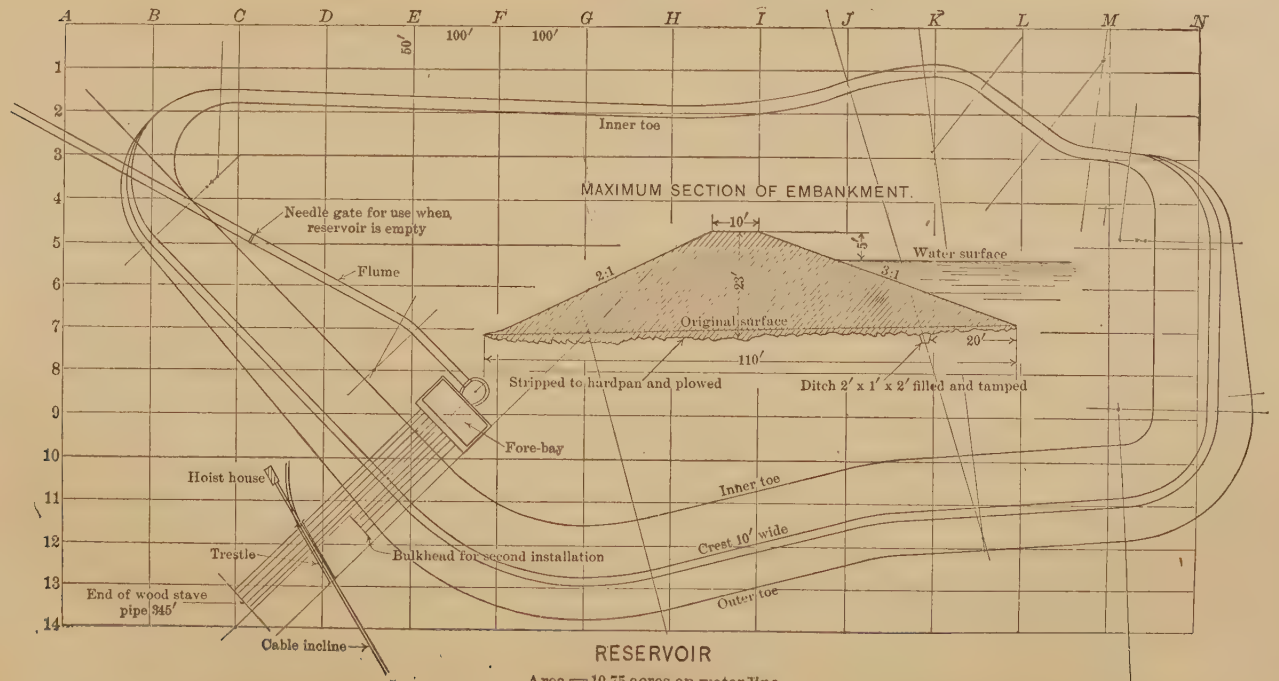
The concrete is a mixture of 1 part of cement, 3 parts of sand and 6 parts of broken stone. The cement used was Alsen's German Portland. The sand, from a pit on the power-house spur, is a very clean, coarse, sharp, river sand. The broken stone is a basic trap, microcrystalline in structure, with very sharp edges, and was quarried and crushed at the power-house site, loaded on cars, and hauled up the incline. The concrete, of which 586 cu. yd. were used, was mixed very wet, in a double-cone, continuous mixer. The contract for this work was let to Messrs. Bihler and Rydstrom, of Tacoma, Wash., for a lump sum of $7 425, including excavation below the level of the reservoir, with unit prices of $8.50 per cubic yard for concrete, and $1.75 per cubic yard for excavation.

Reservoir.—The site selected for the reservoir is on a bench sloping at the rate of 4.5 in 100. This prescribed a long narrow form, which will become more pronounced when extended for future requirements. The first operation was to cut the large cedar and fir timber; the stumps were then blasted out and removed.

The material excavated was a highly indurated boulder clay, requiring powder to loosen it for handling with a steam shovel; this was overlaid, to a depth of from 2 to 4 ft., with sand and gravel and loam. As it was impracticable to save and mix the sand and gravel with the clay in any definite proportion for use in the embankment, all loose material was stripped off and wasted. Excavation to a face

FIG. 1.—MAIN ANCHORAGE.

FIG. 2.—PIPE LINES, WITH AUXILIARY ANCHORAGES.


A
B
C
D
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L
M
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1
2
3
4
5
6
7
8
9
10
11
12
13
14
50′
100′
100′
Inner toe
MAXIMUM SECTION OF EMBANKMENT.
Needle gate for use when reservoir is empty
Flume
10′
5′
Water surface
2:1
3:1
Original surface
110′
20′
Stripped to hardpan and plowed
Ditch 2′ x 1′ x 2′ filled and tamped
Fore-bay
Hoist house
Inner toe
Crest 10′ wide
Trestle
Bulkhead for second installation
End of wood stave pipe 345′
Outer toe
Cable incline
RESERVOIR
Area = 10.75 acres on water line.

FIG. 5.

of 4 ft. was made by hand and run out on small cars by team; the remainder was done by steam shovel and train. The embankment site was stripped and plowed, and a ditch, 20 ft. inside the inner toe line, was excavated. A trestle at the proposed height of the embankment was built, from which the material was dumped. A steel scraper of special form, hauled by two hoisting engines from toe to toe of the embankment, was used to spread the material. The wet condition of the clay, due to the continuous rains in this section, and its treatment by scraper made an embankment which has proved practically impervious to water. An attempt was made to roll the material in layers, but as it resulted in the disappearance of the roller, the puddling process was substituted. Care was taken to isolate all boulders, which form a considerable percentage of the mass and add materially to its stability. The excavation amounted to 132 166 cu. yd., of which 105 131 cu. yd. were built into the embankment.

For 4 weeks water was admitted to the reservoir at the rate of 0.75 ft. per week, and held at the 3-ft. level for one week; the 7-ft. level was reached in 3 weeks more, and on July 15th reached the 8-ft. mark. Continuous inspection has failed to reveal any leak, while the seepage is insignificant in amount, and does not increase with the increase in depth. The maximum section is 20 ft. high, with a base of 113 ft. and a top 10 ft. wide, and inner and outer slopes of 3 : 1 and 2 : 1, respectively. The embankment was trimmed by hand to the true slope, and the outer slope, top, and inner slope to the water line were sown with clover. The area is 10.75 acres at the water line, and no destructive wave action is apprehended, as the prevailing winds are from the south, and the flume extension and gate-wall reduce the exposed north line to less than 100 ft.

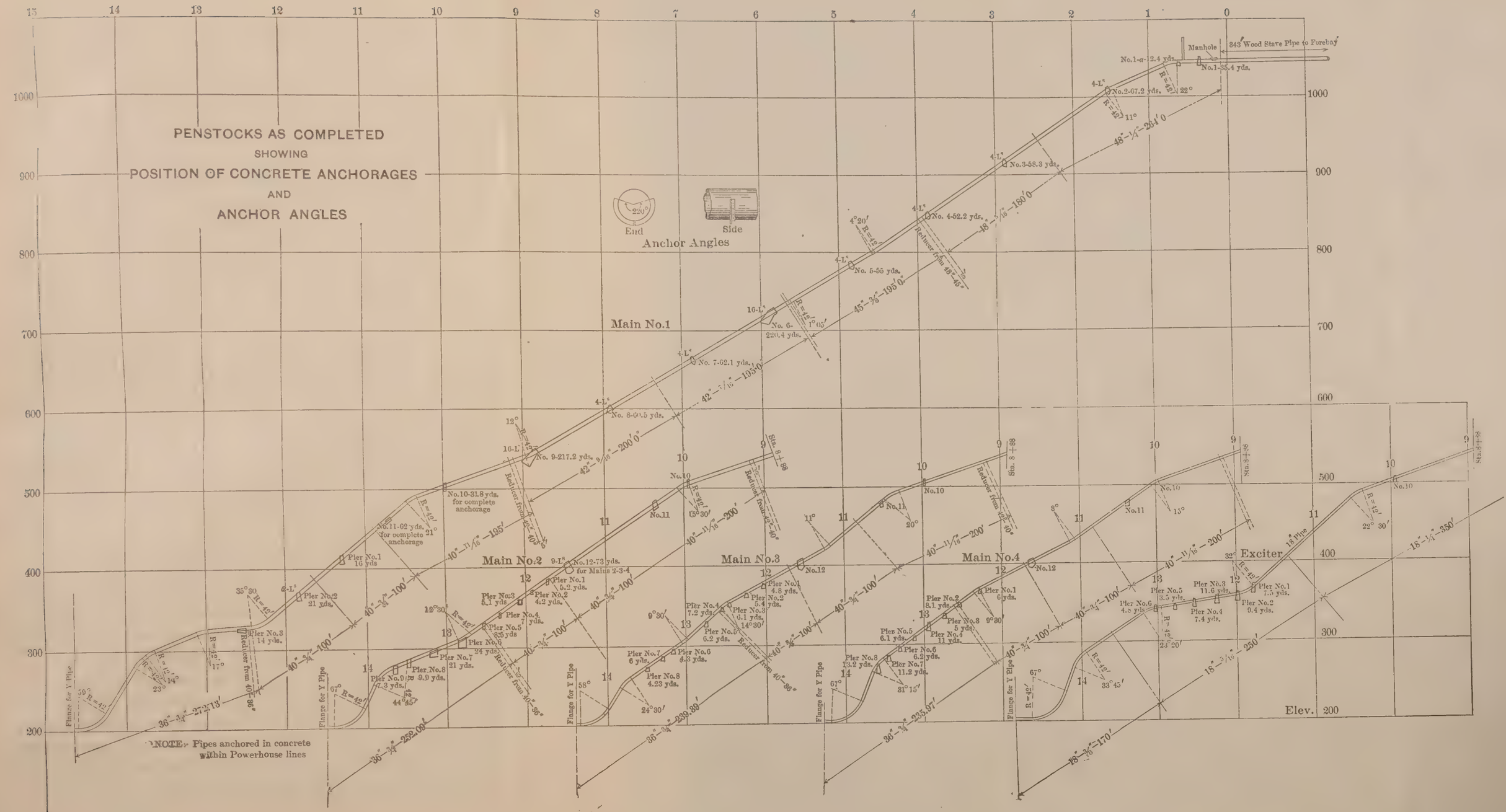
The reservoir gives 6½ hours' storage with 14 ft. depth. This is based on the production of 20 000 h-p., with an effective head of 850 ft.; 276 cu. ft. of water per second; 75% wheel efficiency, and a power factor of 100% for the time given. The contract for the reservoir was let to Messrs. Porter Brothers, for a lump sum of $81 595.50, with a unit price of 71¼ cents per cubic yard for borrowed material.

The excavation of suitable material exceeded the neat section of embankment by 8 000 cu. yd., which were built into the embankment.

PLATE XXIX.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1007.
WARNER ON
HYDRAULIC POWER PLANT.

PENSTOCKS AS COMPLETED
SHOWING
POSITION OF CONCRETE ANCHORAGES
AND
ANCHOR ANGLES

Anchor Angles



NOTE.—Pipes anchored in concrete within Powerhouse lines

Wood-Stave Penstocks.—The wood-stave penstocks consist of eight 4-ft. mains and one 18-in. main. Four of these are carried 175 ft. through the embankment, and bulkheaded. They will be reserved until the second installation is made.

The others, 345 ft. in length, connect with the steel pipe. They are built-up wood pipes. The staves were cut from 2 by 6-in. clear lumber, dressed to radial lines, with a circular curve inside and out, and have oak tongues in the butt joints. Round-iron bands, § in. in diameter, coated with asphalt composition, hold the staves in place. The bands are spaced at 8-in. centers, except at the butt joints, where the space is reduced to 4 in. Two concrete cut-off walls within the embankment enclose the pipe and extend 2 ft. above. This work was let to the National Wood Pipe Company, of San Francisco, Cal., at $2.96 and $1.47 per foot, for 48 and 18-in. pipe, respectively, the power company handling the material from the foot of the incline to the site of the work.

The wood pipe slips into a double-angle steel bellmouth riveted to the taper at the gate-wall, and is caulked with lead. A similar joint connects it to the steel portion of the penstock. The pipe through the embankment is protected by concrete, reinforced over the pipe with expanded metal. Bids were invited for this concrete protection, but the acceptance of the lowest, $7 200, and 30 days' time for completion, was not considered wise, and a company's force was organized. The work was completed in 11 days at a cost of $4 640.

Steel Penstocks.—The main penstocks are compound pipes, 1 730 ft. long, reducing at intervals from 4 ft. to 3 ft. in diameter, and of varying thickness, as shown in Table 1. The exciter pipe maintains its diameter of 18 in. throughout its length. A manhole and an air inlet are provided in each pipe near the wood-pipe connection.

The pipe was delivered at the power station and at the head of the cable incline, and then put in a place by means of a small tramway and hoist. The pipe was laid, holes were reamed and the joints riveted; the seams were caulked inside and out, and painted. Air tools were used for the entire work, with the exception of a small amount of hand-caulking. Anchor angles, 6 by 6 in., were riveted on, enclosing 220° of pipe; four angles were used on each pipe in the main anchorages, and one in the auxiliaries. For purposes of

TABLE 1.—MAKE-UP OF STEEL PIPE.

Diameter, in inches.	Length, in feet.	Thickness.	Diameter of rivets.	Pitch of rivets.	Butt Straps. On straight seams.	Butt Straps. On circumferential seams.	Remarks.
48	264	$\frac{1}{4}$ in.	$\frac{5}{8}$ in.	Straight seams, 2¼ in. Circumferential seams, $1\frac{9}{16}$ in.			Double-riveted lap. Single-riveted lap.
48	180	$\frac{5}{16}$ "	$\frac{5}{8}$ "	Straight seams, 2¼ in. Circumferential seams, $1\frac{9}{16}$ in.			Double-riveted lap. Single-riveted lap.
....	5						Reducer, 48 to 45 in.
45	195	$\frac{3}{8}$ in.	$\frac{5}{8}$ in.	3 in.	$\frac{9}{32}$ x 7 in.	$\frac{3}{8}$ x 7 in.	Double-riveted butt straps.
....	5						Reducer, 45 to 42 in.
42	195	$\frac{7}{16}$ in.	$\frac{5}{8}$ in.	3 in.	$\frac{5}{16}$ x 7 in.	$\frac{7}{16}$ x 7 in.	Double-riveted butt straps.
42	200	$\frac{9}{16}$ "	$\frac{3}{4}$ "	3½ "	$\frac{7}{16}$ x 8 "	$\frac{9}{16}$ x 8 "	Double-riveted butt straps.
....	5						Reducer, 42 to 40 in.
40	195	$1\frac{1}{16}$ in.	$\frac{3}{4}$ in.	3½ in.	½ x 9 in.	$\frac{1}{16}$ x 9 in.	Double-riveted butt straps.
40	100	$\frac{3}{4}$ "	$1\frac{3}{16}$ "	3½ "	$\frac{9}{16}$ x 12 "	$\frac{3}{4}$ x 10 "	Double-riveted butt straps.
40	100	$\frac{3}{4}$ "	$1\frac{3}{16}$ "	3½ "	$\frac{9}{16}$ x 12 "	$\frac{3}{4}$ x 10 "	Triple-riveted butt straps.
....	5						Reducer, 40 to 36 in.
36	273	$\frac{3}{4}$ in.	$1\frac{3}{16}$ in.	3½ in.	$\frac{9}{16}$ x 12 in.	$\frac{3}{4}$ x 10 in.	Triple-riveted butt straps.
18	690	No. 10	$\frac{3}{8}$ "	1¼ "			
18	200	No. 8	$\frac{7}{16}$ "	1½ "			
18	100	$\frac{5}{16}$ "	$\frac{1}{2}$ "	1½ "			
18	350	$\frac{1}{4}$ "	$\frac{9}{16}$ "	2 "			
18	250	$\frac{5}{16}$ "	$\frac{11}{16}$ "	$2\frac{7}{8}$ "			
18	170	$\frac{3}{8}$ "	$\frac{3}{4}$ "	3 "			

Exciter pipe.
Double-riveted lap on straight seams.
Single-riveted lap on circumferential seams.
Joints, flange-riveted.

calculation, a rivet stress of 42 000 lb., ultimate tensile stress, with a factor of safety of 5, was used.

The use of inside riveters, and caulking the seams inside, resulted so satisfactorily that, when tested under full head, only one pipe required touching up, and that was done by one man in less than 3 hours. On completion, the pipes were covered to within 18 in. of the top, scarcity of material preventing the complete covering. This partial exposure is not a source of anxiety in this climate, where the mean annual temperature is high, and the daily variation not considerable. The temperature of the glacial water is low, averaging 50.3° at the fore-bay, and reaching 52.1° in the tail-races at the power-house; the extremes are 44 and 55° at the fore-bay, and 47 and 56° at the power-house. The Risdon Iron and Locomotive Works, of San Francisco, Cal., took the contract for a lump sum of $140 000 for the material and the erection of the pipe complete.

FIG. 1.—CABLE INCLINE; 65.6% GRADE AT THIS POINT.

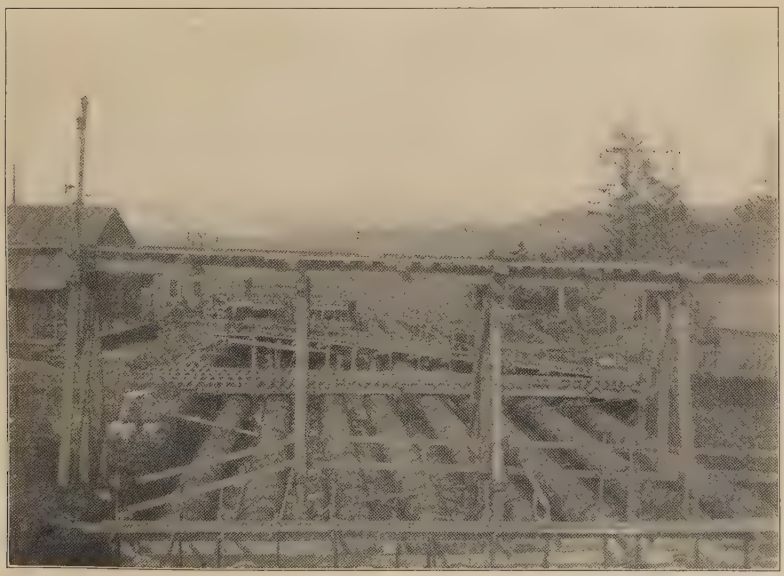


FIG. 2.—WOOD PIPE, WITH TRESTLE IN FOREGROUND FROM WHICH EXCAVATED MATERIAL FOR EMBANKMENT WAS DUMPED.

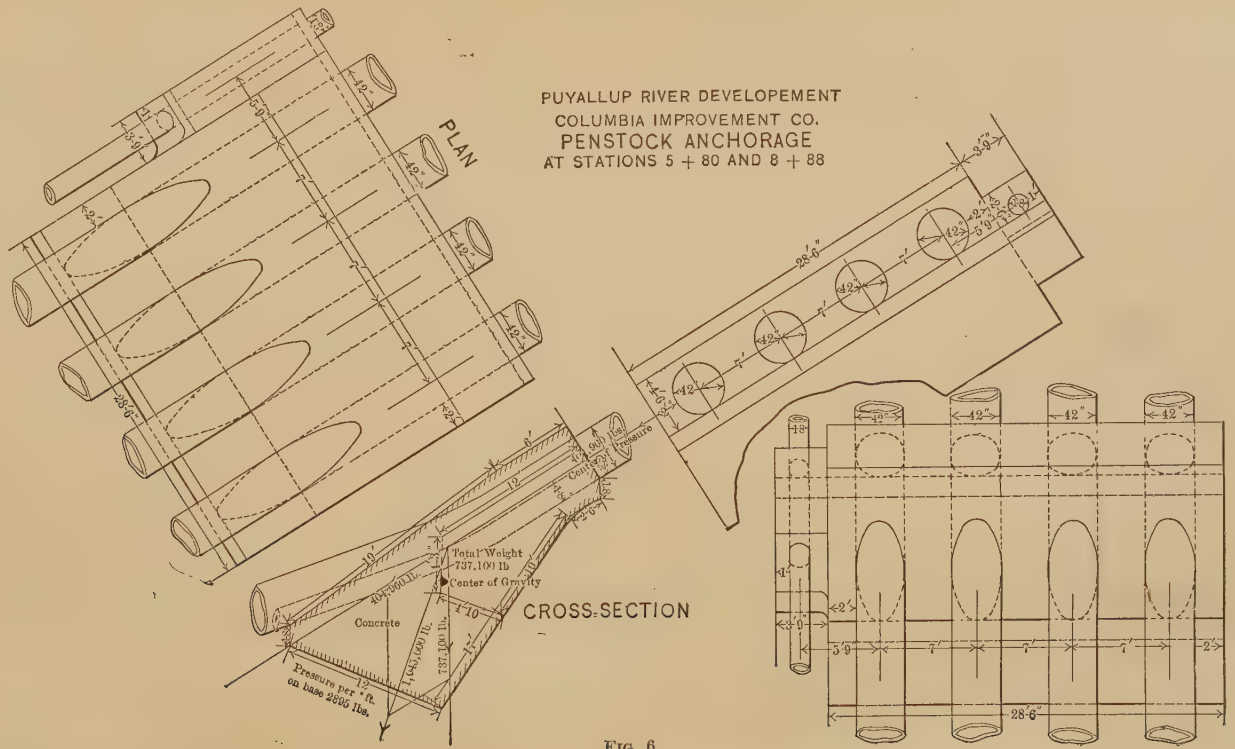

PUYALLUP RIVER DEVELOPEMENT
COLUMBIA IMPROVEMENT CO.
PENSTOCK ANCHORAGE
AT STATIONS 5 + 80 AND 8 + 88
PLAN
CROSS-SECTION
Total Weight
737,100 lb
Center of Gravity
Concrete
Pressure per sq. ft.
on base 2605 lbs.
42"
28'-6"

FIG. 6.

The work was done under difficult conditions of weather, steep slopes and lack of space at the power-house and reservoir ends. The thorough nature of the work, under these adverse conditions, is shown in the very satisfactory test just referred to.

Anchorages.—Concrete anchorages are built around the pipes at intervals of 125 ft. The main anchorages, alone, are designed to resist without movement any stresses which may be imposed; the

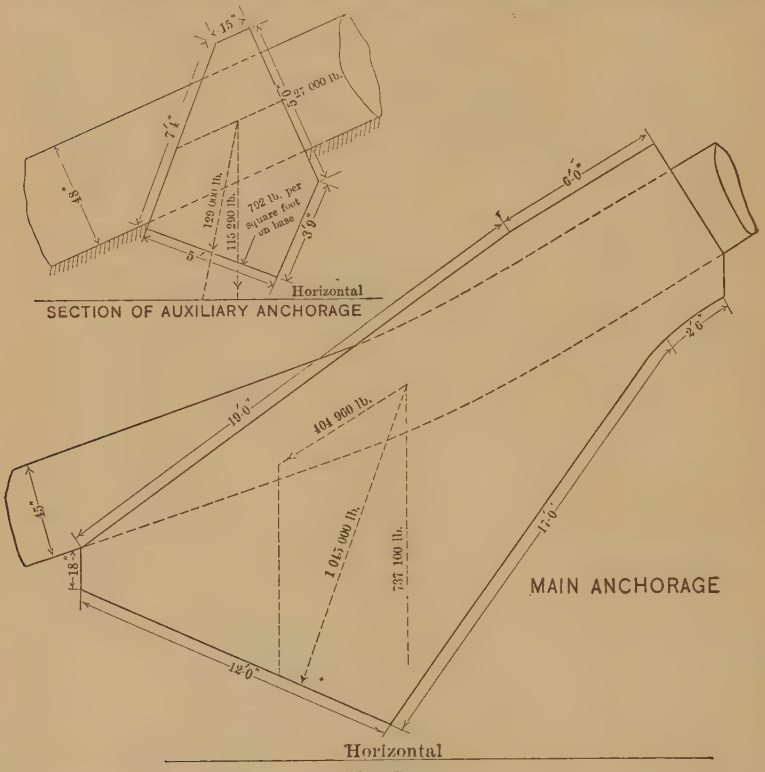
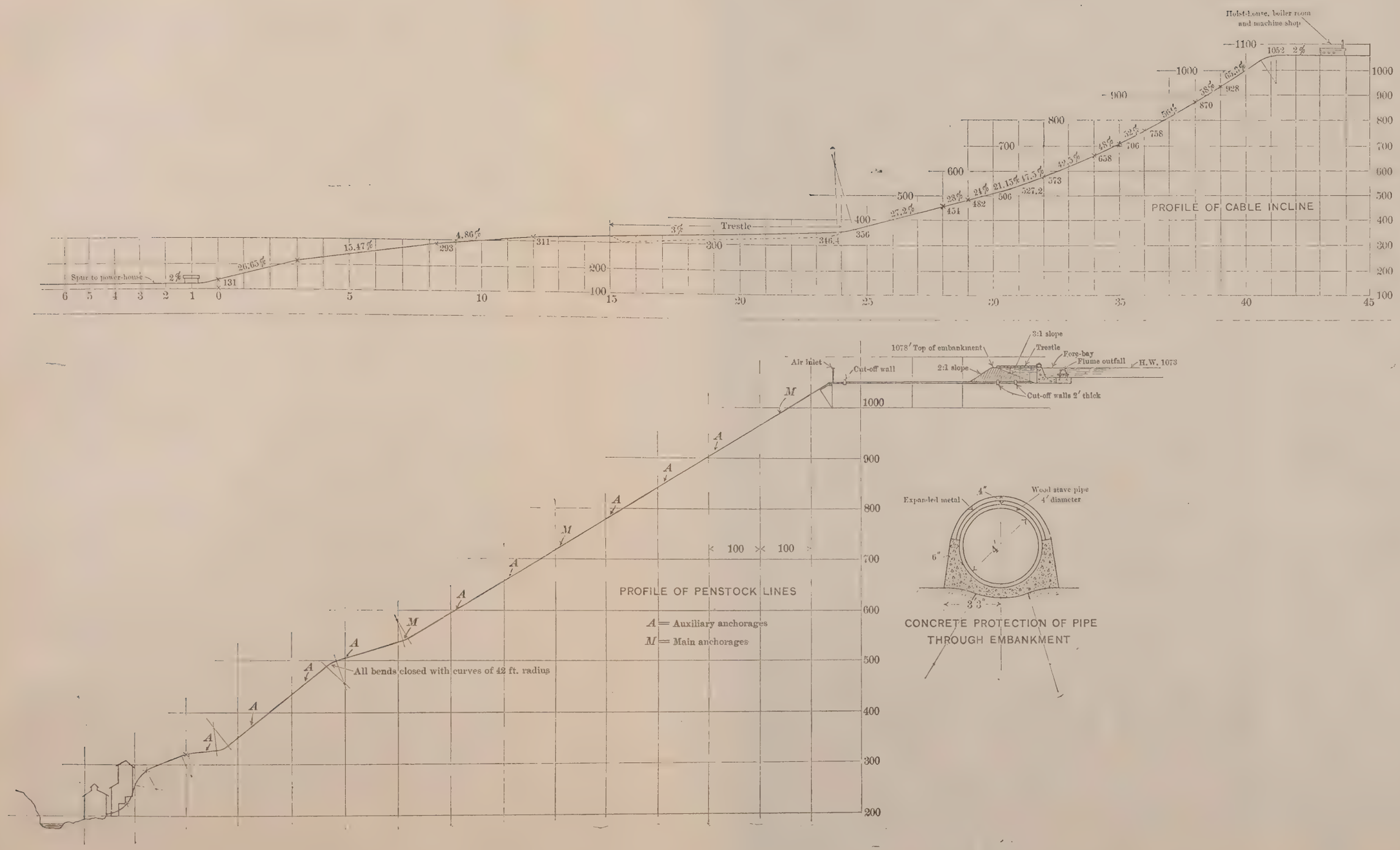


FIG. 7.

auxiliary anchorages are similarly designed to resist these stresses independent of the main anchorages. In designing these anchorages, frictionless surfaces of pipe and ground were assumed. The concrete is a mixture of 1 part of cement, 3 parts of sand, and 6 parts of gravel. The gravel was washed, and the concrete was mixed by hand. The cement and sand were of the same quality as used in the fore-bay work. The concrete amounts to 1 300 cu. yd.,

PLATE XXXI.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1007.
WARNER ON
HYDRAULIC POWER PLANT.



and was let by contract to Messrs. Bihler and Rydstrom at $10.50 per cubic yard. Within the power-house the pipe trenches were back-filled with concrete to a depth of 18 in. above the top of each pipe.

Power-House.—The generator-room, 42 by 150 ft., up to the crane runway, 28 ft. above the floor level, is built of concrete. Above this, to a height of 12 ft., the construction is of brick. The front wall has a thickness of 24 in., and is strengthened at intervals of 29 ft. by pilasters. Below the floor level, the foundation wall consists of three piers, 7 by 8 ft. at the base, and two abutments, 4 ft. thick and 19 ft. long, connected by elliptical arches, of 22 ft. span. The arches are reinforced with 35-lb. steel rails bent to approximate the form of the arch.

One abutment and the adjoining pier are founded on piles driven 30 ft.; a concrete grillage, with steel-rail reinforcement, covers the heads of the piles. A similar foundation supports the footing course for Units Nos. 3 and 4; but, in all other foundation work, bed-rock was reached.

Back of the generator-room there are three transformer-rooms with steel-roller shutter doors. A switching gallery, to which access is had by an iron stairway, occupies 20 ft. of the east end of the building. The gallery is supported on cast-iron columns, and has a 4-in. steel and concrete floor, reinforced with wire netting. The crane runway is of 60-lb. steel rails bolted to a 24-in. I-beam anchored in the wall. The crane, furnished by the Cleveland Crane and Car Company, of Cleveland, Ohio, has a 50-ton main and a 5-ton auxiliary hoist, run by 25 and 12 h-p. motors, respectively; these, with a 25 h-p. motor for the bridge travel and an 8 h-p. motor for the trolley, complete the crane equipment.

The roof is of steel trusses spaced at 14 ft. 6 in. centers, with 6 by 10-in. purlins bolted to the trusses, and covered with matched lumber and galvanized corrugated iron. The contract for the power-house was let to Messrs. Bihler and Rydstrom at the following prices: Excavation, $19 860; concrete, $7 per cubic yard below floor level, and $8 per cubic yard above floor level; brick laid in cement mortar, $25 per thousand; erection of crane and runway, roof, doors and windows, $4 567.

In the generator-room are installed four units and two exciters.

Each unit consists of a General Electric 3 500 kw., three-phase, 60-cycle generator, with rotating field separately excited. The rotor is mounted on a nickel-steel shaft, running in water-cooled bearings, and lubricated under pressure. On the overhang, at each end, is a Pelton wheel, 8 ft. 8 in. in diameter at the flow line, and making 225 rev. per min. The shaft is hollow, with a 7-in. bore; its diameter is 14 in. at the wheels, 16 in. at the bearings, and 20 in. at the rotor. The rotor and wheels were pressed on over feathers with a 200-ton hydraulic jack; the pressure at which they went on averaged 125 tons for the rotor and 120 tons for the wheels.

The wheel discs and buckets are solid cast steel, the latter bolted on with forged-steel bolts.

Water is admitted to the wheels through Y-branches, 25-in. in diameter, bolted to the flanges on the 36-in. penstocks. Each branch is provided with a motor-operated gate and a by-pass. A needle deflecting nozzle, with a Lombard governor, type L, and automatic control, regulates the flow to the wheels. The contract for the wheels, shafts, bearings, Y-branches, gates, regulating devices and floor plates, was let to the Pelton Water Wheel Company, of San Francisco, Cal., for $75 000, erected, the makers guaranteeing an efficiency of 80% and 75%, respectively, at full and half load.

The governor water is supplied by a tank, 14 ft. in diameter and 13 ft. 6 in. high, made of 3½ by 4-in. staves, with a 3-in. floor, located 460 ft. above the power-house floor. A 2½-in. header, with valves, connects the four mains and the exciter main at the upper end of the steel penstocks. From the header a 4-in. pipe discharges into the tank, from which a 5-in. pipe leads to the governors.

Provision has been made for an auxiliary supply, from a small stream near the reservoir, in order to insure clear water if floods in the river should render the flume supply unsuitable.

Each exciter consists of a General Electric, 150-kw., 125-volt, direct-current generator, driven by a 36-in. Pelton wheel making 600 rev. per min. For use independent of the wheels, an induction motor is mounted on the same shaft and takes current from the generators at 2 300 volts. Provision is made for a third exciter to accompany the next installation.

The exciters, in addition, furnish current for the crane, the station lighting and the machine shop. Motor-driven pumps, for

FIG. 1.—TRENCHES FOR PIPE LINES AT POWER-HOUSE; SHOWING BLOCKY NATURE OF GROUND.

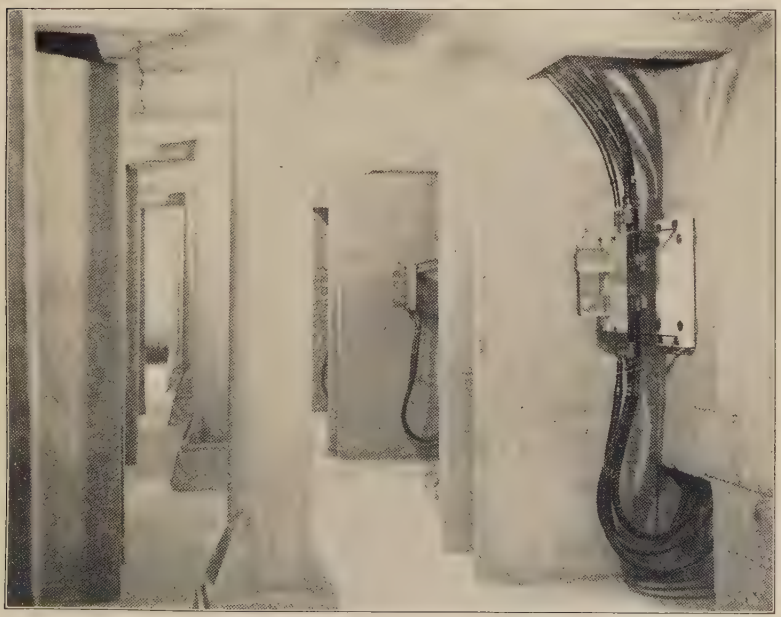


FIG. 2.—LOW-TENSION DISCONNECTING SWITCHES, WITH BARRIERS.

water circulation and pressure oil-lubrication, are located beneath the floor at the end of the building. The direct and return pipes are carried in a duct below the wheel and generator pits running the length of the building.

Each transformer-room contains three General Electric, 2 333-kw., oil-insulated, water-cooled transformers, which raise the initial voltage of 2 300 to 55 000.

In this work, 122 piles were driven, and 3 000 cu. yd. of concrete put in. A 1:3:6 concrete was used. The cement was a California Portland, giving, on a 7-day test, 400 lb. ultimate tensile strength. The sand and stone were of the same quality as described previously.

Switching-House.—This building is 52 by 150 ft., and is located back of the generator-room, with a 4-ft. air space between. It is of concrete, with the exception of the top floor, which is a wooden frame covered with galvanized corrugated iron. The concrete was a 1:3:6 mixture, as described previously, and was used throughout, including that for the 4½-in. floors reinforced with wire netting.

Pieces of this flooring, 4½ by 12 in. and 6 ft. long, with the reinforcement, were made from the run of the mixer, and were broken subsequently, with the following results: The test pieces were broken with a lever and weights, the center bearing being 4¼ by 12 in. and the span 5 ft.

Number.	Broken after:	Broke at:
1	21 days	720 lb.
2	28 "	856 "
3	36 "	1 037 "
4	111 "	1 424 "

The wire netting was made up of two No. 10 wires twisted into one strand. The strands were 4 in. apart, and were tied with one No. 12 wire, making a triangular web.

Rather than impose a further burden of work on the power-house contractors, a company's force was organized, and the work was done by day's labor. The cost of the concrete laid in this way, including 4-in. barriers, shelves for low-tension bus-bars, and 4½-in. floors, was $10.40 per cubic yard.

The first floor is the low-tension disconnecting-switch room, with switches separated by 4-in. concrete barriers. The second floor contains the oil-break switches and a double set of low-tension bus-

bars carried on concrete shelves. The third floor contains the high-tension disconnecting switches, and the fourth floor, the high-tension bus-bars, oil-break switches, and lightning arresters; and from here the cables pass out of two towers, to the pole line.

Transmission.—A double pole line is used for transmission. The poles are from 90 to 115 ft. apart. A cross-arm carries two cables, and the head of the pole a third, making a 6-ft. triangle. Each cable is composed of 19 copper wires, equal to one 0000 wire.

Triple-petticoat insulators, furnished by the Thomas and Locke Companies, and tested to 140 000 volts, with satisfactory results, were used.

The company did all the work preliminary to construction, clearing the flume line and the reservoir site, the flume substructure, and the grading for the penstocks, the switching-house and the concrete protection of the wood-stave pipe, and the back-filling of the pipe lines.

Prior to April 15th, 1903, a wagon road to the reservoir camp, and a pack trail from there to the head-works site, had been built, and the reservoir site and penstock line cleared of heavy timber. On April 15th, 1903, the serious work of construction began. On March 21st, 1904, one exciter was furnishing power for the crane motors and the station lighting. On April 12th, 1904, Unit No. 1 was delivering power to Tacoma for commercial purposes, and on July 23d, Unit No. 4 was put in operation, completing the installation.

TABLE 2.—Elevations.

	Elevation.
Crest of dam	1148.0
Floor of intake	1142.4
Flume, at outfall	1065.5
High water in reservoir	1073.0
Top of embankment	1078.0
Bottom of reservoir	1059.0
Center of nozzle, at power-house	201.5

Only three fatal accidents occurred during the progress of the work, none of which could have been prevented, by even more than the care ordinarily exercised. Minor accidents, such as cuts and

FIG. 1.—HIGH-TENSION ROOM, SHOWING BUS-BARS, OIL-BREAK SWITCHES AND LIGHTNING ARRESTERS.

FIG. 2.—LOW-TENSION ROOM, SHOWING BUS-BARS AND OIL-BREAK SWITCHES.

FIG. 3.—HIGH-TENSION DISCONNECTING SWITCHES.

bruises, were, in number, only such as might be expected with a force of 1 500 men. The total cost of the work approximates $125 per horse-power, a portion of which is properly chargeable to preparation for the second installation of 20 000 h-p., which, when made, will reduce the cost to approximately $90 per horse-power developed. The Columbia Improvement Company is a subsidiary company, of the Stone and Webster Syndicate, formed for construction purposes.

Table 2 shows the elevations of the principal points on the works.

The static head on the Pelton wheels is the difference between 1 073, the elevation of the surface of the water in the reservoir, and 201.5, the elevation of the center of the nozzles, equal to 871.5 ft.

The Pelton Water Wheel Company contracted to supply and erect the wheels, guaranteeing under penalty an efficiency of 80% at full load and 75% at half load for the four 3 500-kw. units, under an effective head of 850 ft.

The writer's connection with the work began in April, 1903, and ceased on August 15th, 1904, or three weeks after the fourth unit had been installed and put in service, and he is not aware that any determination of efficiency has yet been made. It is unfortunate that the company failed to respond to a request for means to standardize the gauges as a preliminary to the tests. By inspection of the gauges under varying conditions, however, the writer was able to approximate a correction which resulted in the following:

Surface of water	1 059	ft.
Elevation of center of nozzle	201.5	"
	857.5	"
By gauge corrected	850.3	"
Loss of head	7.2	"

This is not very satisfactory, and it is to be regretted that means were not furnished for flow measurement in so large a pipe under such conditions of high head. These measurements must be made later to determine whether or not the Pelton Water Wheel Company has met its guaranty. When made, the writer hopes to be able to obtain the results.

DISCUSSION.

Mr. Post. WILLIAM S. POST, ASSOC. M. AM. SOC. C. E. (by letter).—This paper is of particular interest to engineers of the Pacific Coast. At a recent meeting of engineers in Los Angeles, the point was brought out that, in flumes and conduits, considerable variation is found between the calculated and the actual velocities and cross-sections, if not at first, certainly after a year or more of service. It would be of interest if Mr. Warner would state how the calculated velocity is borne out in the finished work, at the beginning as well as after some months of service, with the resulting determination of n in Kutter's formula, in these cases; also whether his experience in this work leads to the conclusion that work by the company's force is cheaper than by contract?

Mr. Cunningham. JOSEPH H. CUNNINGHAM, M. AM. SOC. C. E.* (by letter).—Mr. Warner's description of this plant is most interesting, and, as a whole, quite accurate. There are some points of minor importance, however, which are not correctly stated—the final dimensions of the flume, for instance. It was the original intention to finish all members to the sizes shown on the plans, but, owing to the fact that this was not stated in the Chief Engineer's specifications, the finished pieces were from $\frac{1}{4}$ in. to 1 in. less.

The flume sills were double members, dressed on two sides, $3\frac{3}{4}$ by $7\frac{3}{4}$ in. by 15 ft. long, bolted to posts with $\frac{5}{8}$ by $15\frac{1}{2}$-in. wrought-iron bolts, with cast-iron washers. The posts, dressed on two sides, were $5\frac{3}{4}$ in. square by 9 ft. long, and spaced at 4-ft. centers. The beams were $5\frac{3}{4}$ in. square and 10 ft. 6 in. long, and were gained $\frac{3}{4}$ in. and spiked to the posts with one 10-in. wire spike. The sides of the flume were $2\frac{1}{4}$ by 11-in. tongued and grooved stuff, dressed on the inner surface. The bottom was double, the outer plank being $1\frac{3}{4}$ in. thick, and dressed on the outer side, and the inner plank being $\frac{3}{4}$ in. thick, and dressed on the inner side. At the junction of the sides and bottom a chock, $2\frac{1}{2}$ in. square, was used to confine a 7-in. strip of three-ply tar-paper which extended between the bottom outer and inner floors. This arrangement was quite effective in stopping leaks. Within five days after the water had been turned into the flume, the leakage amounted to a trifle less than 1% of the total discharge.

The bottom was sawed to a true curve, the transverse joints of the upper and lower floors being 18 in. apart, and considerable care was used to break joints longitudinally. No difficulty was experienced in bending the side planks to the sharpest curve, and, consider-

* This discussion was presented before the San Francisco Association of Members, Am. Soc. C. E., at its meeting on August 19th, 1905.

PENSTOCK LINES.

ing the fact that lumber was purchased from a dozen different mills, and that much of it was imperfectly sized, and, also, that all work above the stringers was done by contract, the flume is remarkably tight and well built. Mr. Cunningham.

As the country was too rough to admit of building a wagon road within less than ¼ to ½ mile of the flume line, a standard-gauge railway was laid over the lower 8 miles, and all lumber above the caps was delivered by rail.

On about half of the line the flume posts were used for cross-ties, but, as the spike holes left after the rails were taken up would weaken the post somewhat and tend to produce rot, it is probable that sawed cross-ties for the entire line would have been preferable, since salvage on the ties could have been realized in most cases. After the flume was completed the rails were replaced on top of the tie-beam, the intention being to use light cars for patroling the line and delivering lumber for repairs, etc. This is certainly a novel feature, and, in most cases, could be used to advantage, although in this case the tie-beams are rather light for this work.

Eventually, the flume will be 8 ft. wide and 8 ft. deep. At present the depth is only 4 ft. 4¼ in., and is intended to carry water to a maximum depth of 4 ft. The upper 548 ft. is built on a grade of $S = 0.00393$, or 1:254; on the following 1 051 ft., $S = 0.00294$, or 1:340. On the remainder of the line, 51 940 ft. in length, $S = 0.001366$, or 1:733. The latter grade was intended to be used on the entire line, but, owing to a change in the location of the intake after work had been started on the flume, a broken grade had to be used. The greater portion of the flume was located on a very steep hillside, the average slope on fully half the line being greater than 50 degrees. The soil on this portion is from 1 to 2 ft. deep, beneath which lies hardpan and basalt rock. On the lower portion, the cliffs rise to a height of 500 ft. above the flume line, and in many places are almost vertical, while below there may be an almost vertical drop of nearly 600 ft. to the river. On this portion the flume is located from 100 to 300 ft. above the brink of a cañon, the walls of which rise vertically from the river from 200 to 400 ft. Through this cañon the river tumbles in a wild succession of cataracts, plentifully interspersed with large boulders, making navigation impossible; and, as the vertical cliffs make it practically impossible to descend to the river, it is probable that this portion, which is about 4 miles in length, has never been explored.

The entire line being covered with heavy fir, hemlock and cedar, dense underbrush and logs, with many cliffs, made it a tedious and hazardous undertaking to locate the line and clear the right of way.

Owing to the difficulty and expense of building a wagon road near the flume line, it was decided to use the timber on the ground

Mr. Cunningham.

for trestle caps, posts, sills and mud-sills. All the mud-sills were of cedar; other members were of fir, hemlock and cedar. Drift-bolts and nails were delivered by wagon as near as possible to the flume, and then carried by pack-horses and men. Trestle bents were held upright by braces of light poles. Crabs were used for erecting bents on portions of the line, these sliding over bents previously erected. These, however, did not prove successful on all parts of the line, and, where the trestles were low, better progress was made with gin-poles.

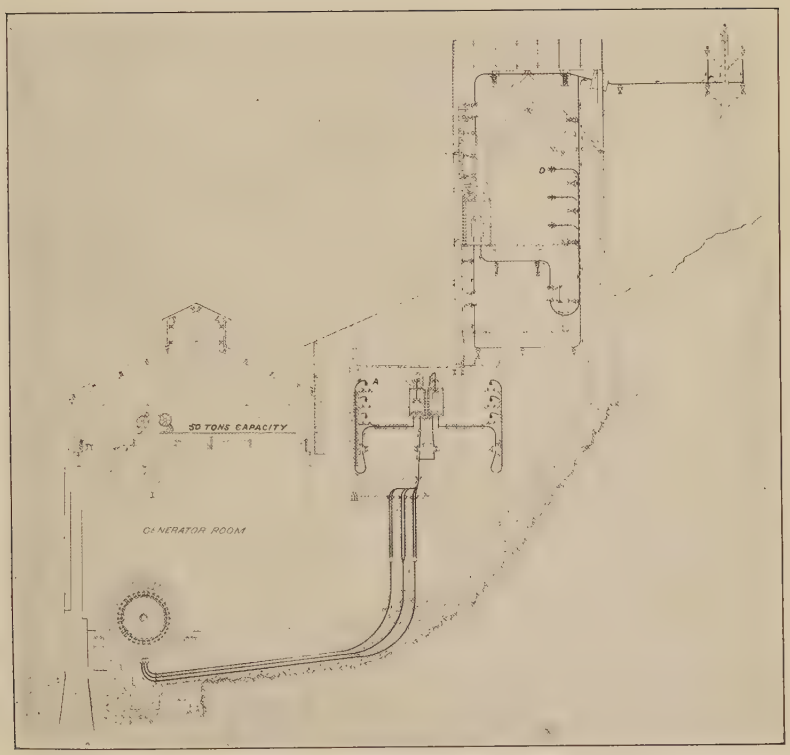
As the ground throughout this region has a strong tendency to slide when disturbed, it was sought to avoid all side-hill cuts as much as possible, and place the flume on a cripple-bent trestle for practically the entire length.

In establishing the gradient of the flume, the friction factor, n, in Kutter's formula, was assumed at 0.0105, in order to cover defective lumber and workmanship. After the flume was completed and water was turned in, the writer made a series of experiments, the results of which are shown in Table 3.

TABLE 3.—EXPERIMENTS ON THE HYDRAULICS OF THE TIMBER FLUME.

Date, 1903.	Section.	Depth, in Feet.	Hydraulic Gradient, "S."	Hydraulic Radius, "R."	Mean Velocity, "V."	Friction Coefficient, "n."	Coefficient, "C."	Discharge, Cubic Feet per Second.
Dec. 15	1	2.67	0.00393	1.60	13.5	0.0098	170.6	288.0
" 18	1	2.44	0.00393	1.51	13.22	0.0097	171.9	258.3
" 18	2	2.60	0.00294	1.57	12.03	0.0095	178.0	250.2
" 19	1	3.23	0.00393	1.79	14.27	0.0100	170.9	369.0
" 19	2	3.45	0.00294	1.85	13.23	0.0095	180.0	365.6
" 20	1	1.33	0.00393	1.00	10.70	0.0093	171.0	118.9
" 20	2	1.42	0.00294	1.05	9.78	0.0090	176.2	114.2
" 22	1	3.25	0.00393	1.79	14.55	0.0098	173.3	378.3
" 22	2	3 55	0.00294	1.88	13.45	0 0095	181.0	382.2
" 24	1	2.85	0.00393	1.66	14.08	0.0097	174.3	321.3
" 24	2	3.15	0.00294	1.76	12.08	0.0100	167.9	322.9

The mean friction coefficient, n, for dressed lumber, from Table 3, is 0.00962, and it will be noticed that in nearly every case n increases slightly with the sine of the slope, S, also with the hydraulic radius, where S is constant. The velocities were measured with a Price current meter, No. 131, at two stations 1 000 ft. apart. The observations taken on the same date were made when the depth was constant. The values of the coefficient, C, in Kutter's formula, $V = C\sqrt{R\,S}$, were deduced from the mean velocities, and the values of the friction factor, n, were taken from the diagram prepared by Messrs. Hering and Trautwine. No measurements were



POWER-HOUSE. CROSS-SECTION LOOKING EAST, SHOWING CIRCUITS NOS. 1 AND 4.

made on the lower grade of the flume, for the reason that at this time it was not quite finished at the lower end. Assuming a friction factor of 0.010 for the lower grade of $S = 0.001366$, the carrying capacity of this flume when running 7 ft. deep will be 576 cu. ft. per sec., and, for the present development, 4 ft. deep, 284 cu. ft. per sec. Mr. Cunningham.

The time required to complete the flume, including the trestle substructure, was 7½ months. The trestling and grading required about 4 months, and was by far the most expensive, not for the reason that it was done by day labor so much as on account of difficulties in the transportation of materials, supplies, etc.

In reference to the dam, it is quite evident that the line of sheet-piling under the crest is of no benefit whatever, as any water which finds its way past the upper line of piling is free to flow over the top of the second line, and, while this dam is a good, safe structure, it is the writer's opinion that an equally safe and much cheaper plan could have been adopted; but in this, as in a number of other instances, money seemed to be of no object to the company, and the mechanic who was later in charge of the work had no difficulty in introducing a number of freak designs in gates, sand-boxes, rack-supports, etc., etc. In fact, the policy of the construction manager of this work was to "strain at a gnat and swallow a camel"; small details, costing little, but of great benefit, were cut out, while extravagant plans and locations were adopted without question. During the construction of this work the writer called attention to the fact that owing to the high velocity of the water in the sand-boxes (about 5 ft. per sec.), little sand would be deposited, and for all practical purposes the sand-boxes might have been omitted, since, at the present writing, most of the sand entering the intake at times of freshet finds it way to the reservoir, and it is necessary to dredge it out occasionally. It would seem to the writer that in a case where the velocity in a flume is high it would be better to provide sluice-ways for removing the sand at the intake, which can be made wide enough to give a low velocity and permit the deposit of sand, etc.

The grades of the several pipe lines from the reservoir to the power-house were fixed by the chief engineer, and, as an example of "how not to do it," are conspicuous. At one point each alternate pipe is elevated about 6 ft. above the others on a narrow ridge not wide enough for the riveters to stand upon.

The power-house was originally located some 60 ft. farther out from the hill, but as bed-rock was not found for a considerable depth at certain points it was moved back against the hillside. Later, it was conceded that this was a mistake involving large expense in moving rock and earth; furthermore, the danger of land slides is largely increased by cutting off the toe of the slope. In the writer's

Mr. Cunningham.

opinion, the original location of the power-house was the better in every way, and, by sinking concrete piers to bed-rock, a more economical foundation could have been secured.

On the cable incline a minimum grade of 3% was adopted, but, through an error by the purchasing department, a 1-in. plow-steel cable of six strands, with seven wires to the strand, was purchased instead of the more flexible nineteen-wire strand. The hoisting engine, which was purchased by the company under a guaranty from the makers, was geared too low, and would not handle the standard cars on the 65% grade, hence the necessity of using small flat cars, made by the company, and weighing about 4 000 lb. each. The weight of these cars on the 3% grade was not sufficient to overcome the friction of the 2 300 ft. of cable on a descending car, therefore it was necessary to divide the incline into two sections, using a large donkey engine at the upper end of the 3% grade for hauling standard flat cars up to that point, where all loads had to be transferred to small cars. The writer was in favor of making 4% the minimum grade, in view of the uncertainty in the amount of friction to be overcome. This road is 4 396.5 ft. long, and cost $40 000, and at least $30 000 could have been saved by adopting the original location and making it a single track for the entire length. The lower end of the road as originally located was about 700 ft. north of the long trestle shown on the map, Fig. 1, and the writer proposed to connect here with the railway to the power-house, which was to have a maximum grade of 3%, with two switch-backs to enable cars to run directly into the power-house. It was admitted that this would be the more economical construction, but, as time was limited and the management skeptical about a single-track line having sufficient capacity, it was decided to build the more expensive line, which was double-tracked for one-half its length, after which but one track was used. This plant cost fully $150 000 more than was necessary, and was a repetition of the old story of the business manager and laymen mixing in the engineering department, with the usual results.

Mr. Adams.

ARTHUR L. ADAMS, M. AM. SOC. C. E.* (by letter).—The design of works of the character described in this paper gives rise to interesting and important hydraulic and economic questions, to two of which the writer wishes to call attention.

The first is the carrying capacity of large, open, box-flumes, such as used in the construction described by the author.

Although this type of construction has been common for so many years, engineering literature records no experiments on flumes of comparable size adequate to enable one to determine satisfactorily the effect on carrying capacity to be expected from using dressed or

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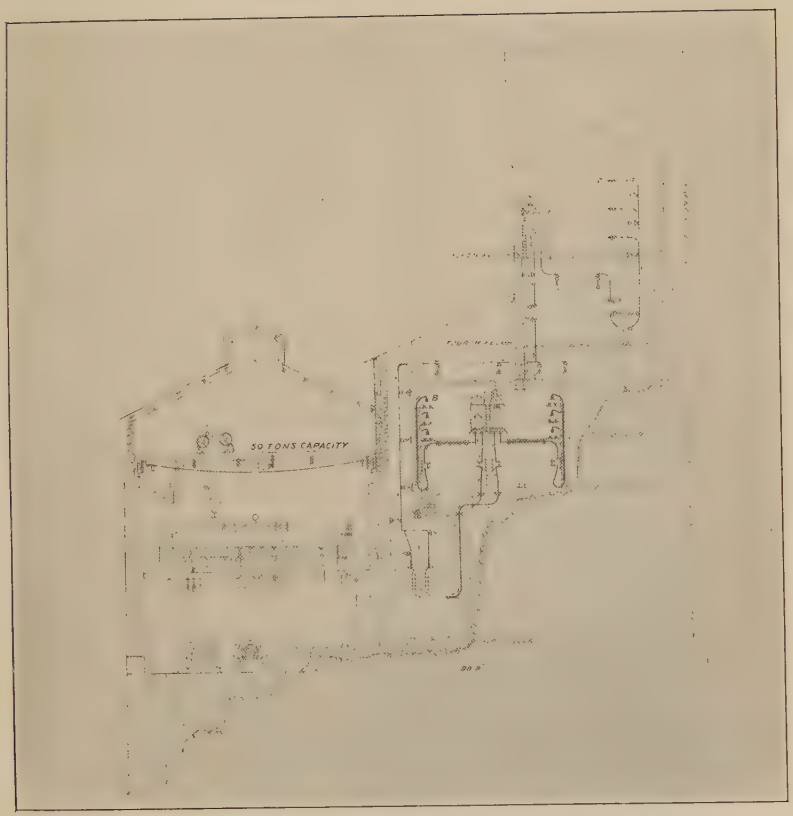


POWER-HOUSE. CROSS-SECTION LOOKING EAST, SHOWING CIRCUITS NOS. 2 AND 3.

rough inner surfaces, the effect of age on such surfaces, and especially the effects of curvature and angles. Mr. Adams.

Our knowledge of flume capacities, thus far, is based largely upon a few observations made on straight experimental channels, supplemented by inferential deductions from observations made on iron pipes, and channels of other character. What is needed is careful determinations of results achieved in actual practice.

If the flume described has been built with close adherence to established grade, a condition unfortunately not often met in accomplished work, it would seem to offer, by reason of its size and length, an unusually favorable opportunity for a valuable addition to professional knowledge on this important subject. The expectation that 280 cu. ft. per sec. will be delivered with a depth of 4 ft. seems to the writer doubtful of realization.

The second question to which the writer would call attention is the problem involved in determining the permissible sacrifice in frictional head in flumes and pressure pipes in order to reduce the size and keep the cost in accord with the requirements of the most economic expenditure.

In the case of a flume, it is true, of course, that the grade is often necessarily determined by other considerations, such as the necessity for diverting at a certain point, and the desirability of delivering the water to the pressure pipes at a certain advantageous place, or it may be influenced by the character of the intervening country; but none of these factors will influence the permissible loss of head in selecting the size of pressure pipes to be used.

Whatever the other limitations may be in any specific case, it should always be borne in mind that the same principle, long ago enunciated for securing the highest economy in the quantity of copper used for electric transmission, applies to other forms of transmitting energy; and that the total investment in flume or pressure pipe should, when practicable, at least approximate to the value of the power sacrificed in losses of head therein.

The writer believes that this broader application of the rule has been very often overlooked by engineers, and he recalls no instance where attention has been directed to it in professional literature.

In the case of power installations where regulation is obtained by automatic gates controlling the discharge, low velocities in the pressure pipes may be desirable, in the avoidance of ram and in securing ease of speed control; but, under high heads, where deflecting nozzles are used for the automatic regulation, whether or not needles are used for adjusting by hand the rate of discharge, there can be no danger of excessive water ram, and, consequently, no mechanical objection to the use of high velocities in the pipes if the water is reasonably free from grit, as it must be for other reasons.

Mr. Adams. In the installation described by the author insufficient data are given to ascertain whether or not a study of the problem has been made in determining the size and grade of the flume; and nothing is said as to whether local conditions did or did not decide the question. But, concerning the pressure pipes, enough data are given to throw some light on the subject.

The cost of these pipes is given as $140 000; the loss of head therein as 7.2 ft. Calling the installation a 20 000-h. p. plant indicates an expected efficiency of 75% for the power-house equipment. The total power lost in the pressure pipes, therefore, is

$$\frac{276 \text{ (cu. ft. per sec.)} \times 62.4 \text{ (lb.)} \times 7.2 \text{ (ft.)} \times 75 \text{ (\%)}}{550} = 169 \text{ h. p.}$$

and the cost of pipe per horse-power sacrificed in the pipe, therefore, is

$$\$140\,000 \div 169 = \$828.$$

There is, therefore, power supplied, in using pipes of the size chosen, costing at this very high rate for the pressure pipes alone, whereas the average cost of the entire installation, including transmission lines and all, is given as about $125 per horse-power. It appears, therefore, that, in keeping the frictional loss in the pipe as low as 7.2 ft., a price is being paid for some power saved thereby of nearly seven times the average cost of producing and transmitting it.

Would it not much better have been used in inducing a more rapid rate of flow and a correspondingly lessened investment in pipe?

The writer would be glad to be informed if in this case there were factors in the problem, not mentioned in the paper, which overruled the considerations he has just mentioned and made necessary the course adopted.

Mr. Havens. VERNE L. HAVENS, JUN. AM. SOC. C. E. (by letter).—The criticism by Mr. Arthur L. Adams brings out some questions of unusual importance to engineers. He states that:

"The total investment in flume or pressure pipe should, when practicable, at least approximate to the value of the power sacrificed in losses of head therein."

It must be remembered that it is the power saved that is paid for, and the $140 000 was not expended in losing 7.2 ft. head, but in saving the available head less 7.2 ft.

The true question is not what power one can afford to lose, but what power one can afford to save, the remainder of the static head being what must be lost because of excessive cost. Therefore, after reducing it to a practicable minimum, the friction head has no direct relation to the investment.

POWER-HOUSE, SHOWING FOUR GENERATORS IN OPERATION, AND TRENCHING AND GRADING, AT THE RIGHT, FOR FOUR ADDITIONAL UNITS.

Mr. Havens.

In any pipe line there are local details, such as anchorage, damages, etc., which will approximate the same cost for any rational design. These should be added to the summation of all costs of plant, giving the pipe line such a margin for possible expenditure that it will leave the total investment an attractive one for capital. The pipe then becomes a local problem, having no relation, within the marginal limits, to the cost of the entire plant. It is then governed by capitalized value, and not cost of power.

In any pipe design there are two extremes: One lies where the cost is infinitesimal and the loss of power great; the other lies where the loss is infinitesimal and the cost great. The maximum cost cannot exceed the marginal limits noted above. Between these two extremes the design must lie, and the line of low cost should be chosen as the more favorable working basis, and then increase the cost as the loss decreases until a condition is attained where the added cost of any added power would exceed the capitalized value of that power. The remainder of the available head, in the case considered, is 7.2 ft., and one cannot arrive at the unit cost of power saved by dividing the total cost of power saved by the total amount of power lost.

Should not the rule given, then, be altered to read as follows?

"The total investment in pressure pipe should approach that point as a maximum, where any added power will cost more than the capitalized value of that power, provided this investment, when added to the summation of other plant-costs, does not exceed an amount which is an attractive investment."

Mr. Warner.

Edwin H. Warner, M. Am. Soc. C. E. (by letter).—The question of company force account *versus* contract work, raised by Mr. Post, is one on which the writer has always held a fixed opinion. Under ideal conditions a company can do its work as cheaply as by contract. Under usual conditions it cannot. The ideal conditions are: An engineer in responsible charge, possessed of the necessary administrative ability; his absolute freedom in the selection of subordinates; and full control of all departments, including the timekeeping and accounting. This latter is important, since a rational distribution of cost shows at once any departure from the fixed normal for each class of work, and permits a correction to be applied before the trouble becomes serious. This distribution cannot be made by the professional bookkeeper, who, in spite of his willingness to help, is not sufficiently informed to enable him to do so. The usual conditions are: A divided responsibility; relatives and friends of directors and shareholders, for whom places must be made; a system of accounting which gives no detailed information; and a commissary and supplies department handled by inexperienced men. If, to the above, is added the fact that the "equipment and tools account" is closely scrutinized, while the labor roll

Mr. Warner. arouses no interest, it will be readily understood why a company pays more than when the work is let by contract. This plant suffered somewhat from the usual conditions, but not to an abnormal degree.

Under ideal conditions, as outlined above, the writer is endeavoring to justify his statements; so far, the results have been satisfactory, but, "call no man happy until he is dead." Later, the writer hopes to prepare a paper on the subject.

Mr. Cunningham, whose friendship is valued by the writer, and for whose attainments as an engineer the writer has a very just appreciation, has unfortunately allowed his feelings to warp his usual good judgment, to the extent of injecting matters into this discussion which would more properly have been omitted.

While his corrections are accepted in such minutiæ as the difference between a 6 by 6-in. and a 5¾ by 5¾-in. post, the writer does not think these differences necessarily misleading.

As to cheapening the dam, the quantity of lumber saved at $10.50 per thousand would have been offset by changes in drawings, multiplication of correspondence, and loss of time. The diversity of opinion and division of responsibility, with the consequent difficulty of securing approval of any design during the first few months the writer was on the work, would have justified the building of a dam along less favorable lines, rather than re-open a matter once settled. In this diversity of opinion and lack of harmony the writer cannot hold the engineering department blameless. With the reorganization of the department, the writer became Principal Assistant Engineer, and held this position for thirty days after the complete plant was in operation. Thus Mr. Cunningham's criticism touches the writer to this extent: all plans passed his inspection, and had he made decided objection, the plans would have been modified. His relations with both chief engineers, and with the construction manager were invariably pleasant, and, while differences of opinion did arise, they were reconciled without difficulty, as must always be the case when the best interests of the work are held to be paramount.

The writer recalls no "freak" designs. The gates in the forebay were made by the Coffin Valve Company. The needle-gates for the checks below the waste-ways in the flume, and the radial gate at the flume intake, are certainly conventional. The rack supports, Plate XXVII, are composed of a grillage of channels and I-beams, and are certainly not "freaks." The sand-boxes served a useful purpose while the writer had opportunity to judge. During sudden freshets, some finely divided material found its way to the reservoir, but it is also true that the sand-boxes intercepted a part of the solid matter. That they did not intercept all the sand, led

the writer to suggest, after personal knowledge of the conditions gained during five months' operation of the plant, that the flume be widened at the intake, and that scouring sluices be introduced. Mr. Warner.

On reviewing mentally the various features of the plant, the writer can see opportunity for beneficial change, as must any engineer who analyzes his work carefully, but this statement does not alter the fact that the plant is well designed, is efficient, and, up to date, has given no serious trouble in its operation. Its cost is not above the normal for a plant of its size, and, when increased to 40 000 h. p., will compare favorably in every way, cost included, with other plants in the country.

The criticism of the tramway details is idle, since a tramway should never have been considered; a switchback to the reservoir site was the practical and cheap solution of the transportation question.

The power-house was moved back from its original location in spite of protests by Mr. Cunningham and the writer. The criticism of the adjustment of the pipe line grades is eminently correct.

Table 4, showing the flow in the flume, answers the questions by Mr. Post and Mr. Adams:

TABLE 4.

No.	Date, 1904.	Bent.	Depth, in feet.	S.	R.	C.	V.	N.	Q.	Area.
1.....	April 15.	364	2.25	0.001366	1.44	137	6.08	0.0118	109.42	18.00
2.....	May 13.	364	3.50	0.001366	1.87	123	6.26	0.0137	175.45	28.00
3.....	" 17.	364	3.50	0.001366	1.87	125	6.34	0.0133	177.35	28.00
4.....	" 20.	364	3.46	0.001366	1.85	124	6.26	0.0136	172.27	27.68
5.....	" 9.	8 066	3.07	0.001366	1.73	143	6.98	0.0125	171.43	24.56
6.....	" 15.	499	3.35	0.001366	1.82	129	6.47	0.0130	173.3	26.80
.....	.....	.....	.....	.....	.....	.....	.....	Curve Radius = 286.5 H.		

Assuming "n" = 0.0105, and $V = C\sqrt{RS}$, we get for No. 2, $V = 8.05$ and $Q = 225.4$.

The measurements were made at the points indicated on the chart showing the cross-section of the flume. The observations were made on straight portions of the flume, with the exception noted. Levels were taken to determine the slope of the water surface, and agreed with the prescribed grade of the flume. A hook-gauge was used in measuring the depth of water, both before and after the flow was metered. A small current meter was used, and registered the number of revolutions in 30 seconds in each position shown on Fig. 8. The observations were made with care, and the value of "n" deduced therefrom. It will be noted that these values differ materially from those in Table 3, given by Mr. Cunningham.

Mr. Adams lays down the law "that the total investment in

Mr. Warner. * * * pressure pipe should * * * approximate to the value of the power sacrificed in losses of head therein." He deduces from the law the fact that in this particular case the permissible sacrifice of power greatly exceeded that actually lost, whereby an uneconomic size of pipe was selected.

Applying Mr. Adams' method:

$140 000 ÷ 125 = 1 120 h. p., permissible sacrifice. Power in Seattle and Tacoma is vendible at $20 per annum.

$20 × 1 120 = $22 400.

It is proposed to sacrifice this revenue, which, capitalized at 10% = 224 000 to reduce $140 000 to some lower figure. There is no relation, fixed or otherwise, between the cost of the plant as a whole, and the cost of the pipe, nor between the cost of the plant and

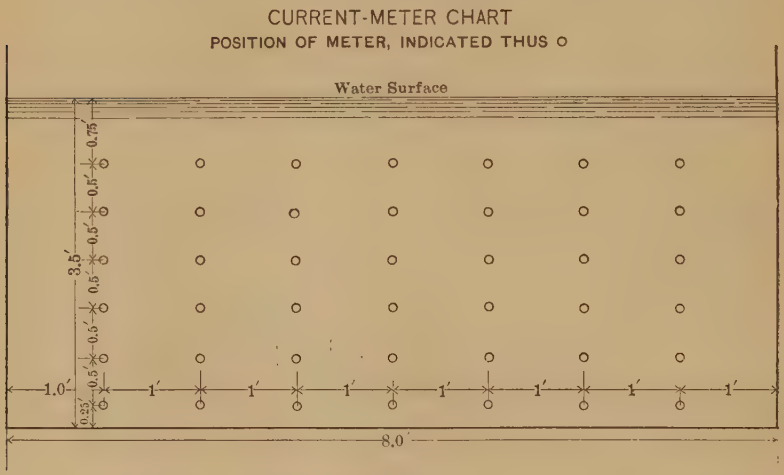


FIG. 8.

the power sacrificed in friction losses. The facts are: A loss of 169 h. p. at $20 = $3 380, loss of revenue, which, capitalized at 10% = $33 800. It would seem as if this sum could be added to the pipe investment with advantage, but the loss of revenue represents only $\frac{8.5}{100}$ of 1% of the total, and this is certainly less than will be the difference between the calculated and actual income, or the yearly difference in a series of annual incomes.

140 000 ÷ 20 000 = $7, cost of pipe per horse power. The writer concludes from the foregoing:

(*a*) That $828 does not represent the value of any part of the power lost, saved, sold or otherwise disposed of.

(*b*) That the economical dimensioning of this pipe line was closely approximated.

(*c*) That an inspection of Table 1, showing the make-up of the pipe, will impress one with the fact that the difference between the actual friction head and the calculated head of 15 ft. is a matter of congratulation, and not a reproach. The calculated loss of power, based on 15 ft. friction head, was 352 h. p. Mr. Warner.

$20 × 352 = $7 040, annual revenue lost.

$140 000 × 5 (%) = $7 000, annual interest, which seems to be in fair agreement with Mr. Adams' statement of equated values.

In spite of this apparent agreement, the writer still cannot see the connection between loss of power and total cost of pipe, for if $20 per horse-power be capitalized at 10%, and this divided into the cost of the pipe, $140 000 ÷ $200 = 700 h. p. sacrificed to obtain a pipe line the cost of which is zero. Therefore the pipe line cost disappears entirely, but 20 000 h. p.—700 = 19 300 h. p. remain, a condition a trifle difficult to realize in practice; continuing the process, however, with this plant, we reach a point where the efficiency is reduced to 25% and the total cost of the plant has disappeared, while there remain 5 000 h. p. on which to collect revenue. It will be noted that the lower the efficiency, the less money is required. If it be urged that $20 is not net, the answer suggests itself that the revenue from 169 h. p. certainly is net, while, with respect to the 5 000 belated h. p., there can certainly be no charges, fixed or otherwise, on a plant possessed only of whatever is the equivalent, in power plants, of an astral body.

The writer cannot rid himself of the idea that the pipe line cost $7 per horse-power, equated values and capitalized revenue to the contrary notwithstanding. If increased investment increases the returns, and it is possible to beg or borrow the money, it would seem to be wise to do it. This is neither new nor original, but it applies to a pipe line, in common with much else which lack of space forbids to mention.

Lord Kelvin's law, first stated in 1881, later given a qualified application by Kapp, with tables to facilitate calculations by Forbes and Carhart in 1885, was intended to apply to a direct-current distribution system from a central station. All such calculations have been dropped since the introduction of the net work system of distribution made it possible to maintain a constant potential with two or three sizes of mains and one or two sizes of feeders.

The writer is not at all sure how this law will work out if applied to an alternating-current transmission line, where a choice, involving varying amounts of copper, may be made of one of seven systems; and has not at hand the data necessary to consider its application to the three-phase system of the present plant.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1008.

SOME SPECIALTIES OF THE SYSTEM FOR FLUSHING THE NEW SEWERS OF THE CITY OF MEXICO.*

By Roberto Gayol, M. Am. Soc. C. E.

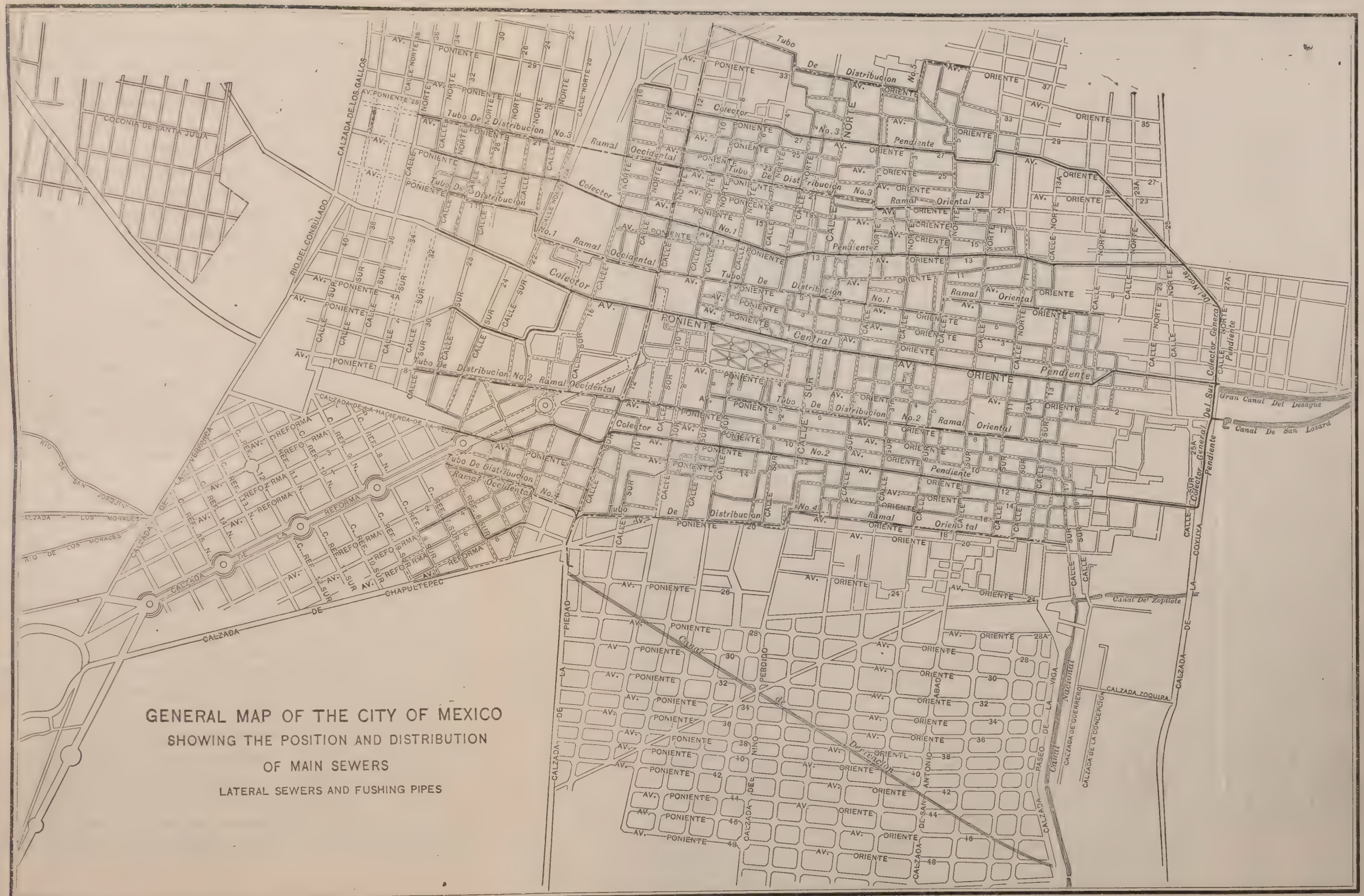
With Discussion by Messrs. Alexander Potter and Roberto Gayol.

General Remarks on the Old and New Sewers.

In 1885 the writer entered the service of the City of Mexico as Chief Engineer, and was at once instructed by the Alderman in charge of Public Works, the Engineer, Manuel M. Contreras, to study the proper construction of the sewers, as he was desirous of modifying the old system of construction by establishing modern methods.

A study of the drainage of the city was undertaken, and it was found that the sewers, consisting of an extensive network, had many defects. They were of rectangular section, without any established grade, because the city being on an almost horizontal plane, all the sewers had been constructed without any plan or order, and the slight grade at the bottom was often found to be contrary to that which was required by the flow of the water; the materials used in the construction of the walls were in some places brick of a very poor quality, and in some places stones, of irregular form; and,

* Presented at the meeting of October 4th, 1905.



GENERAL MAP OF THE CITY OF MEXICO

SHOWING THE POSITION AND DISTRIBUTION OF MAIN SEWERS

LATERAL SEWERS AND FUSHING PIPES

as hydraulic mortar had not been used in any case, the joints were empty, and thus presented very rough surfaces.

On the other hand, the absence of a systematic plan for the sewers, the construction of which had been commenced in the middle of the eighteenth century, resulted in their being given insufficient dimensions for the volume of water which they were required to carry, for each sewer had been built in accordance with the caprice of the person who constructed it, and as if its only duty was to drain the block in which it was situated. The result was that, in many cases, and on the same line, the cross-section of the sewer decreased as it approached the connecting canal. The latter passed among the houses on the east side of the city, and was a noisome ditch, full of putrid water.

The level of the water in this ditch varied according to that of Lake Texcoco, and also depended on the rainfall in the Valley of Mexico. This valley, having no outlet whatever, the volume of water was decreased only by evaporation. Therefore, whenever the rains were heavy, the evaporation was not sufficient to lower the level of the lake, and the city was flooded, so that many blocks remained under water, sometimes for six months or more.

These extensive inundations took place about every ten years, coinciding with the cycles of the periodical variation of the rains; but, besides this, as heavy rains fall every year in Mexico, whenever the rainfall amounted to 15 or 18 mm. or more, many streets and houses were flooded for 6 or 8 hours. In these cases, the filthy waters gushed up from the sewers in the lower streets, and on subsiding left them covered with a pestilent and loathsome mud.

Besides these objections, the sewers, on account of their bad hydraulic conditions, were always full of mud, and exhaled such a strong and vile smell that any person approaching the city by rail could perceive the bad smell at a distance of 4 or 5 km.

A study of the condition of the city led to the conclusion that, in order to get rid of the sewage and avoid the injurious inundations, it would be necessary, first, to complete the drainage of the valley, which work had been commenced three hundred years before, and, for many different reasons, had not been finished.

On the other hand, as these works were not completed, and bearing in mind that the permanent sewers would have to be built at a

lower level than the bottom of Lake Texcoco, it would be necessary to drain them by pumps, and these pumps should be erected at once, if the drainage works of the valley were not executed with rapidity, as it was urgently necessary to commence the construction of the new system of sewers.

Fortunately, the Federal Government actively pushed forward the completion of the drainage works of the valley; but the very magnitude of these works delayed that object, and, as the rainfall in 1887 and 1888 was exceedingly heavy, the city was again inundated. The writer then proposed the erection of four large centrifugal pumps, which would liberate the city from the risk of extensive inundations.

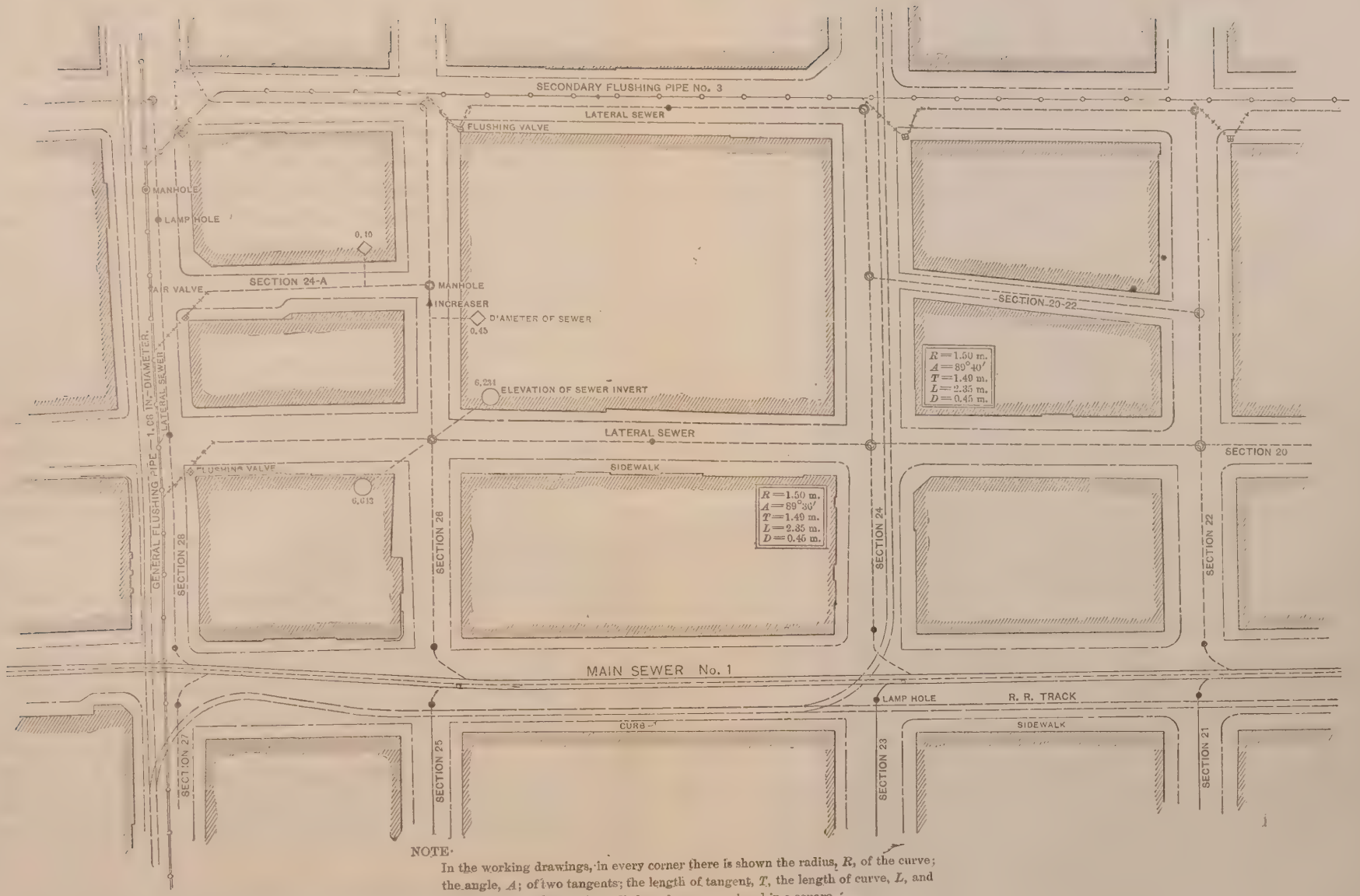
This scheme having been approved, the pumps were erected under such conditions that the water in the city could be lowered to the level which it would have when the drainage works of the valley were completed.

Having secured an outlet for the water by pumping, a scheme for the construction of a sewer system was presented to the Municipal Corporation. In studying this system the writer visited several cities in the United States, and thus had the opportunity of forming valuable friendly relations with such engineers as Messrs. Rudolph Hering, S. M. Gray, F. P. Stearns, Eliot C. Clarke, Howard Carson, Benezette Williams and others, whose reputation as sanitary engineers is generally known.

From an analysis of the peculiar conditions of the City of Mexico, the conclusion was reached that the combined system of sewerage should be used, and, from a general study of this system, the writer was convinced that it could only give satisfactory results if the sewers were provided with a perfect system of flushing, because this is the only accepted means for cleaning sewers, not only as regards economy, but also because those sewers which are frequently flushed are those which are maintained in the cleanest condition.

Governed by this idea, it was planned that the new sewers should be provided with a thoroughly efficient system of flushing, so as to avoid the complaints and difficulties which would arise with sewers of the combined system, in a city in which it often happens that not a drop of rain falls during seven months of the year.

PLATE XXXIX.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1008.
GAYOL ON
FLUSHING SEWERS.



NOTE.

In the working drawings, in every corner there is shown the radius, R, of the curve; the angle, A; of two tangents; the length of tangent, T, the length of curve, L, and the diameter of the pipe, D; all these data are enclosed in a square.

Limits of the masonry of the houses
Curb stone
R.R. tracks
Center line of completed sewer
" " " projected "
Number of decameters from the starting point

Elevation of center line of pipes. 5.60 6.234
Elevation of sewer or pipe invert
Topographical pegs
Reference line
Manhole in main sewer
Tiles showing elevation. The number on the tile, shows the elevation above datum of the divison line between the two colors. 10 11

Flushing pipe 1.08 m. in diameter.
" " 0.76 m. " "
" " 0.20 m. " "
" " 0.16 m. " "
Gate-valve of 0.76 m.
" " 0.20 m.
" " 0.16 m.
Manhole in flushing pipes.

Manhole where two lateral sewers cross
Manhole with gate where a new lateral starts
Manhole where two sewers join
Air valve
Lamp hole
Diameter of sewer. 0.15 0.40
Elevation and diameter. 4 6.332
Increaser

This idea led the writer to propose a flushing system differing from any that he knows of in other cities. For this reason he desires to have the honor of bringing it to the attention of the members of this Society, now that it is not only in actual service, but has been shown, by nearly two years' experience, to work without any difficulty whatever, and with great economy, as will be demonstrated by the statistics of the working expenses.

In November, 1891, the writer had the honor of attending one of the meetings of the Society, and there explained the principal grounds on which he had proposed the scheme of drainage for the City of Mexico. He also described in general terms the system proposed for flushing the sewers. Shortly afterward, he forwarded a printed paper* in which he described the scheme in detail, and which is now in the Library of the Society. Subsequently, however, improvements were made in the original plan.

The incidents which delayed the construction of the sewers need not be mentioned, as they are of no scientific interest; but it should be stated that the scheme encountered much opposition, and the writer had to struggle for six years before the work was undertaken, although it was carried out afterward with great rapidity.

During this time, and while fighting to carry out the plan, the writer introduced improvements which, undoubtedly, contributed very much toward producing good results; and these are now appreciated, even by those who opposed the project.

It would take too much time and be of little use to enter into a description of the changes made and the reasons for their introduction, and therefore this paper will describe the work as it is, and state the results obtained.

The city is divided into five zones, which, as far as permitted by the irregularities of the streets, cover a width of six blocks, as indicated in Plate XXXVIII, to which the following description refers.

Along the longitudinal center of each zone there is a main sewer of large dimensions, which receives the waters carried by the lateral sewers in that zone, and which for this reason is termed a "Collector," and has the same number as the zone to which it belongs. Each lateral runs along a sinuous general line, straight between

* "Proyecto de Desagüe y Saneamiento de la Ciudad de Mexico," Roberto Gayol, Mexico, 1891.

corners, but turning every corner alternately so as to follow a street from north to south, and then another from east to west, until it reaches the boundary of the zone, where there is a 76-cm. cast-iron pipe, which is called a distribution pipe because it distributes the water for flushing the sewers.

The water from the National Canal is carried in the Derivation Canal to the flushing pumps at the crossing of the Piedad and Chapultepec Roads. These pumps force the water into the distribution pipes with an initial pressure of 12 hectogrammes per square centimeter, equivalent to that of a column of water 12 m. high.

At every street crossing where the distribution pipe passes, two 15-cm. cast-iron pipes issue from the main pipe and run to the starting points of two lateral sewers, thus serving to inject into each of the latter a volume of water of about 120 liters per second (1 600 gal. per min.). The water enters with an initial velocity of from 8 to 10 m. per second, which is soon reduced by friction to 1½ or 2 m. per second, but this velocity is sufficient to make the sewer perfectly clean.

Of the five zones into which the city is divided, one is called the central zone; the two on the north side are numbered 1 and 3, and the two on the south side 2 and 4, in this way following the general rules that govern the nomenclature of the streets.

Each lateral sewer is considered as forming a section of a zone. Those on the north side have even numbers, and those on the south side have odd numbers, both rising from east to west.

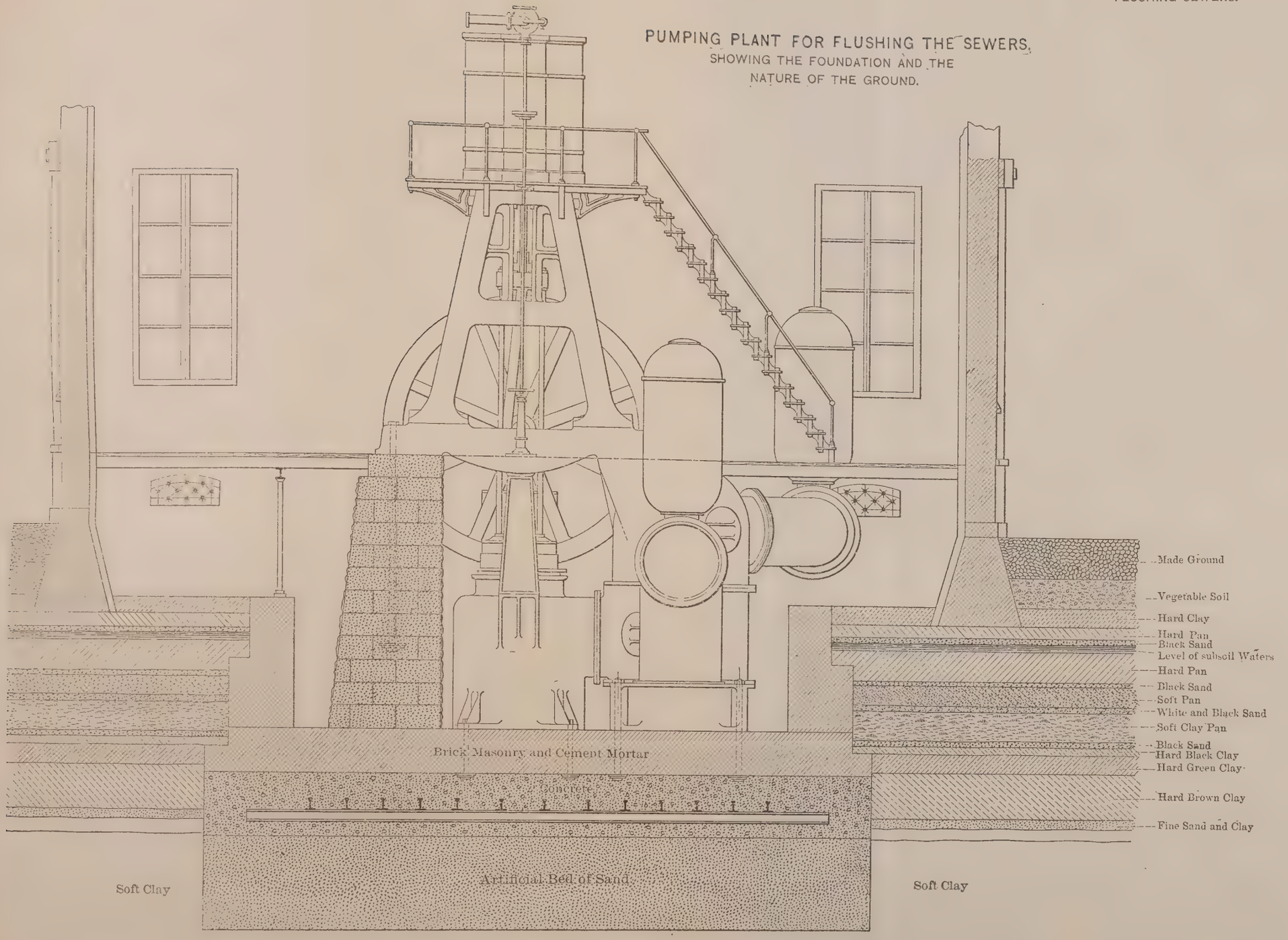
The general distribution pipe, having a diameter of 1.08 m. (42 in.), starts from the pumping plant and crosses the city from south to north. From this pipe there are nine branches at present, five to the east and four to the west; those to the east in the northern region bear the numbers 1, 3 and 5, and those in the southern region are numbered 2 and 4.

The city has not spread much to the west of the general distribution pipe, and for this reason only Pipes 1 and 3 are laid in the northern and Pipes 2 and 4 in the southern region.

This would be the system in its most elementary form, if the city had been laid out in a regular manner, but as there are many irregularities, it became necessary to adapt the system and the location of the sewers to these irregularities. The general idea, which

PUMPING PLANT FOR FLUSHING THE SEWERS,

SHOWING THE FOUNDATION AND THE NATURE OF THE GROUND.



A SECTION OF THE CITY OF MEXICO SHOWING DETAILS OF ARRANGEMENT OF FLUSHING PIPES, SEWERS, COLLECTORS, ETC.

Fig. 1

has just been explained, undoubtedly renders it much easier to understand Plate XXXVIII, which shows graphically the location of the main sewers, lateral sewers and distribution pipes, just as they are now constructed and in active service. In this plan the following irregularities may be observed:

I.—Several sewers, such as those numbered 15 and 17 in the central zone, do not reach the distribution pipe.

In this case, and in each other in which the lateral sewers discharge directly into the main sewer, it is considered as an independent section, and bears a special number, but receives the flushing waters, not directly from the distribution pipe, but from some other sewer with which it is connected by a manhole in which a sluice-gate can be placed, in the manner described further on.

II.—In certain cases, such as that of Sewer 25 of the central zone, one or more sewers which commence at the distribution pipe do not reach the main sewer, but discharge their contents into another lateral sewer.

In this case, as in that of No. 25, above mentioned, the longest line is considered as the principal one, because it is the line that serves to fix the grade of the others which discharge into the longest; the first on the east side is numbered 25 *A*, the second 25 *B*, and so on, successively.

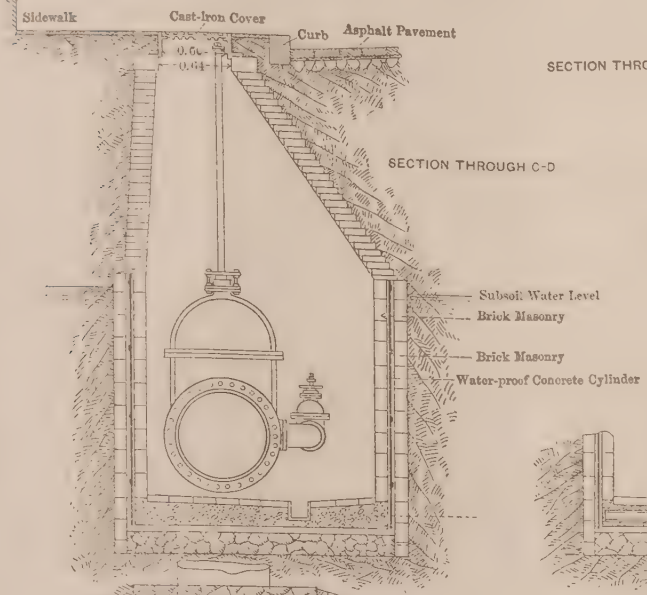
III.—The third irregularity is found in those sewers which neither commence from a distribution pipe, nor terminate in a main sewer; a case which is frequently repeated. An example can be seen between Sewers 30 and 32 of the central zone.

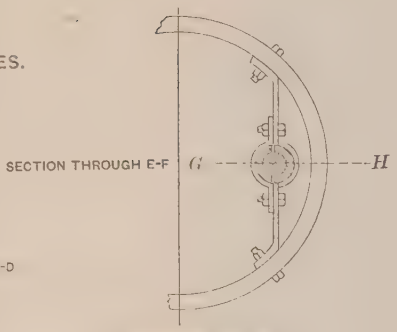
Each sewer which is subject to these conditions is designated by two numbers, that of the sewer from which it receives the flushing water, and that of the sewer into which it discharges its contents. For this reason, the sewer just mentioned is numbered 30-32 of the central zone.

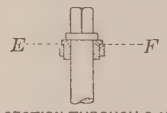
With these data, it is easy to understand the meaning of the numbers and locations shown on Plate XXXVIII.* The diameters are indicated in centimeters, the lengths in meters, and the grades in decimal fractions, or otherwise by the natural sine of the angle of descent.

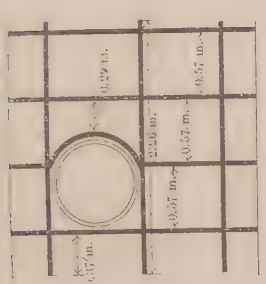
* All the data relating to the diameter of the sewers, their grades, the length to which each diameter is carried, and the total length, as well as the level of the bottom at the starting point, and the slope of the main sewer, are filed with the plans in the Library of the Society.

MANHOLE AND CYLINDRICAL MASONRY BOX
FOR THE 0.76-M. VALVES OF THE FLUSHING PIPES.

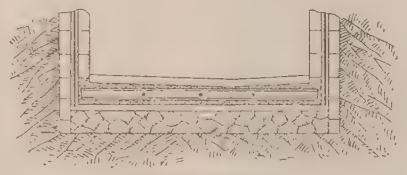


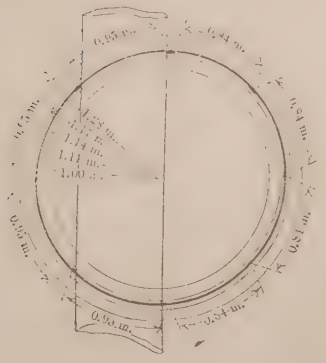






WROUGHT-IRON FRAME FOR THE
WATER-PROOF CONCRETE CYLINDER




0.95 m.
0.64 m.
0.64 m.
0.57 m.
0.57 m.
0.95 m.
0.95 m.
0.57 m.
0.57 m.
0.57 m.
0.57 m.
0.57

Details of the New Sewers.

Plate XXXIX is a diagram which shows in detail the reference signs used in the plans for the construction. It explains the meaning of the data contained in the other plans presented herein, and serves to give an approximate idea of the details of the sewerage system.

As all the water used in flushing the sewers has to be pumped, and as fuel is extremely expensive in Mexico, the writer was very careful in the selection of the proper kind of pump.

Specifications in Spanish, English and French were forwarded to the principal countries of the world, and tenders were invited, and, when received, were carefully analyzed.

To commence with, all tenders in which the annual expense exceeded seven-tenths of the maximum expense were at once excluded, and then a careful comparative study was made of the special conditions of the best tenders, the results of which were tabulated. Then, from these data, was formed the diagram, Fig. 2. The object of this diagram is to show at a glance the economical conditions, and the engines that comply best with the specification. The diagram is divided into two parts by a horizontal line, the first section of which represents the specified duty; the second represents the average price of the machinery, which average has been obtained by taking the mean value of the twelve proposals; the third section represents the annual expense of $28 000, which gives very closely the average of all those in the diagram. The upper part of the diagram shows the favorable, and the lower part the unfavorable, conditions, as regards economy; for this reason the scale of the duty is ascendant and those of the cost and annual expenses are descendant. In those cases in which a manufacturer presented various tenders, the average of the data referring to the same was adopted.

On the horizontal line there is a point through which the lines of the tenders which agreed with the specifications are made to pass; then follows the line of the minimum acceptable duty, that of the average of the prices submitted by the manufacturers, and that of the maximum annual cost that could be admitted.

There were only two tenders having lines which maintained themselves continuously within the zone of favorable conditions, namely, that presented by the E. P. Allis Company, and that of the

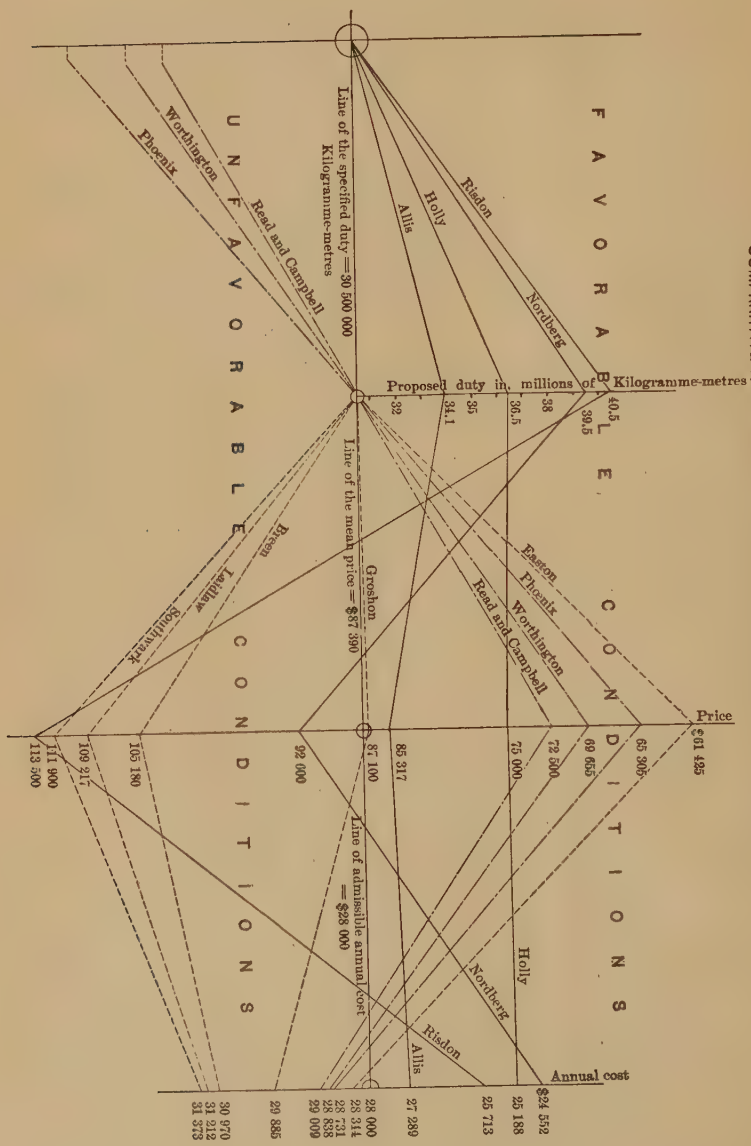
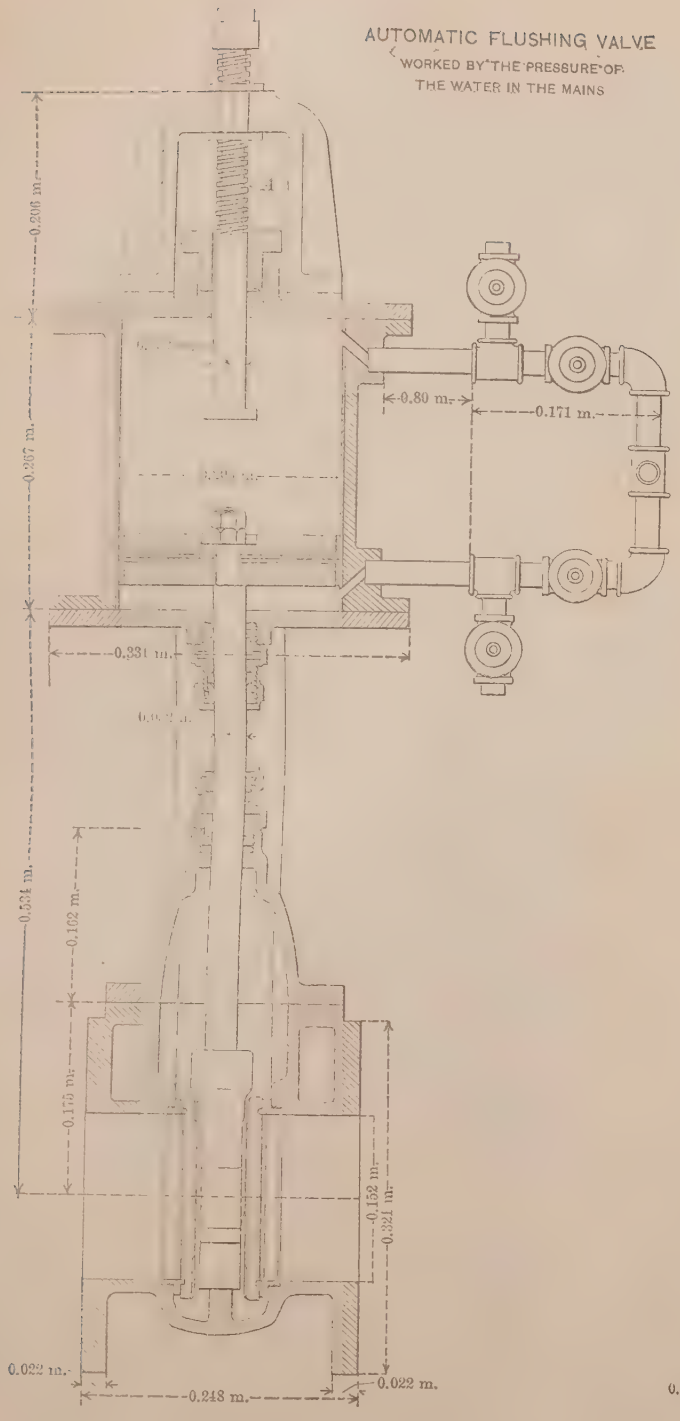


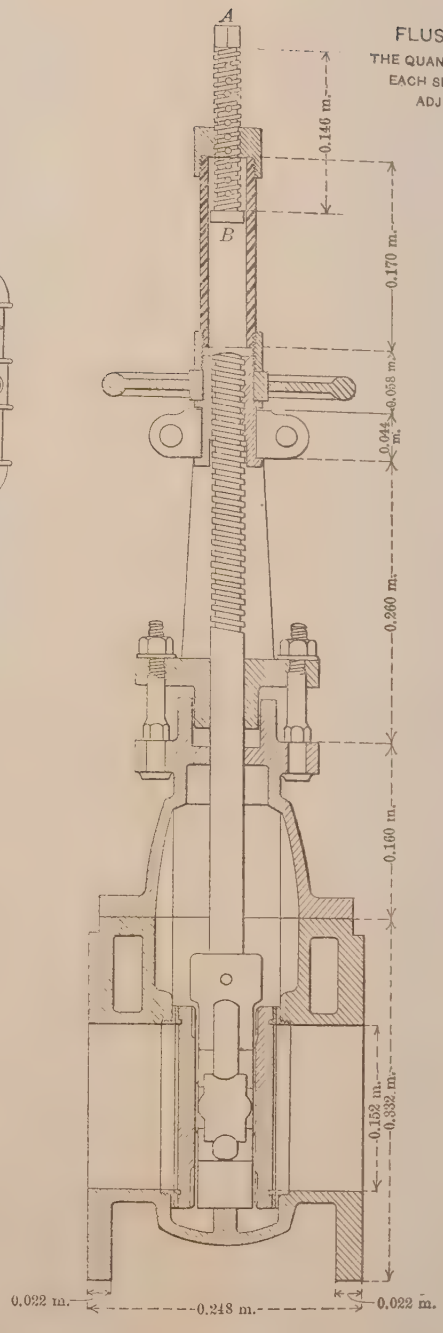
DIAGRAM SHOWING DETAILS OF THE PRINCIPAL CONDITIONS FOR ESTIMATING THE COMPARATIVE ADVANTAGES OF THE TWELVE TENDERS FOR MACHINERY

FIG. 2.

AUTOMATIC FLUSHING VALVE

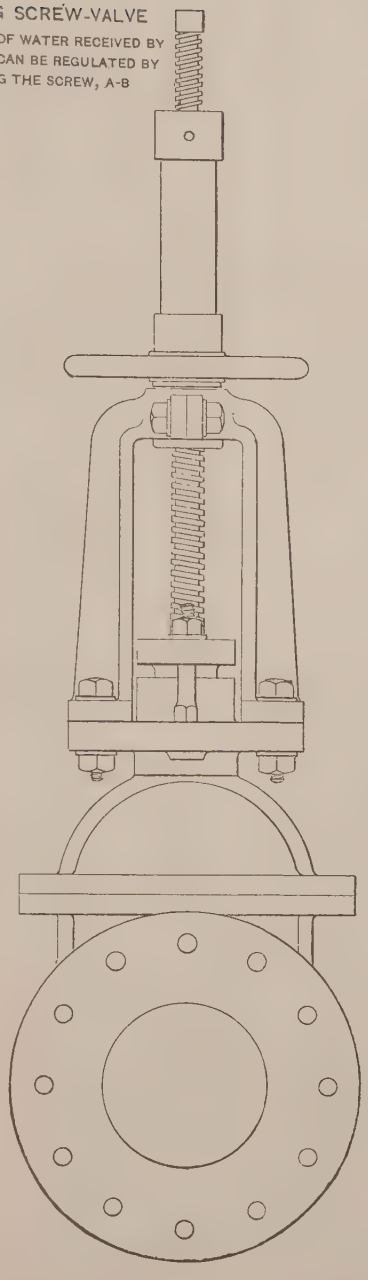
WORKED BY THE PRESSURE OF THE WATER IN THE MAINS



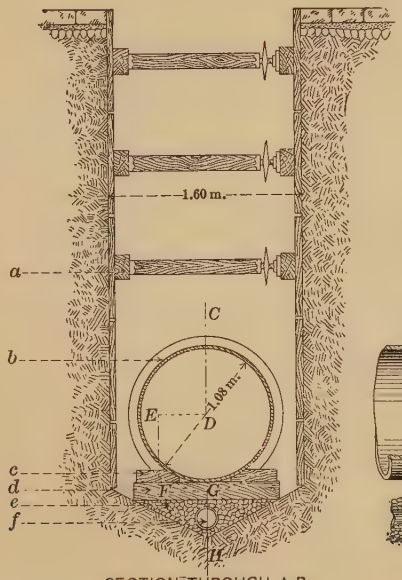


FLUSHING SCREW-VALVE

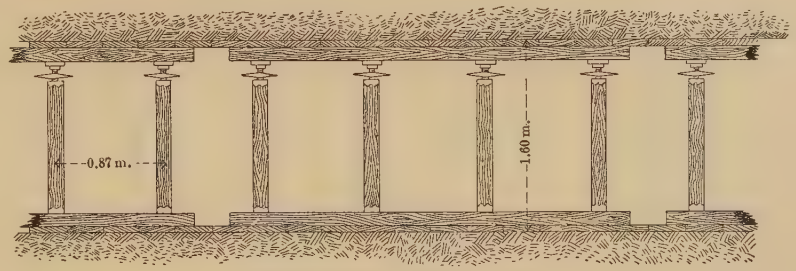
THE QUANTITY OF WATER RECEIVED BY EACH SEWER CAN BE REGULATED BY ADJUSTING THE SCREW, A-B



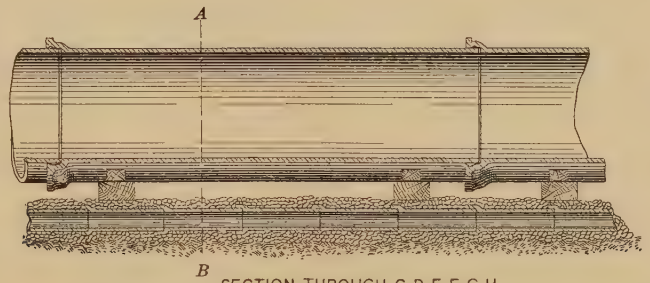
SYSTEM OF LAYING THE CAST-IRON FLUSHING PIPE.



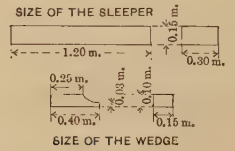
SECTION THROUGH A-B.



PLAN OF EXCAVATION AND LINING



SECTION THROUGH C-D-E-F-G-H.



a—Provisional lining.
b—Cast-iron pipe.
c—Cedar-wood wedge.
d—Cedar-wood sleeper.
e—Puzzolana pebbles.
f—Vitrified clay pipe.

FIG. 3.

Holly Manufacturing Company; but the line of the latter was the highest, and the pump cost $10 000 less, for which reason it was accepted, and the writer is entirely satisfied with its working.

Plate XL shows the general arrangement of the pump, a geological section of the ground on which the machinery was erected, and the nature of the foundations on which it rests.

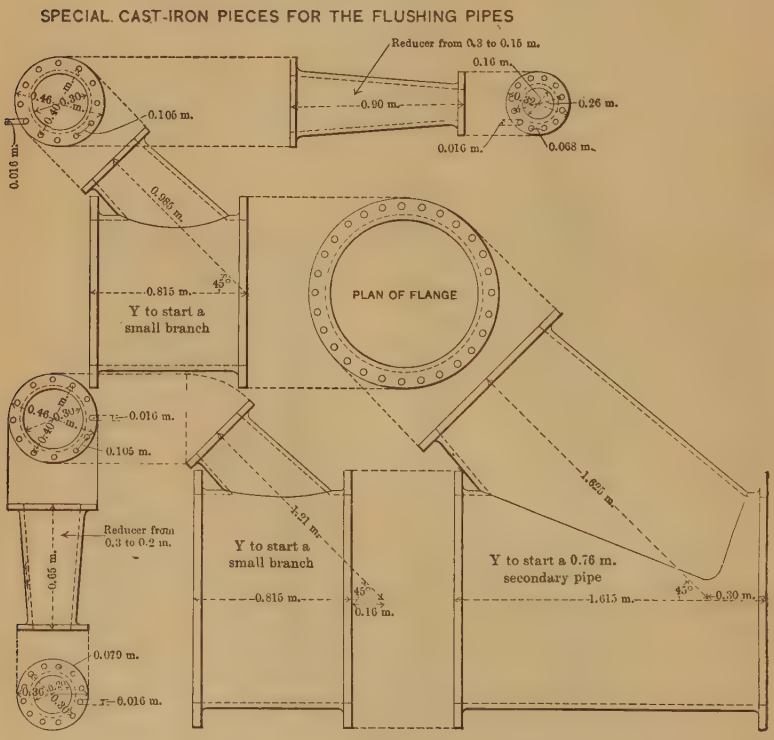
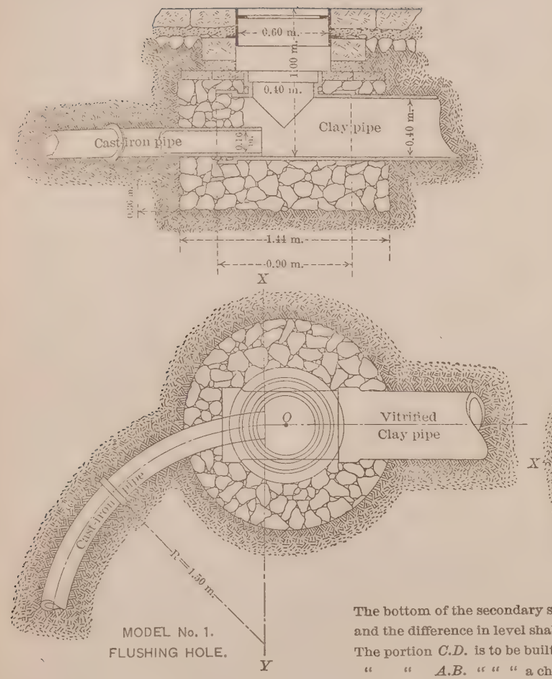


FIG. 4.

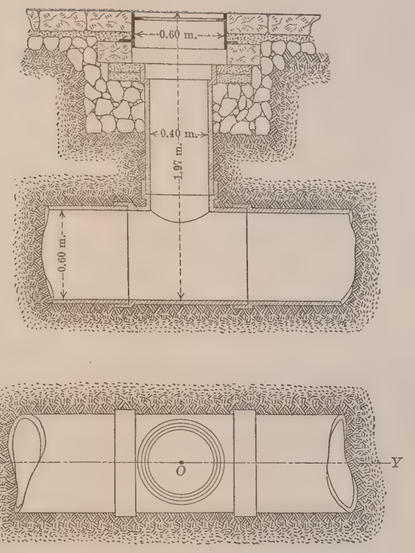
Fig. 3 shows the details of the manner of laying the distribution pipes. In every case it was necessary to provide a drain to collect the subsoil water, which is very abundant.

Plate XLI shows the details of the method of constructing the masonry around the 76-cm. valves in the flushing pipes, work which had to be carried out with special precautions, to prevent the subsoil water from penetrating the interior of the manholes, because,

ACCESSORY CONSTRUCTIONS OF LATERAL SEWERS.



MODEL No. 1.
FLUSHING HOLE.



MODEL No. 2. - LAMP HOLE.

The bottom of the secondary sewer shall be higher than the bottom of the principal one, and the difference in level shall be eight-tenths of the diameter of said principal.

The portion *C.D.* is to be built in concrete.

" " *A.B.* " " " a channel of vitrified pipe.

" " *E.F.* " " " a complete vitrified pipe, or, in some cases, a slant without socket.

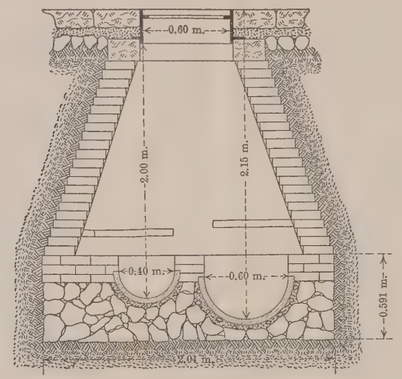
G.H. shows the position of cast-iron sluice-gate guides.

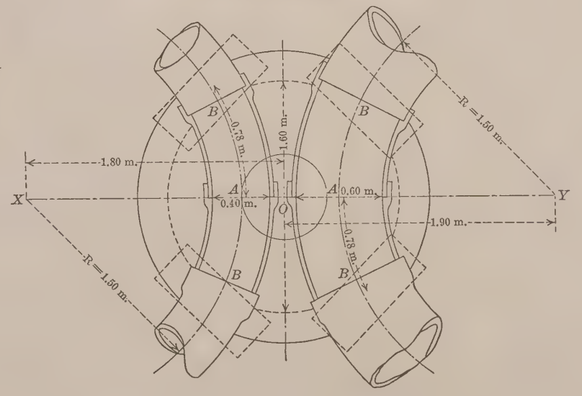
The portion *I.J.* shall be a piece of clay pipe without socket joint.

" " *L.K.* " " built either with a channel of clay pipe, or, in concrete.

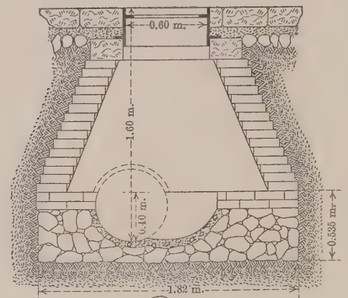
The depth, *RS*, of the channel shall be about eight-tenths of the diameter of the principal sewer.

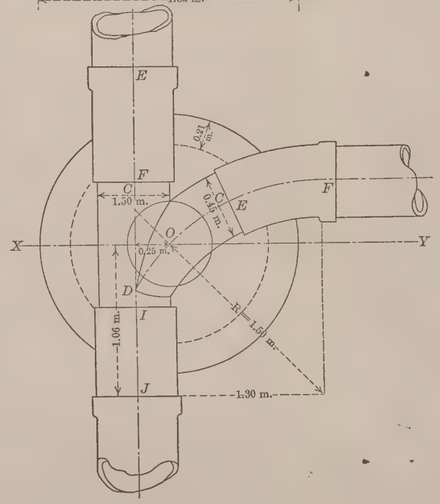
In order to fix on the ground the position of every manhole, the working plans of the sewer shall give the necessary data for running the line, ***XY***, fixing the center, *O*.



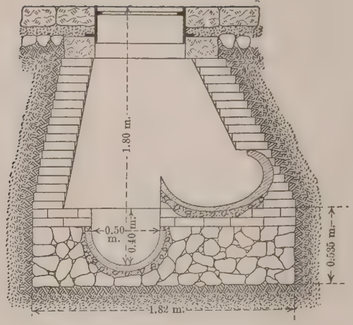


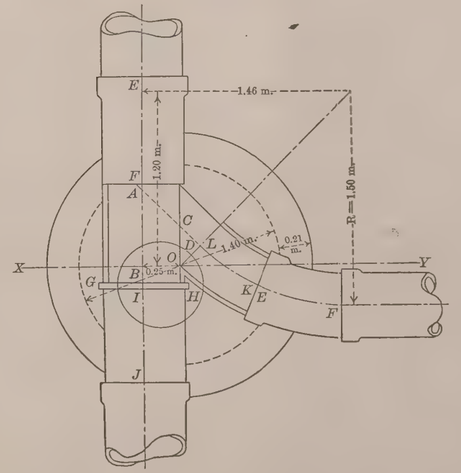
MODEL No. 3. - MANHOLE WHERE TWO SEWERS PASS.





MODEL No. 4. - MANHOLE WHERE TWO SEWERS CONNECT.





MODEL No. 5. - MANHOLE WHERE THE BRANCH SEWER FORMS AN ANGLE OF 90° WITH THE MAIN SEWER.

as can be seen from the drawing, the subsoil water stands at a much higher level than the bottom of the manhole.

In those manholes which are in bad ground there is a pressure around the bottom which has a tendency to raise the structure, and, for this reason, the lower masonry of these manholes is reinforced with steel beams.

Figs, 4, 5, 6 and 7 show the details of the special pieces of the flushing pipes and manholes which allow those pipes to be entered.

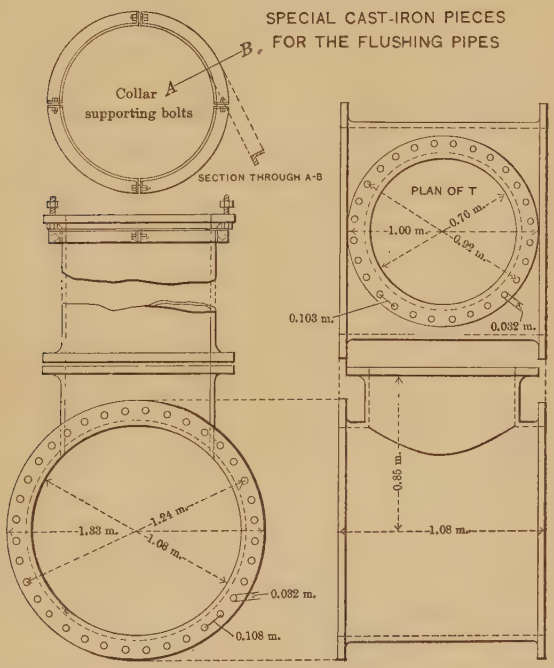


FIG. 5.

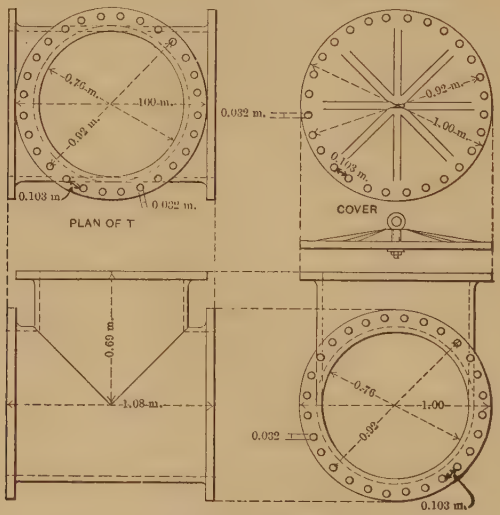
In order to simplify the work of construction, and of repairs which might be required, an effort was made to reduce to a minimum the number of models of special castings.

In studying the theoretical part of the flushing system, it appeared to be desirable to arrange it in such a manner that the valves would open automatically, and that the length of the stroke

of the piston should be limited by means of the screw, *A* (Plate XLII), with the object of having the volume of water required for flushing enter each sewer continuously, independent of the will of the operator who handled the valve. This valve is opened and closed by the pressure of the water circulating in the pipes. The screw, *A*, allows the volume of water flowing into each sewer to be increased or diminished to correspond with the diameter and length of the conduit.

In practice, it was found that the use of other and simpler valves, as shown by the drawing of the screw-valve on Plate XLII,

SPECIAL CAST-IRON PIECES FOR THE FLUSHING PIPES.



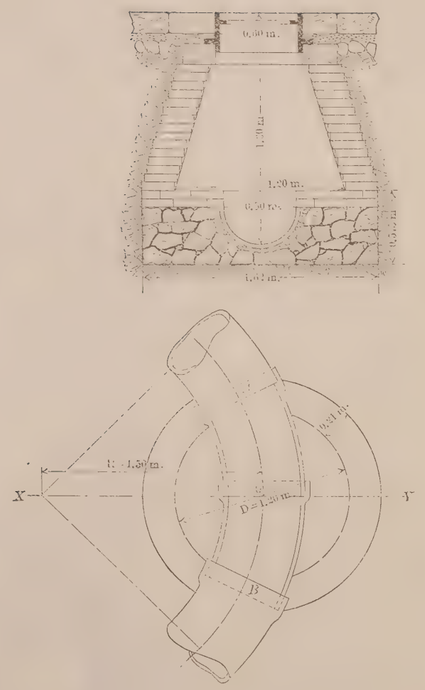
SIDE AND FRONT
ELEVATION OF MANHOLE T IN 0.76-M. PIPE

FIG. 6.

presented no difficulty, as in these the operation of opening and closing the valves is effected by a simple screw, *B*, while, at the same time, the screw, *A*, permits the control of the maximum quantity of water to be admitted into each conduit.

There are now 80 automatic valves and more than 120 screw-valves, and as the latter, on account of their simplicity, are subject to fewer accidents, and work perfectly, the automatic valves are gradually being converted into screw-valves.

ACCESSORY CONSTRUCTIONS OF LATERAL SEWERS.



MODEL No. 6.—MANHOLE WHERE A SINGLE SEWER PASSES.

The bottom of the secondary sewer shall be higher than the bottom of the principal one, and the difference in level shall be eight-tenths of the diameter of said principal.

The portion *C.D.* is to be built in concrete.

" " *A.B.* " " " a channel of vitrified pipe.

" " *E.F.* " " " a complete vitrified pipe, or, in some cases, a slant without socket.

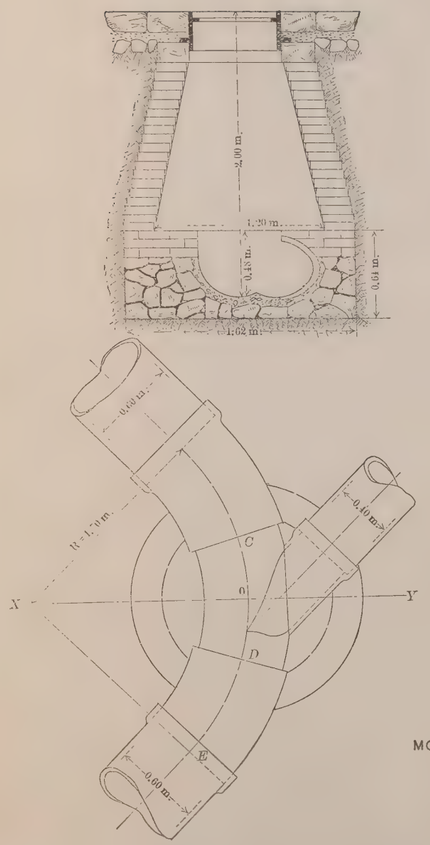
G.H. shows the position of cast-iron sluice-gate guides.

The portion *L.J.* shall be a piece of clay pipe without socket joint.

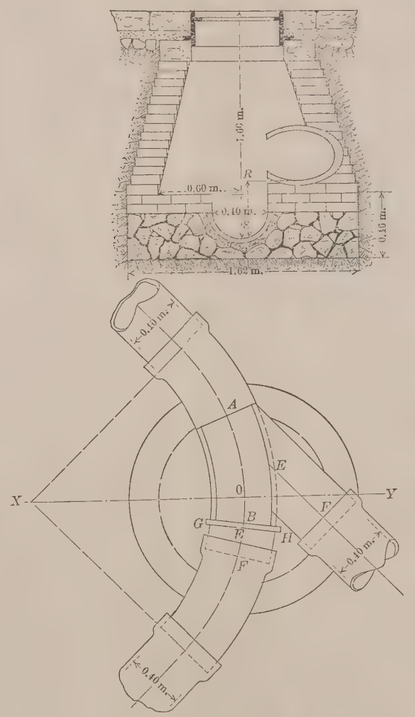
" " *L.K.* " " built either with a channel of clay pipe, or, in concrete.

The depth, *RS*, of the channel shall be about eight-tenths of the diameter of the principal sewer.

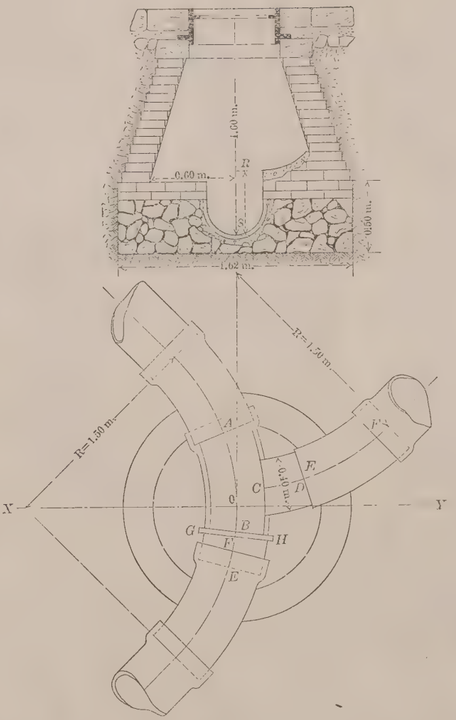
In order to fix on the ground the position of every manhole, the working plans of the sewer shall give the necessary data for running the line, *XY*, fixing the center, *O*.



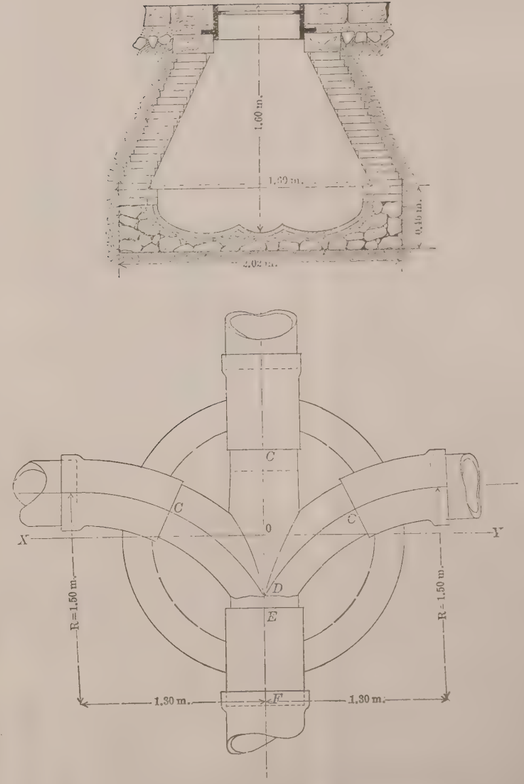
MODEL No. 7.—MANHOLE WHERE TWO SEWERS JOIN—THE PRINCIPAL ONE BEING ON A CURVE.



MODEL No. 8.—MANHOLE WHERE A NEW SEWER BEGINS: THE PRINCIPAL SEWER, BEING ON A CURVE, AND THE SECONDARY SEWER RUNNING IN THE SAME DIRECTION AS THE FIRST LINE OF THE PRINCIPAL ONE.



MODEL No. 9.—MANHOLE WHERE A NEW SEWER BEGINS, AND THE SECONDARY ONE RUNS IN AN OPPOSITE DIRECTION AS COMPARED WITH THE SECOND LINE OF THE PRINCIPAL SEWER.



MODEL No. 10.—MANHOLE WHERE THREE SEWERS MEET.

Working drawings were supplied to the contractors for laying the flushing pipes. In addition to other details, these drawings showed the exact position to be given to the special pieces from which the 15-cm. branch pipes were to start. The contractors were also provided with a working drawing for each street crossing for

WORKING DRAWINGS FOR THE CONSTRUCTION OF MASONRY AND CAST-IRON COVERS FOR MANHOLES OF THE FLUSHING PIPES.

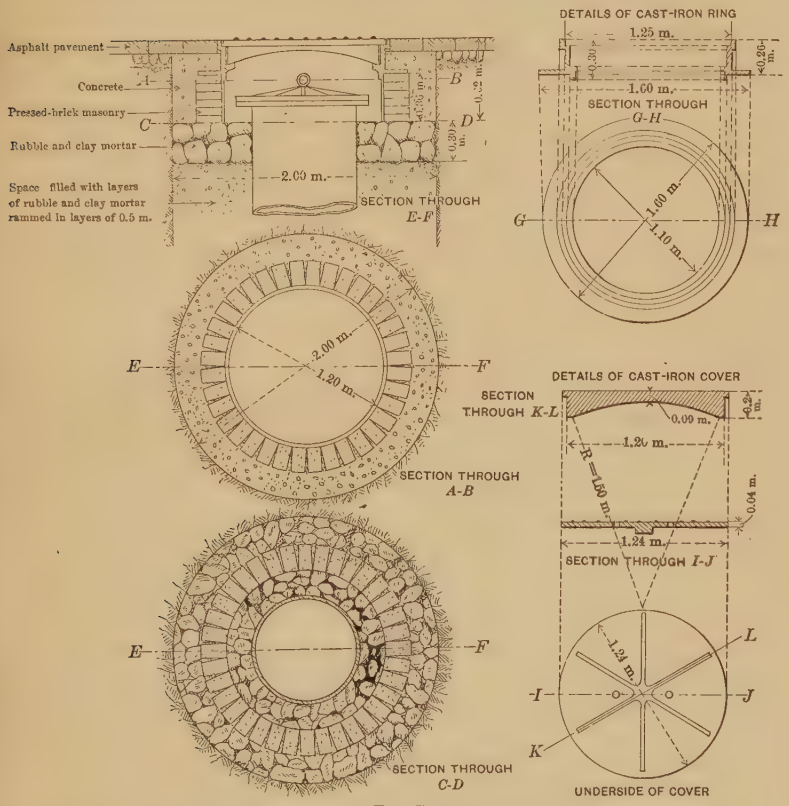


FIG. 7.

placing the intakes and flushing valves. All the flushing valves are placed in the sidewalks, and it was necessary in each case to study the proper situation of the Y of the main pipe, which is a special casting, in order that the 15-cm. pipe should always be laid at angles of 10, 15, 20°, etc., to the horizontal, so as to adapt curved

pieces at angles which varied from 5 to 6°, which greatly facilitated the construction.

Plates XLIII and XLIV show the details of the flushing hole and the principal varieties of manholes. It is not necessary to give any special description of each one, because each drawing shows for which case each model was constructed. It should be stated, however, that Model No. 3 was built in at all street crossings through which two sewers passed, in such a manner that the conduits could be inspected in four different directions from each manhole.

Models 8 and 9 were built at those points where a sewer started outside of the lines on which the flushing pipes were laid, and which, therefore, could not receive the flushing water directly from these pipes. In these models, the secondary or derived sewer has its bottom at a higher level than the principal sewer, so that when the latter receives water, it always flows through the principal sewer; but inside the manhole there are guides or vertical channels in which a sluice-gate is set for a few minutes to detain the flushing water, the level of which rises until it flows through the derived sewers and runs through their whole length.

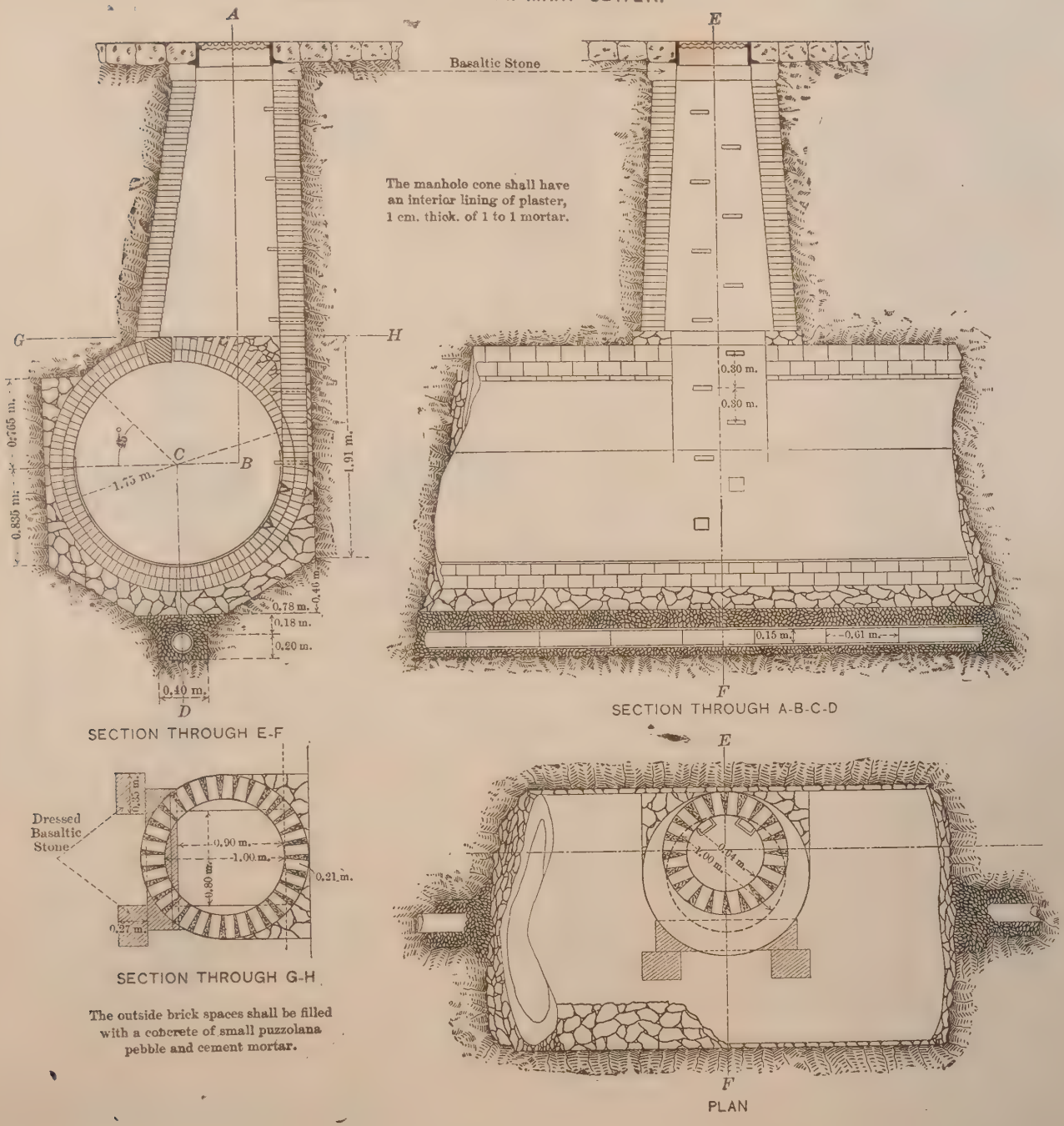
At every point where it was necessary to put in a manhole, the working drawing gave the number of the model governing the construction of that manhole, and, as the contractors were already in possession of the drawing for each model, with all the necessary data for building each one without any difficulty, it was only in exceptional cases that it became necessary to supply details for manholes in which the conditions differed very much from those of the others.

Plate XLV shows the details for a manhole on a 1.75-m. main sewer; Fig. 8 shows the method of putting in the catch-basins and connecting the rain-water pipes; and Fig. 9 shows a junction of two main sewers.

Plate XLVI and Fig. 10 show the details of the structure for connecting the main sewers with the Grand Drainage Canal of the Valley of Mexico. This canal is 30 miles in length, passes through a tunnel 6 miles long, and furnishes for the sewage an outlet beyond the mountains that surround the Valley. The sewage then passes down the slopes that run to the Gulf of Mexico.

The structure connecting the main sewers with the Grand

MANHOLE FOR 1.75 m. MAIN SEWER.



The outside brick spaces shall be filled with a concrete of small puzzolana pebble and cement mortar.

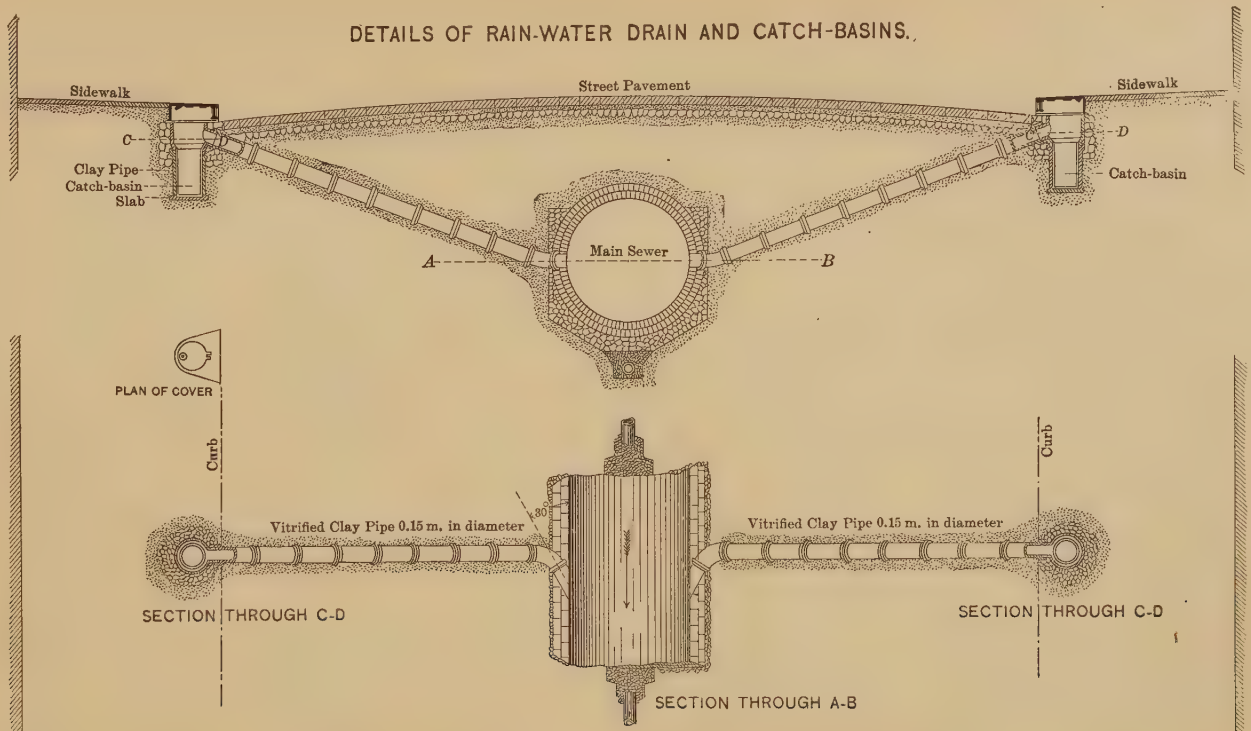

DETAILS OF RAIN-WATER DRAIN AND CATCH-BASINS.
Sidewalk
Street Pavement
Sidewalk
C
D
Clay Pipe
Catch-basin
Slab
Catch-basin
A
Main Sewer
B
PLAN OF COVER
Curb
Curb
Vitrified Clay Pipe 0.15 m. in diameter
30°
Vitrified Clay Pipe 0.15 m. in diameter
SECTION THROUGH C-D
SECTION THROUGH C-D
SECTION THROUGH A-B

FIG. 8.

Drainage Canal was built in very poor ground, and for this reason every effort was made to give the work the lightest weight possible. It was placed on a very solid foundation, capable of resisting the earth pressure at all those points where the structure was necessarily heavy. When the canal was opened, the ground at the bottom bulged upward in several places, because, being in a semi-fluid condition, it was forced up by the pressure of the ground which had not been excavated.

The level of the bottom of the main sewers is below the level of Lake Texcoco, and for this reason, foreseeing that by some accident, the waters of that lake might invade the canal, and with the object of preventing the damage that the city might suffer, sluice-gates were placed in the structure, so that in case of need they would prevent the passage of the water from the lake into the main sewers. In the event of its becoming necessary to close these sluice-gates, the main sewers could be discharged by means of the pumps erected in 1888.

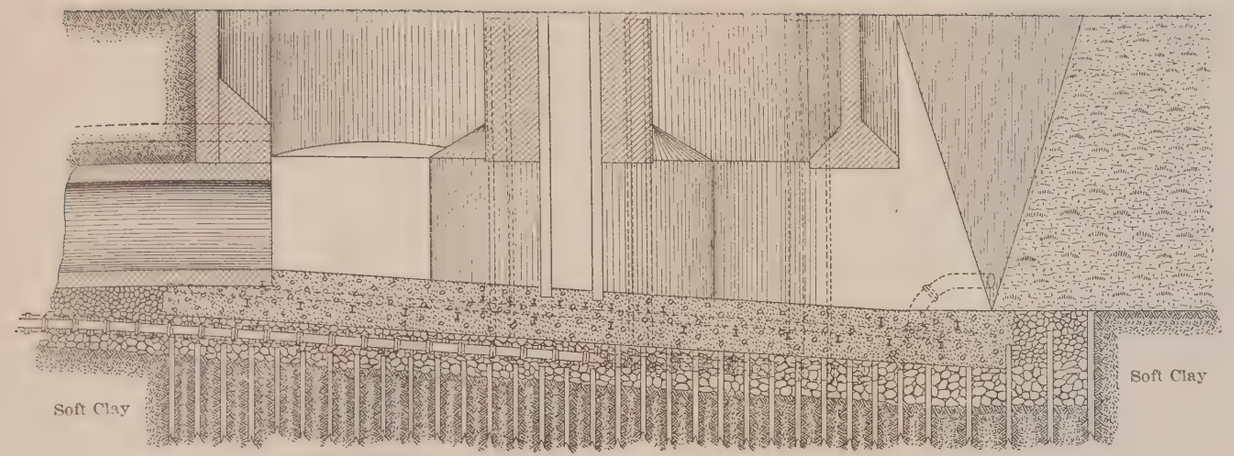
An examination of Plate XLVI and Fig. 10 will show the resources of which the writer took advantage to counteract the horizontal thrust of the soil. The walls were given a curved form, and the curves were combined in such a way that the pressure is finally transmitted through the wing walls to the solid ground. By this means, the vertical walls, which receive the thrust of the soil for a height of 18 ft., have a thickness of only 2 ft., up to a height of 10 ft., and of 1 ft. for the remaining height.

It is now five years since these works were completed, and, as yet, not the least sign of settlement has been observed.

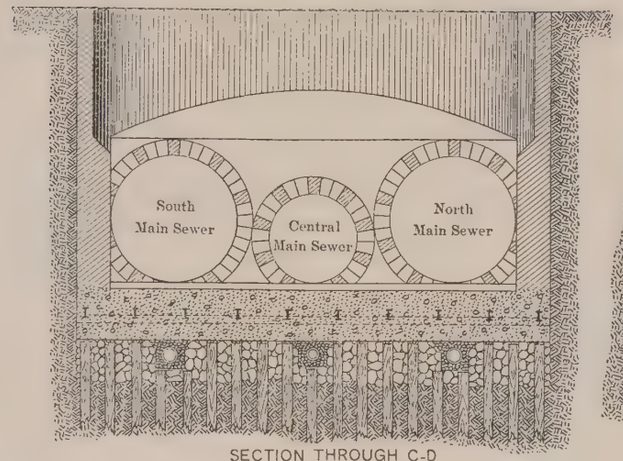
Details of the Methods of Flushing and Cleaning the Sewers.

The pumps and pipes which distribute the water for flushing the sewers are reserved exclusively for that purpose, and, for this reason, the pumps work only during the time required for that operation, or when it is necessary to clean the sewers with brushes in the manner described further on.

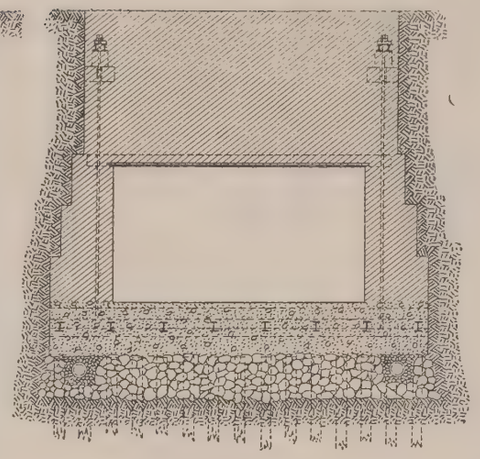
There are two small gangs of laborers. The first gang commences work at 7:20 A. M., opening nine valves in the distributing pipes numbered 2 and 4 so as to concentrate the whole volume of



SECTION THROUGH A-B

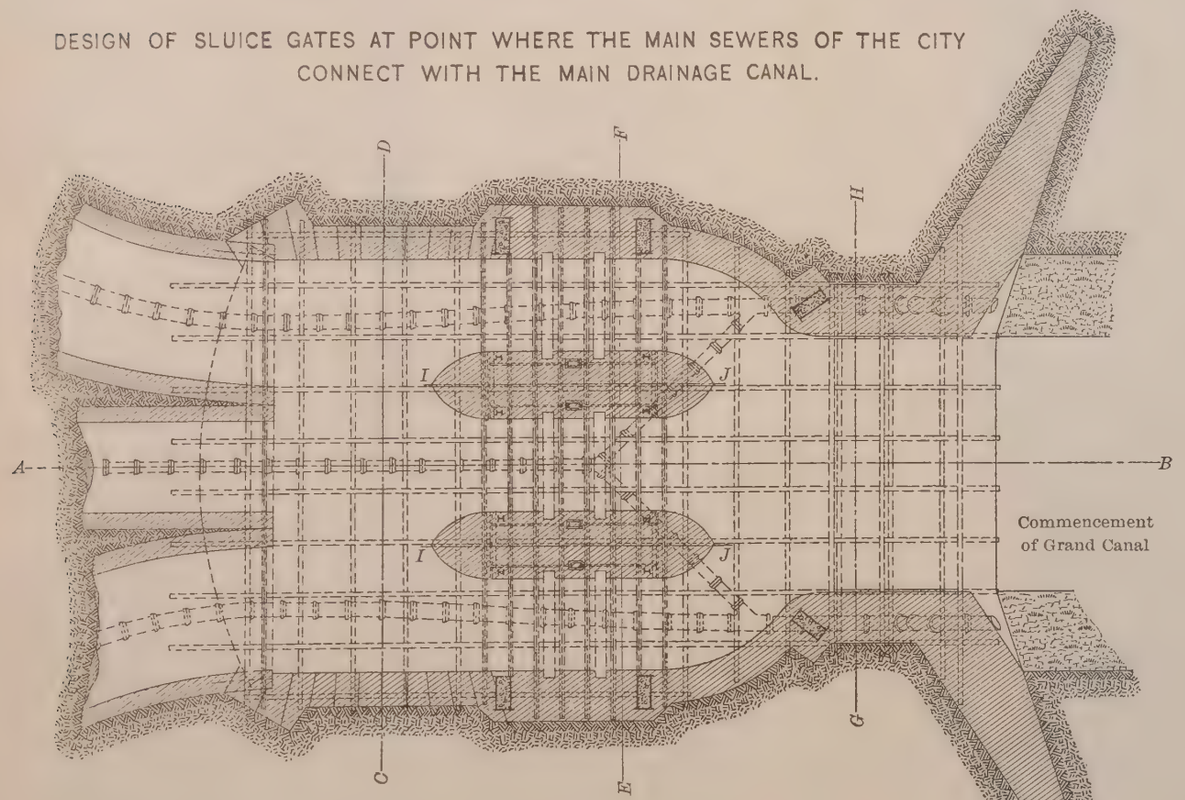


SECTION THROUGH C-D

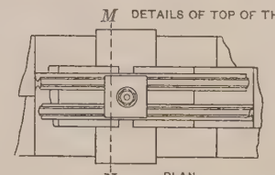


SECTION THROUGH G-H

DESIGN OF SLUICE GATES AT POINT WHERE THE MAIN SEWERS OF THE CITY CONNECT WITH THE MAIN DRAINAGE CANAL.

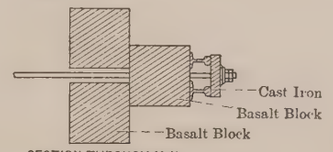


PLAN SHOWING DISTRIBUTION OF I-BEAMS AND RAILS.

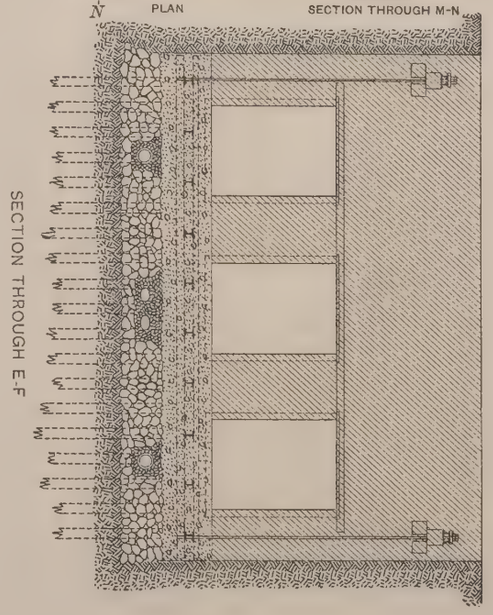


DETAILS OF TOP OF THE TIE-RODS

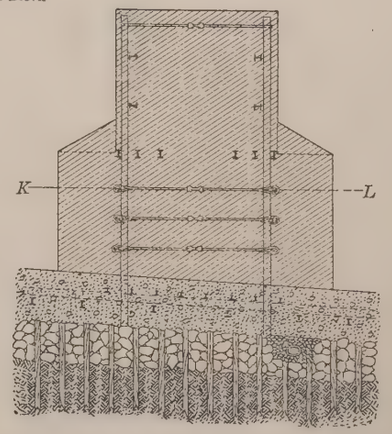
PLAN



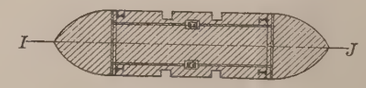
SECTION THROUGH M-N



SECTION THROUGH E-F



SECTION THROUGH I-J



SECTION THROUGH K-L

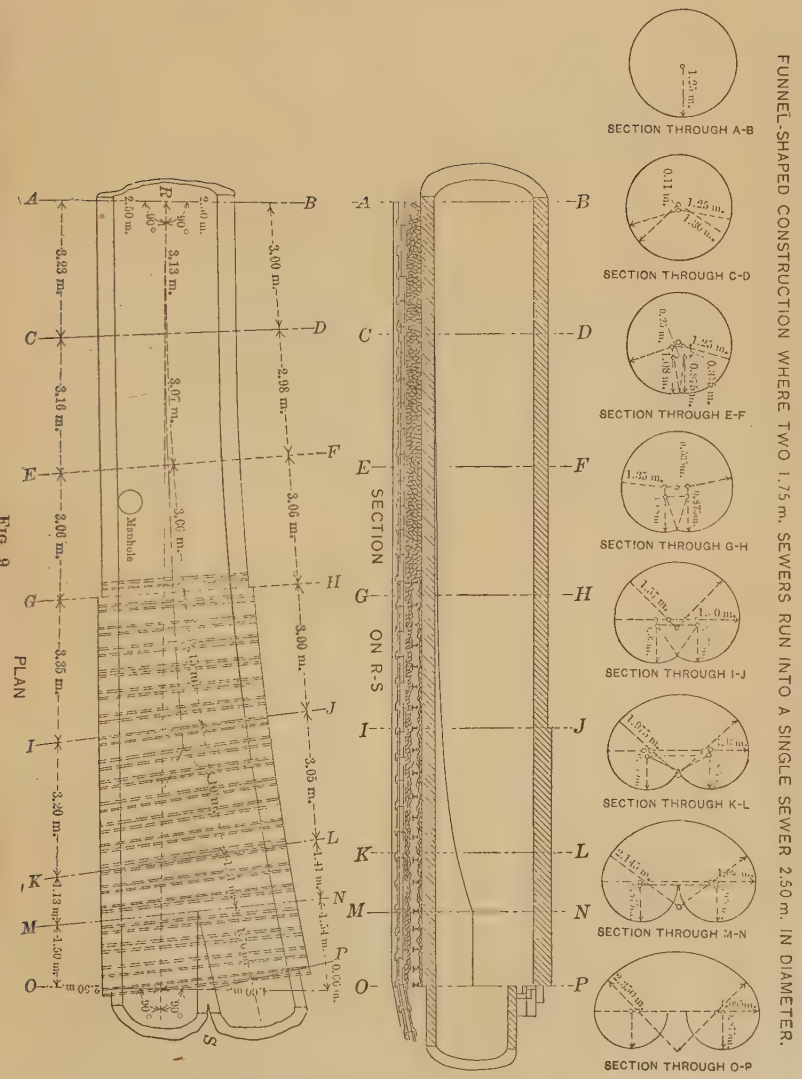

FUNNEL-SHAPED CONSTRUCTION WHERE TWO 1.75 m. SEWERS RUN INTO A SINGLE SEWER 2.50 m. IN DIAMETER.
PLAN
SECTION ON R-S
SECTION THROUGH A-B
SECTION THROUGH C-D
SECTION THROUGH E-F
SECTION THROUGH G-H
SECTION THROUGH I-J
SECTION THROUGH K-L
SECTION THROUGH M-N
SECTION THROUGH O-P
Manhole

Fig. 9.

water available, consisting of 1 cu. m. per sec., in the sewers which connect with Collector No. 2. Two men then advance, opening valves, one on the line followed by Tube No. 2 and the other along Tube No. 4, but they open only the valves corresponding to the sewers which discharge in Zone No. 2.

Each of these men takes with him a list showing the exact hour and minute at which each of the valves under his charge is to be opened, and, besides this, the hour and minute for opening the corresponding valve is painted in red on the lower face of the iron cover which protects the valve; the hour and minute at which the valve is to be closed is also painted in the same place, but in white.

A list has been prepared which shows in detail the hour at which each of the valves is to be opened and closed, and which has been fixed with reference to the length and diameter of each sewer. As a general rule, each valve remains open from 12 to 20 min., but the time is reduced to 4 min. and sometimes increased to more than 20 min., according to the length of the sewer.

In practice, it has been observed that the flushing water is discharged in a foul condition for only the first 20 or 30 sec.; but in order to render the flushing more efficacious, the stream of water is kept running for several minutes, undoubtedly much longer than is absolutely required for the cleaning of the conduits. The operation of flushing all the new sewers shown on Plate XXXVIII is accomplished in about 5 hours. This is done once every day, thus maintaining the sewers in good condition and completely free from all kinds of sediment.

Besides the flushing, the conduits are also cleaned with the apparatus and utensils shown in Plates XLVII and XLVIII. A heavy mop is passed through each sewer, and, acting as a brush, separates from the walls of the sewers all the grease and other substances which adhere to them and cannot be washed off by a current of water.

In order to carry out this operation, the valve through which the water enters the sewer is opened slightly and a float is introduced which is carried by the current from one manhole to another at the next street crossing. The float carries with it a hemp rope, $\frac{1}{2}$ in. in diameter, to which a wire rope of the same diameter is fastened.

As soon as these cables are fastened, the hemp rope is drawn

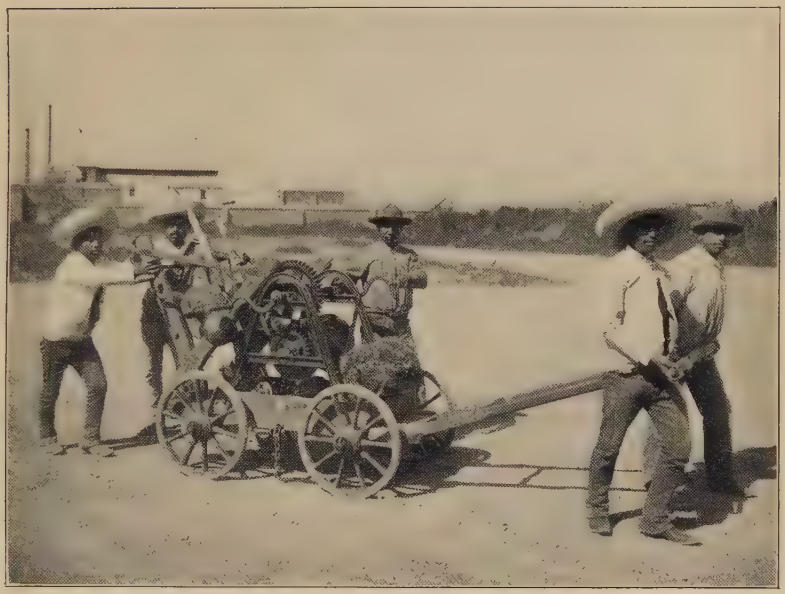


FIG. 1.—APPARATUS USED IN CLEANING SEWERS.

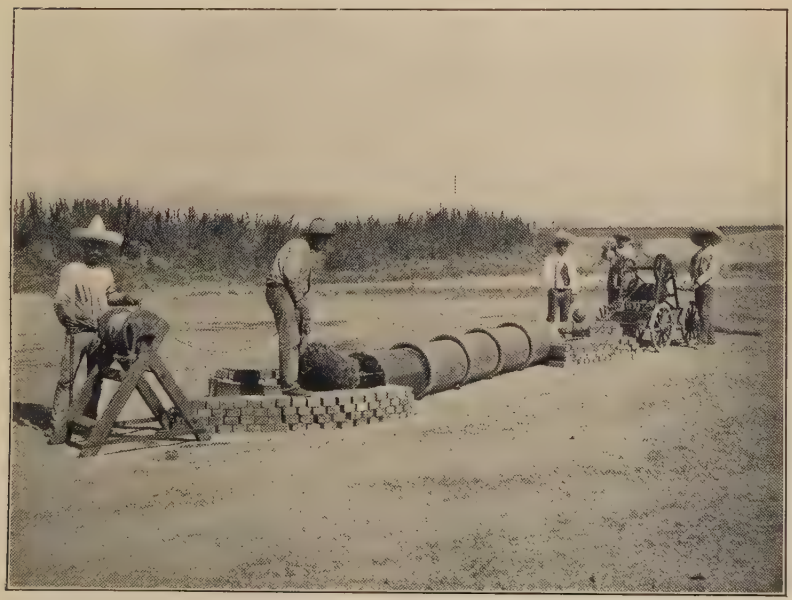


FIG. 2.—THE INTRODUCTION OF THE MOP FOR CLEANING A SEWER.

PLAN SHOWING THE SHAPE OF THE VERTICAL RETAINING WALLS.

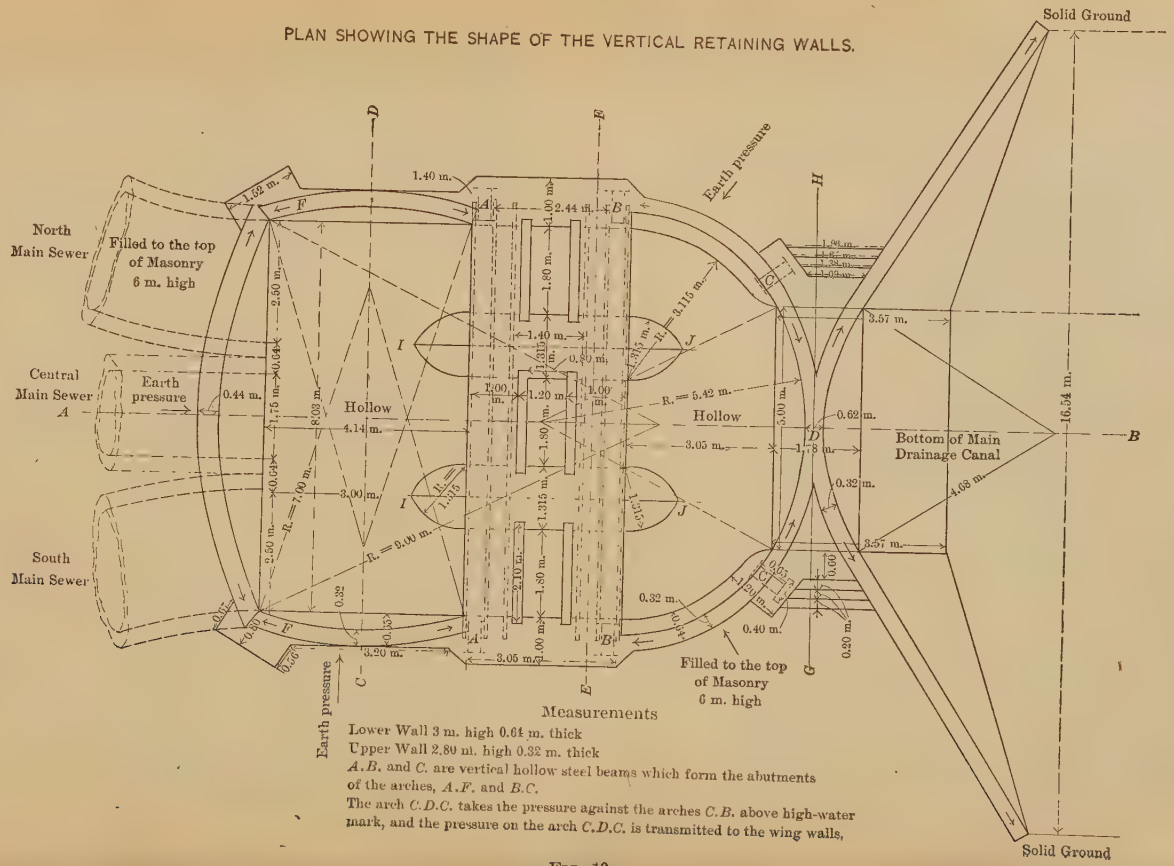

Solid Ground
North Main Sewer
Filled to the top of Masonry 6 m. high
Central Main Sewer A
Earth pressure
0.44 m.
Hollow
4.14 m.
3.00 m.
South Main Sewer
Earth pressure
Filled to the top of Masonry 6 m. high
Hollow
Bottom of Main Drainage Canal
Solid Ground
Measurements
Lower Wall 3 m. high 0.64 m. thick
Upper Wall 2.80 m. high 0.32 m. thick
A.B. and C. are vertical hollow steel beams which form the abutments of the arches, A.F. and B.C.
The arch C.D.C. takes the pressure against the arches C.B. above high-water mark, and the pressure on the arch C.D.C. is transmitted to the wing walls,

FIG. 10.

through the sewer by a winch placed over the second manhole. When the end of the steel rope reaches the manhole through which the float was introduced, the mop is fastened to the steel rope and drawn to the second manhole.

While the mop is passing through the sewer, a certain quantity of water is introduced which carries off all the substances loosened from the walls of the conduit by the mop.

This operation costs $82 per mile, but by it the sewers are kept perfectly clean.

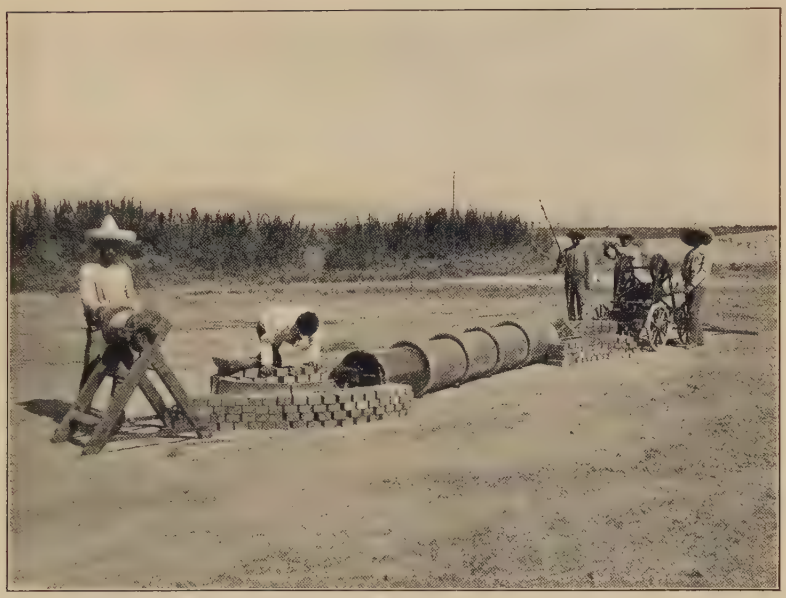


FIG. 1.—PASSING THE MOP THROUGH THE SEWER, BY MEANS OF A STEEL CABLE AND WINCH.

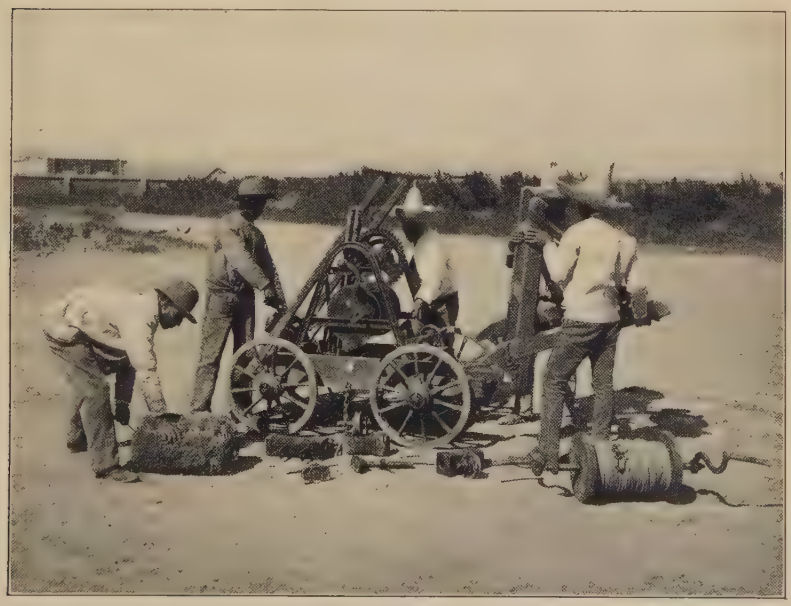


FIG. 2.—APPARATUS FOR CLEANING SEWERS, PREPARED FOR REMOVAL.

DISCUSSION.

Mr. Potter.

ALEXANDER POTTER, ASSOC. M. AM. SOC. C. E. (by letter).—Anyone acquainted with the City of Mexico before the installation of the sewerage system will appreciate the great work accomplished by Mr. Gayol and what this work means for the health and welfare of that city.

His paper describes a method of sewer flushing which is unique, but which, to the average engineer, may seem to be too expensive for common use. The question, therefore, which first suggests itself is: How far is the present excellent condition of the sewers of the City of Mexico dependent upon this apparently expensive but effective flushing system?

Will the author briefly outline the cost of this flushing system, and the cost of its maintenance, and also give a brief outline of other plans considered by him?

The writer would also like to ask the author if he has made any observations as to what extent the flushing water is augmented by infiltration, and how much the ground-water plane has been lowered since the system was put into general use.

In examining the open trenches in the City of Mexico, from time to time during the construction of the sewers there, the writer noted considerable water in the trenches. It would be of value to know the rate and amount of infiltration during the dry and rainy seasons, respectively.

The author states that, besides the flushing, the sewers are cleansed by hand at a cost of $82 per mile. Is this estimate in gold or Mexican currency? How often is it necessary to perform the hand-cleansing, and is this necessary on account of the slime from the sewage, or on account of a growth of sewer fungus on the interior walls of the sewers?

In some sewers running from two-tenths to three-tenths full, the theoretical velocities for these depths being from $2\frac{1}{2}$ to $2\frac{3}{4}$ ft. per sec., the writer has found that a year's growth of fungus is sufficient to cause a decrease of 20% in the velocity of flow found in such sewers when clean.

In having it possible to discharge fixed amounts of water into the upper ends of the sewers, Mr. Gayol has a splendid opportunity of comparing actual observations with results obtained from the commonly accepted, but more or less theoretical, formulas for sewer discharge, at various depths of flow, and it is hoped that he will write a paper on this subject.

Many observations conducted by the writer indicate that in pipe sewers, for depths of flow of less than one-half the diameter, the commonly accepted formulas give results which disagree with actual observations. For instance, observations on a 22-in. pipe, laid on a

Mr. Potter. grade of 0.22%, with a depth of flow of 0.44 ft., showed a velocity of 1.44 ft. per sec., as compared with a theoretical velocity of 2.38 ft. per sec., using $n = 0.013$. After cleaning this sewer a velocity of 1.79 ft. per sec. was secured when it was flowing at the same depth.

Observations on a 24-in. pipe, running 47½% full, gave a velocity of 3.476 ft. per sec., as compared with a theoretical velocity of 3.60 ft. per sec., an approximate agreement between the practical and theoretical determinations.

Mr. Gayol. ROBERTO GAYOL, M. AM. SOC. C. E. (by letter).—It is with great pleasure that the writer answers the questions and observations by Mr. Potter relative to the system for flushing the sewers of the City of Mexico. For the sake of clearness, his questions will be answered in the same order in which they are presented.

The writer is convinced that the present good condition of the sewers is entirely due to the flushing system, because, whenever the flushing is omitted, owing to some special circumstance, they begin to give out a bad smell within a few days, and as this is disagreeable to the public, it causes complaints. On the other hand, as long as the flushing system works with regularity, no bad smell is perceived, and the conduits are preserved in an excellent state of cleanliness.

Before the new sewers were constructed, one of the serious evils was the insufferable smell during the dry season, not only within the city, but even at a distance of 3 or 4 miles. In constructing the sewers, if special attention had not been given to the establishment of a simple system of flushing and cleaning, it is certain that, with the prevailing climatic conditions, combined with the slope and diameter of the sewers, bad odors would have been produced. If this had occurred, the public would have blamed the writer, on the ground that after having spent several millions on the works, these had proved to be a failure, because the great evil of the bad smell remained, and with this the bad sanitary condition of the sewers. Although it is true that no one now speaks about the sewers, saying that they are in first-class condition, it is also true that no one remembers that there are any sewers, which proves that these indispensable works are fulfilling their office without causing offense or injuring the residents.

The total cost of the flushing system as now constructed was as follows:

Pumping plant	$ 172 382
42-in. cast iron pipe, laid and ready for operation	380 645
30 " " " " " " " " "	1 115 329
6 " " " " valves, covers, special castings, masonry, etc.	114 820
Total	$1 783 176

Mr. Gayol.

This is in Mexican money, and, as the total cost of the works was $8 210 150, the flushing system represents 0.217 of the total cost. It should be noted, however, that the plant already built is capable of flushing a much larger system than that which has been constructed, as indicated in one of the plans accompanying the paper. The pumping plant, also, is capable of flushing a length of sewers four times as great as it serves now. At present only 5½ hours are required to flush the sewers already constructed, and therefore in 24 hours the plant could flush a length more than four times as great.

Bearing these facts in mind, the writer is of the opinion that when the entire system of sewers is developed the cost of the flushing system will be only 0.18 of the total.

The maintenance of the flushing system costs $1 242, Mexican currency, per month, and, with this amount, 128 km. of main and lateral sewers are flushed every working day.* The cleaning out of the main and lateral sewers costs $586 per month.

As regards the flushing systems studied by the writer before deciding to propose the one now established, the following observations must be made: The City of Mexico is on such a level site that the slopes of the main and lateral sewers are slight and artificial; the potable water supply that could be relied on only amounted to 30 gal. per capita per day; and, lastly, it was absolutely necessary to use the combined system, in order to take care of the storm waters, which are precipitated in heavy showers, but only from the end of May to the beginning of October, it being by no means a rare thing for 5 or 6 months to pass without a drop of rain falling.

Under these conditions, if the sewers were not supplied with an efficient flushing system, the corruption therein would be intolerable during the dry season.

Profoundly convinced of this fact, the writer made a careful study of the eight flushing systems used in Berne, Würzburg, Breslau, Zürich, Bremerhafen, Brighton, Danzig, Karlsruhe, Frankfort, Stuttgart, Dortmund, Budapest, Berlin, Memphis, and other cities, and found that some of these systems were inapplicable to the City of Mexico, while others would be entirely useless.

For this reason, the flushing system described in the paper was designed, an attempt being made to follow strictly the principles laid down by Lindley, and avoid all dead ends. In this design it was considered that, although the supply of drinking water was small, the system could dispose of a million gallons of water per hour, which, although not drinkable, was quite good enough for flushing the sewers.

The flushing water is not increased to an appreciable extent by

*It must be borne in mind that in the City of Mexico coal costs more than $22 per ton, and that the consumption is about 1 ton per day.

Hr. Gayol. infiltration. Before the present system of sewers was constructed, the soil was saturated with water, and the city literally stood over a swamp. A tank full of stagnant and putrid water was often found under the wooden pavements of the ground floors of dwellings, and in any excavation that was made foul water was sure to be found at a depth of 18 or 24 in.

The writer has always maintained, and has been able to demonstrate, that this water was caused by leakages from the old sewers, which were in a very bad condition. In fact, previous to the construction of the new sewers, and judging by his experience, he asserted that the sub-soil waters would be reduced as soon as the new sewers were built, and this has proved to be the case.

The level of the sub-soil waters fell as soon as excavations were made for the new sewers, especially the main sewers, which are the deepest. Two or three days after opening a trench the common wells for the adjacent houses became dry and did not again contain any water unless they were sunk to a greater depth.

The level to which the water has been lowered in different parts of the city varies, and is dependent on the depth of the nearest sewer. With only slight differences, it is always at the same level as the bottom of the nearest sewer.

With reference to the humidity of the soil, the city has improved in an extraordinary manner. In former times, the walls of buildings were covered in their lower parts with large stains caused by dampness, the plastering frequently falling off, owing to the effects of the saltpeter which crept upward. Whenever any room on a ground floor was left closed for a single day, the smell characteristic of confined humidity was perceived as soon as it was opened, all of which rendered the lower floors of dwellings unhealthy. All these difficulties have disappeared, and cellars are now built under the houses at depths at which water formerly rose from the sub-soil.

The cost of cleaning the sewers by hand, $82 per mile, is in Mexican money, all the figures given being based on the Mexican dollar as a unit.

The hand cleaning is done, not because it has been found indispensable, but as an additional means of preventing any bad smell in the sewers. It is for this reason that, with the small appropriation of $586 per month, mops are passed through each and every sewer in a systematic manner until every one has been cleaned. The hand cleaning, therefore, keeps the conduits in good condition, each sewer being cleaned in its proper turn (once in every year and a half), even though there may be indications that this cleaning might be omitted.

It has also been found desirable to clean the main sewers by hand at the close of the rainy season, because, at that time, in those parts

of the city in which the pavements are not good, they contain a certain quantity of coarse sand and gravel, but never any mud or fine sand. This sand, however, has never accumulated to such an extent as to form any obstruction to the current. Mr. Gayol.

No sand or even mud is ever found in the lateral sewers, but only stones, sticks or pieces of wood, which are thrown in by boys or fall accidentally through the gratings of the manholes or lamp-holes.

As already stated, the hand cleaning is considered desirable because it serves to wipe from the walls of the conduits the grease and particles of organic matter which have a tendency to adhere thereto; but wherever the cover of a manhole or lamp-hole is opened no deposit of mud is ever found, nor have any fungi or algæ ever been found.

The writer, having seen the sewers of the City of Mexico constructed in accordance with the plan which he proposed, and against which there was much opposition, has withdrawn into private life, and no longer has anything to do with the management of the works. These circumstances, as well as the writer's many occupations, prevent him from making the observations relating to the comparison of actual quantities with the results of theoretical flow formulas mentioned by Mr. Potter; but he is of the opinion that such observations would unquestionably be of great use, and could be easily made, because the system is especially adapted to such a purpose.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1009.

NOTES ON THE IMPROVEMENT OF RIVER AND HARBOR OUTLETS IN THE UNITED STATES.*

By D. A. Watt, M. Am. Soc. C. E.

With Discussion by Messrs. L. J. Le Conte, William W. Harts and D. A. Watt.

The experience available at the present time, from improvements of the mouths of rivers and harbors, is in no single country as extensive as in the United States. For more than 30 years, work of this nature has been carried on, embracing the coasts of the Atlantic and Pacific Oceans and of the Gulf of Mexico, and varying from rivers draining a small portion of one State to rivers like the Columbia or the Mississippi, and from harbors of local magnitude to those having an importance which has grown to be world-wide. The experience with these works has been very varied, and has made plain the fact that the operation of the laws governing the improvement of the outlets of rivers and tidal harbors is complex and very difficult of analysis.

In no department of hydraulic engineering does there appear to be a greater absence of precise knowledge, nor anywhere that a plan for improvement may be more likely to produce divergence of

* Presented at the meeting of September 20th, 1905.

opinion. Thus, with the improvements proposed some years ago, but never executed, for training works in the Estuary of the Mersey, engineers of wide experience advanced almost contrary views; and, for the improvement of the Estuary of the Seine, submitted to the best engineering talent of France, fourteen different projects were brought forth.

The forces at work are usually many and varied, and, while the effect of their action singly upon a plan might be foretold, their action in combination can only be approximated. There are, for example, the following: The transportation and deposit of the sediment present in most rivers; the effects of floods and tides; the presence or absence of currents along the coast; and the gradual effects of storms and of the drift of shore material, which, with small rivers, may change the outlet entirely, as with the Yare, on the East Coast of England, where the outlet has been driven south 4 miles in the course of time. Such causes, in some instances, produce daily changes of channel, as with the Hoogly, which ships can navigate only in daytime and by constantly taking soundings; in others, the movement is more gradual, as with the Estuaries of the Thames and the Seine; in others, again, the movement may occupy many years. Occasionally, other forces are also met, the causes and effects of which are equally obscure. Thus, at the Harbor of Madras, during the construction of the jetties, certain portions of them, in a depth of from 18 to 20 ft. of water, were frequently covered 3 ft. deep with sand during seasons of calm weather, and with no perceptible littoral current, nor any visible cause of disturbance except the ground-swell.*

Seeing the amount of material at hand, an attempt was made by the writer to investigate the conditions at several localities before and after improvement, with the object of ascertaining whether some general principles had not been manifested by the natural forces in the interval, or special experiences gained, which might serve as a guide in future work. During a connection with river engineering he has had frequent cause to study the effects of the laws of flow, and of their influence, as shown by Nature, on the formation of banks and channels, and the question naturally arose as to how far the same laws might be found to hold good when modified by the

* *Minutes of Proceedings*, Inst. C. E., Vol. C.

conditions usually existing at outlets. In investigating the matter he was fortunate enough to have access, among other publications, to a complete set of the Coast Survey charts of the United States, as well as to the data and maps of the Reports of the Chief of Engineers, U. S. Army, for the last 25 years. The principles suggested by this study, and the generally accepted theories of the subject, are summarized herein. They are submitted to the Society in the hope that they will be criticised by members of riper experience in this field.

1.—A body of flowing water, when preserving a constant or comparatively constant width and depth of channel, appears as moving in a curve. It is when the current crosses from one side to the other, or flows in a straight reach, that the channel becomes shallower or uncertain. To produce a permanently satisfactory channel, therefore, by training works, the latter should be located so as to permit the water to travel in a curve where such appears to be its tendency.

2.—In alluvial material there appears to be a fixed relation between the curvature and the best attainable conditions of channel. Where the natural curvature is increased, the channel becomes wider and of less depth; where it is decreased, the channel becomes deeper and narrower.

3.—At every outlet there usually exists, in some part of it, or has existed before improvement, a channel which seems to indicate the limiting dimensions, as regards depth, width and curvature, which the local flow is capable of producing, these dimensions being the final resultant of the velocity, the scour, and other local forces and conditions.

4.—The channel to the bar, commencing at some point within the outlet (or the main channel, where there is more than one), is usually found existing as a reversed curve the arcs of which are united by a tangent, or as a series of reversed curves and tangents; and the direction of its bar crossing is indicated, even in shifting outlets, with more or less constancy, being the resultant of the tides, winds, sand-drifts, etc. If the distance to the bar is short, the reversed curve is often of approximately equal radii. Exceptions to this rule are frequently due to the presence of hard shoals or bars,

or similar obstructions which the water has not been able to scour away, or to the temporary effects of storms.

5.—To secure the full success of the improvement, it seems to be essential that these natural resultant conditions of location and curvature, especially as regards the bar crossing, be interfered with as little as possible. Any attempt to divert or change them, when jetties are used, is likely to reduce the scouring power of the water, which will tend constantly to return to its original and natural location; and, if the diversion is marked, and the jetties are of much length, the power of the water may be dissipated and lost by the time it reaches the bar. If the curvature be reduced, with the object of producing a deeper channel, the latter may be forced against the jetty, and may become restricted in available width until ships are hampered in their movements. If the curvature be increased, it appears that there is a danger of the channel impinging on one or other of the jetties with similar results, in its attempt to preserve its original location.

6.—Where the works of improvement have been located so as to accord with and preserve the natural tendencies of curvature and location, and the jetties have been carried to deep water, the pushing of the bar seaward, the drift of sand along the coast, etc., appear to have produced only a small portion of the trouble usually apprehended. Conversely, where jetties have not been sufficiently extended, or the natural tendencies have been disturbed, these sources of trouble seem to have been more or less in evidence. As long as the waves can disturb the bottom, it is difficult for jetties to produce the removal of the sand brought upon the bar, and which is more likely to accumulate in such circumstances, because the jetties partly shelter it. The effects of waves can disturb rubble to a depth of 20 ft. and more below the surface, where the construction is such as to cause a considerable reaction; and, with ordinary reactions, a light material is no doubt disturbed to a greater depth. The practical value of jetties properly located appears to result from the protection of the flow of water from the disturbing effects of waves, coast currents, and sand-drift, until it has passed the bar and reached water of depth sufficient to neutralize the effects of these forces upon the channel, rather than from any actual training of the channel

itself. If the water is provided with suitable scope and protection, these facilities will usually permit it to work out, of its own accord, the best results obtainable at the outlet.

7.—Where two jetties are to be used, and construction will be rapid, they should be advanced together, so as to preserve the balance of the water in creating its new channel. Where one jetty is built in advance of the other, at localities where marked sand-drift exists, it usually results in deposits on one side or the other, which the water has to deal with later. To obtain the full scour, the jetties should be raised at least to ordinary high tide; results with half-tide or low-tide jetties seem to have been less satisfactory than when the same jetties have been raised later.

8.—Rapidity of construction has come to be recognized as a factor of success. Where the work has been prolonged for several years, the result has usually been that the seaward movement of the bar and foreshore has kept pace with the extension of the jetties, and the latter in consequence have had to be prolonged beyond the original needs of the case. In addition to this, the location of the channel to be improved may be altered entirely by the periodic change going on at almost every outlet, and this change may result in permanent injury to the waterway and greatly increased expense.

9.—If much sediment is carried, or where immediate results are desired, it may be advisable to assist the removal of the bar by dredging. If this is not done, under the former conditions, the sudden contraction of the outlet by the jetties may cause serious deposits of sediment above, such as occurred during the improvement of the South Pass of the Mississippi.

1 and 2.—The first two statements comprise principles well known to everyone who has had cause to study rivers from a practical standpoint, and need no comments. M. Fargue, at the International Congress of Navigation of 1892, was the first, it is believed, to place on record these and other laws in a concise summary of the effects of flow as he had observed them in his studies for the improvement of the Garonne.

3.—According to the third statement, the limit of improvement is often shown more or less clearly by existing natural conditions; and if a certain depth and width is desired for the improved channel,

that depth and width should appear as already existing approximately in the outlet, or to obtain it may require artificial means, such as dredging. In the majority of cases, except those of funnel-shaped estuaries, the charts were found to show the water as entering the sea in a well-defined channel or gorge, which continued toward the bar for a greater or less distance, depending on the slope of the ocean floor, before it shoaled and became uncertain. Where the floor is of steep slope, the waves and other outside forces bewilder the flow soon after it leaves the shore line, but where the floor is comparatively level, the flow seems to continue its course for some distance before it is overcome, and builds out a protection of sand-bars and banks in the shallow water, sometimes for long distances from the shore, in the same way as a delta-forming river builds out into the sea, and protects itself between the banks until the flow can retain its natural elements of location undisturbed. It is the elements of this channel at or near the shore which seem to indicate the limits of an improvement, since they are the resultant of the principal causes, namely, the flow of water and the conditions of the banks, both within and beyond the shore line, which direct that flow; and, unless these causes are altered, their resultant must remain unchanged. Jetties appear to act as protecting banks, and by their shelter they permit the flow of water to regain and establish permanently its natural tendencies.

It was observed that at several unimproved minor entrances, especially on the Gulf Coast, where the normal tide range is about 13 in., good depths of water existed, but the curvature necessary to maintain them was so abrupt and the channel so narrow that only small vessels could navigate them. In such cases a curve of width adapted to an improved channel could only be produced by using a longer radius, with a resulting loss of depth, and danger of spoiling the entrance. Experience with improved channels, however, in rivers improved by regulation, as well as in estuaries or harbors, indicates that the natural curvature can be safely changed only within certain limits, because, as stated further on, flowing water displays a marked persistence in adhering to the location found in its natural condition.

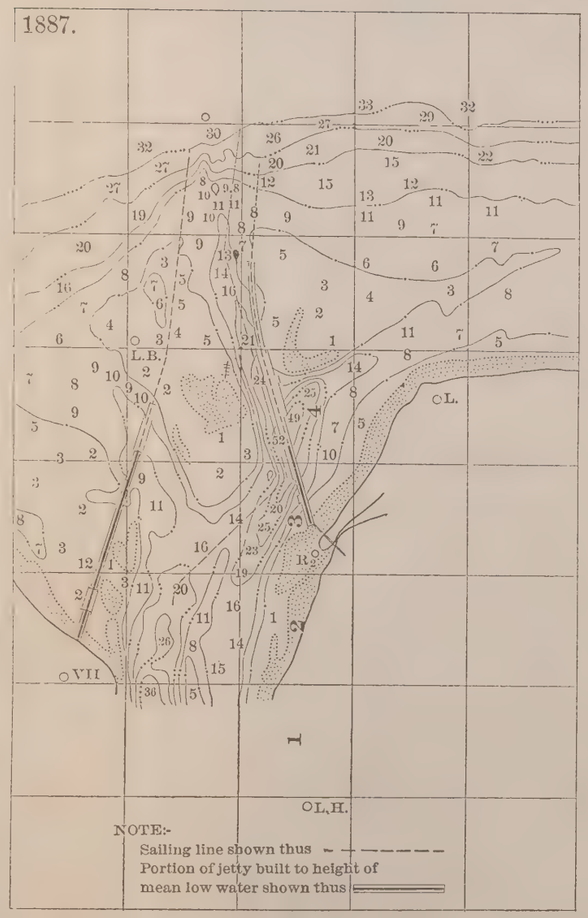
As would be expected, the outlets which carry a large volume of

water appear almost invariably as of large radii, while those of small volume are of small radii, there being apparently an intimate relation between the volume and the curvature.

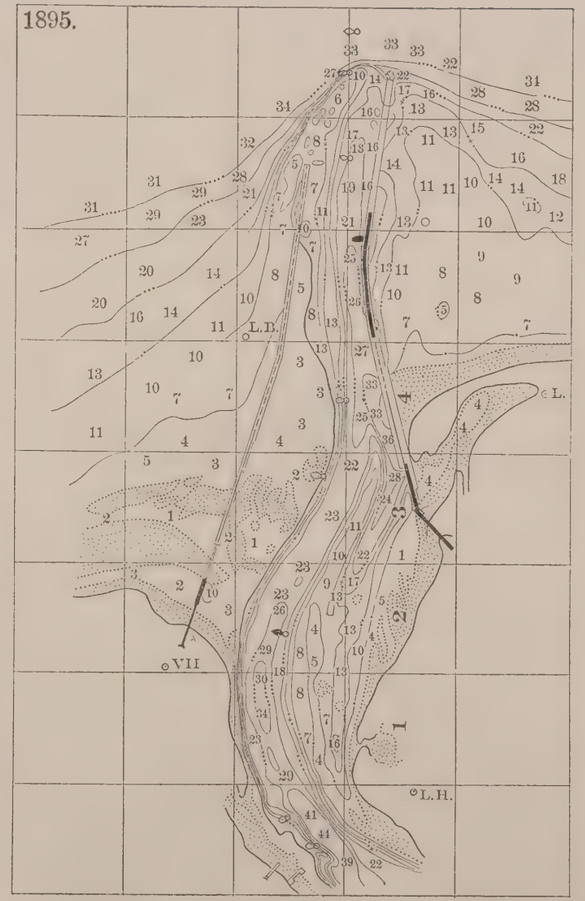
4.—The fourth statement describes natural conditions, which can be verified by an examination of coast charts or similar data. The general resultant tendency of flow toward the bar, in the great majority of instances, appears to be about at right angles to the neighboring coast line, or along the shortest path to deep water, this being ordinarily the line of least resistance and the one which the force of the water would therefore naturally take. The channels on leaving the shore maintain their respective widths and depths with considerable regularity for some distance toward the bar, this distance being generally in proportion to the distance of the bar off shore. Another feature is that, at nearly all the larger outlets, both on the coasts of the two Oceans and of the Gulf, there are, or were, two channels to the bar (in addition to the small side or "swash" channels) which deepen and shoal alternately during a cycle of change, and shift their location within a sector of a circle covering an angle usually from 45 to 90°, but never exceeding fixed limits. On the South Atlantic Coast, for example, the channels swing generally between east and south, one or two cases being in evidence where the main channel, as at Charleston Harbor, ran parallel to and near the shore in a southerly direction. Uusually, however, the channel leaves the shore at once. On the Pacific Coast the variation, as a rule, appears to lie between southwest and northwest. The cycle of change seems to take place as follows: Suppose the main channel to be the southerly one. After it has continued in existence for a few years, it will begin to shoal, possibly on account of a period of gales, possibly from the more obscure causes by which Nature works out her cycles. At the same time the northerly channel will begin to open, and the closing of the southerly one will continue until it has become valueless for shipping. Usually more or less change of location occurs at the same time, the channels sometimes wandering over a considerable portion of their field before the final shoaling or opening occurs. The northerly channel will then pass through a period as did the other, and may shift further north, gradually deteriorating until the natural forces close it, and the water breaks open again along its first direction to the south. As

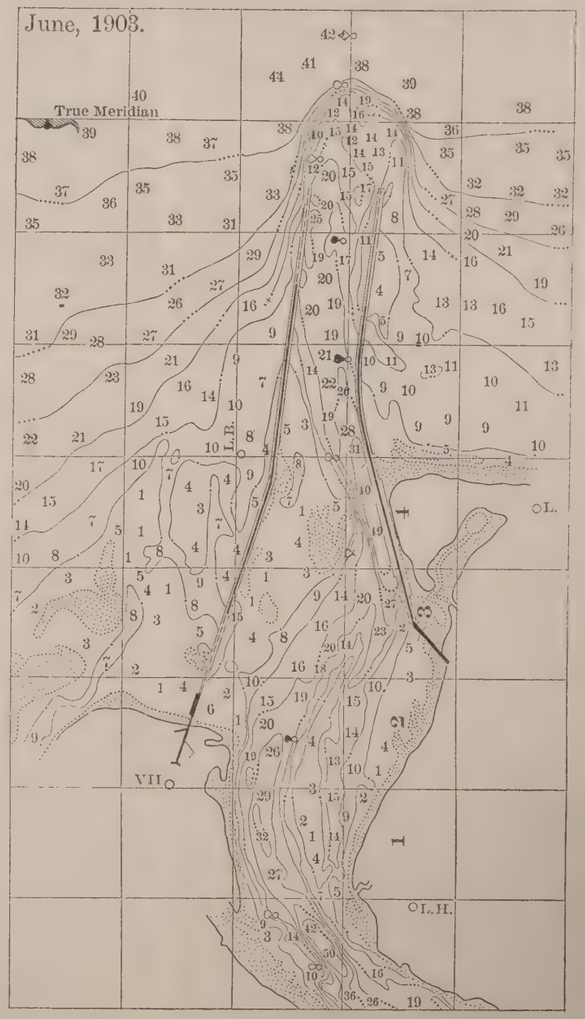
ENTRANCE TO ST. JOHNS RIVER, FLORIDA.

Scale of Feet
0 1000 2000 3000 4000 5000



NOTE:-
Sailing line shown thus
Portion of jetty built to height of mean low water shown thus





NOTE:-
Crest of Jetties below Mean Low Water
" " " at " " "
" " " between M.L.W. and M.H.W.
" " " at or above Mean High Water

far as known, the regularity of these cycles is a matter of doubt; at some outlets, a change such as just described has taken place in a few years' time; at others, the channels have remained almost stationary for a long period. The question could probably be decided only by a very protracted series of observations.

The "middle ground" lying between these channels, and which in closed estuaries is found inside the outlet as well as in the open sea, seems to be the natural result of a sediment-bearing flow meeting still water. The greatest amount of heavy sediment, being carried along the center or deepest portion of the water, is deposited gradually as the flow slackens, and begins to form a bank along the middle of the channel, which builds up and forces the side flow, carrying lighter sediment, to one side, until the water is finally divided into two channels with the more or less stable middle ground between.

5.—The contention of the fifth statement is that the natural conditions as then existing should control the location of the jetties, or of the dredging, where such is to be done, under adherence to the fact that the location and dimensions of the channel are the effects of those natural conditions, and cannot be altered, except within very narrow limits, unless the causes themselves are altered. Experience with the regulation of rivers, as before mentioned, has shown that the more closely the tendencies of flow and channel can be preserved the more successful will be the improvement, and that any forced change produces disturbances in direct proportion, and the same laws seem to be applicable to outlets. It was observed that where, in improving outlets, attempts had been made to divert the flow, as, for example, with the intention of scouring out a portion of the bar not crossed naturally by the channel, or in order to make the channel more direct than shown in its natural location, there was in the completed work an evidence of a disturbed balance of forces, the water striving to keep its former path, and the jetties resisting its efforts, and in extreme cases so breaking its strength that it wandered to the sea between the works in a bewildered and constantly shifting channel. The evidence with dredged channels seemed to be the same, except that the path of the water was less hampered, and it merely refilled the cut or such portions of it as did not accord with the direction of the flow.

As examples of the inherent tendency of the water to follow its

chosen path, the improvements of the St. Johns River, Florida, of the South Pass of the Mississippi, and of Galveston Harbor, Texas, may be cited.

The St. Johns River work belongs to that class where appropriations have always been moderate; and it has suffered in consequence much in the same way as did the work at Cumberland Sound, described further on. The outlet in 1879 presented the usual feature of two channels, one of which ran about northeast, the other about east-southeast. The former was the ship channel at that time, and, accordingly, was chosen for improvement when work was begun, about 1880. The general conditions are shown on Plate XLIX, and it will be noted that the map of 1879 shows the main flow of water striking the future line of the south jetty in the vicinity of the squares marked 3 and 4.* During the twenty-five succeeding years, in spite of the gradually increasing influence of the jetties, the flow has continued to impinge upon the same zone, along almost the same direction; and, even during short periods, when the maps show the channel attempting to divide and to open more fully its northerly arm, the southerly one still kept to its old location, and impinged on the jetty about at the original angle.

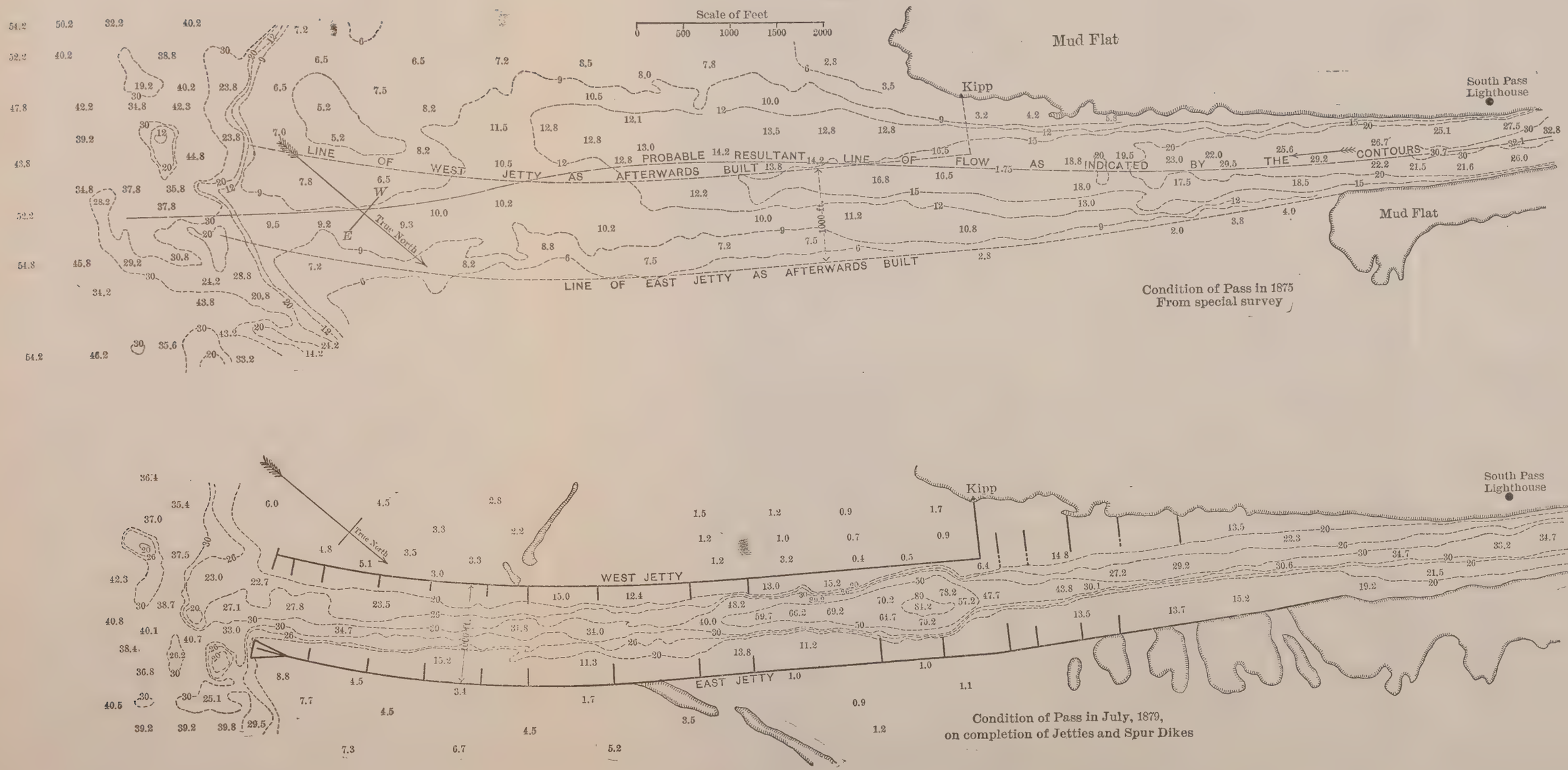
A somewhat similar case is to be seen in the improvement of the Estuary of the Seine, now being carried on. The new training dike on the south side makes an angle with the old dike below Berville in such a manner that it cuts into and partly deflects the natural location of the flow, and the latest map shows a deep reaction hole scoured out at the point of impact (with no doubt a corresponding loss of force), such as appears on the south jetty of the St. Johns.

In the case of the South Pass of the Mississippi, Plate L, the change in the natural location, which was slight, was made at the bar. The writer had been informed some years ago by officers of steamships plying to New Orleans that just in front of the jetty entrance there lay a mud bank which was a source of considerable trouble to shipping, especially when the river was in flood. It necessitated a sharp turn to a course about northwest or southeast, according as a ship might be entering or leaving, which a large vessel could not always make in time, and if caught by the current

* The maps from which Plate XLIX is compiled are taken from an unusually complete and valuable series made by the Engineer Department, U. S. Army, showing the changes annually (with a few exceptions) since 1879.

PLATE L.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1009.
WATT ON
RIVER AND HARBOR OUTLETS.

CHARTS OF THE SOUTH PASS, MISSISSIPPI RIVER.



when going out, she would be driven upon the bank, or, if coming in, would go ashore on the jetties. Of this he had a practical illustration when entering the Pass on a vessel a few months ago. His vessel was preceded by a steamship of some 3 000 tons, which made the turn safely past the bank and headed for the jetty entrance. The current, however, was strong, the river being in flood, and before the pilot could straighten up the ship in the narrow channel, the current began to swing her, or, as sailors say, she "took a sheer," and went hard aground on the east jetty.

While damage to shipping rarely results from such accidents, the expenses for tug hire and the loss of time have been a source of considerable complaint from ship-owners. On making inquiry as to whether this sharp curve in the channel, and the bank which existed in spite of its location opposite the discharge from the Pass, might not be indications of some change having been made in the natural position of the outlet, it was found from the description of the Engineer-in-Charge that such was the case. The tendency of the outlet is described as follows:*

"It is an observed fact that delta-forming rivers, when they enter the sea, have a tendency to direct their currents against the littoral currents of the sea, for the reason that the suspended sediment which they discharge is carried by the sea to the leeward bank of the river, and the accretions, which are made upon this bank from the river discharge, cause it to grow more rapidly than the other, and finally to curve the fluvial current around against the littoral current. * * * The prevailing littoral current (at the Pass) is from the east."

The writer of the foregoing continues to state that it was feared, however, that by forcing the current too far from its natural location the concave or east jetty might be undermined, and that the final location of the ends of the jetties was a compromise between the desire to make the river discharge at right angles to the littoral current, and the apprehension that by so doing too much scour might result along the east side.

The correctness of the views expressed in the quotation has been abundantly verified since the jetties were completed in 1879. The chart of that year† (Plate L) shows the bank beginning to grow on

* "The Mississippi Jetties," E. L. Corthell, M. Am. Soc. C. E., p. 73.

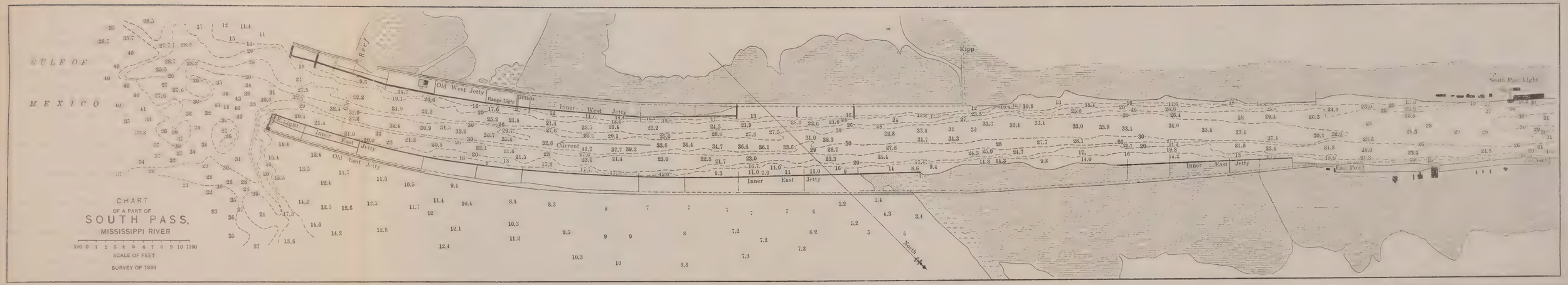
† The charts of 1875 and 1879 are reproduced by permission of E. L. Corthell, M. Am. Soc. C. E., from "The Mississippi Jetties"; those of later years are from surveys by the Corps of Engineers, U. S. Army.

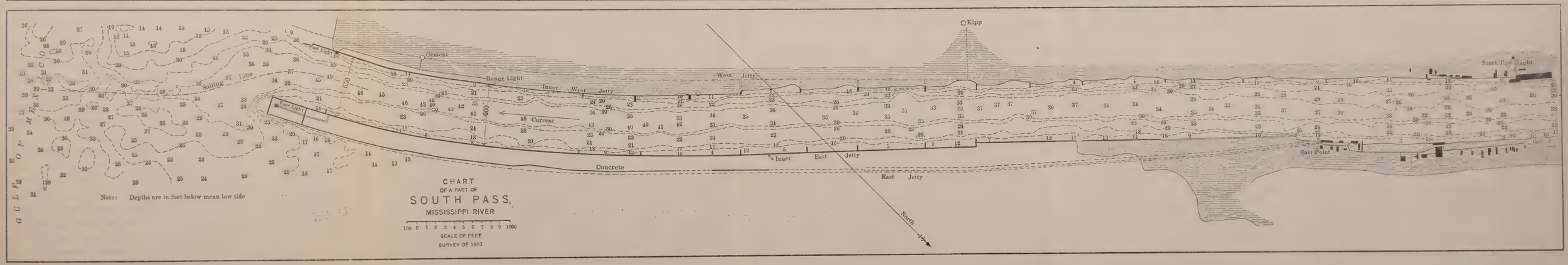
the west side, while the accretion shown there on the chart of 1889 (Plate LI), and its absence on the east side, is very noticeable. It is this shoal which is the cause of the trouble to shipping above referred to, and the direction of the discharge into the Gulf appears on the various annual charts of the outlet, published by the Chief of Engineers, U. S. Army, as adhering to the same course (about southeast) as it pursued in its natural state, and before any effort was made to deflect it.

Although this deflection was slight, the river has persisted in attempting to hold and to regain its former and natural direction, and this tendency and the axis of free discharge, appear to have remained constant for the past twenty-five years. The Pass approaches the sea in a series of very flat reversed curves, the last point of reversion having originally been near the northern ends of the jetties. The latter, trending to the south, transformed the final reversed curve into a simple curve which the river follows until it is freed from their influence at the mouth.

The tendency of the river to build up its west bank is a noticeable feature, as for many miles it is separated from the Gulf only by a narrow tongue of land on the east, while on the west the deposits have built up an area several miles across. The general direction in approaching the delta, however, is steadily southeast.

Another point in connection with the formation of the new channel seems worthy of note. In order to accelerate the scour at the upper end, over a portion of the bed somewhat compact, there were built on the east and west sides, during the fall and winter of 1876, some short spur-dikes above the deep basin shown on the chart of 1879. The main jetties, apparently, were then being built up, so that the stream was beginning to feel their restraint. During the floods of the following spring the deep basin shown on the chart just mentioned began to appear, scouring out to a depth of 95 ft., which, however, was reduced to 75 ft. as the river deposited its sediment during the summer and fall. In the floods of the next year (1878), however, the basin was scoured out again, reaching a depth of 114 feet. The conditions becoming dangerous, the spurs on the west side were cut back to the line of the jetty, alleviating the evil. The basin, however, still continued to exist at about the same


GULF OF MEXICO
Reef
Old West Jetty
Range Light
Pilots
Inner West Jetty
Kipp
South Pass Light
E. Light
Inner East Jetty
Old East Jetty
Current
Inner East Jetty
East Point
North
CHART
OF A PART OF
SOUTH PASS,
MISSISSIPPI RIVER
100 0 1 2 3 4 5 6 7 8 9 10 1100
SCALE OF FEET
SURVEY OF 1889


GULF OF MEXICO
Cement
Range Light
Inner West Jetty
West Jetty
Kipp
South Pass Light
Sailing Line
West Light
East Light
Current
Concrete
Inner East Jetty
East Jetty
East Point
North
Note: Depths are in feet below mean low tide
CHART
OF A PART OF
SOUTH PASS,
MISSISSIPPI RIVER
100 0 1 2 3 4 5 6 7 8 9 1000
SCALE OF FEET
SURVEY OF 1903

point, as shown on the charts for 1879 and the years following, up to 1882. That for 1883 shows the basin with a depth of about 74 ft., and moved up a short distance above the commencement of the west jetty. This change was probably due to the approach of the inner east jetty, which, in the interval, had been built up stream until it impinged on the lower end of the basin and forced it up the river. In 1884 the depth was about 57 ft., and the location the same as in 1883; by 1886 it had shoaled to about 43 ft., and the approach of the inner west jetty was causing the final break-up of the reaction. By 1888 the inner jetties had so narrowed the stream that it was forced at last into a straight flow, and the basin filled up to the general profile of the bed.

At the time of its appearance, this tremendous scour was attributed to the effects of the spur-dikes just constructed. Whatever the cause, the effects were precisely those which have appeared in similar cases where the flow of a river has been deflected more or less from its course, and where, there being no spur-dikes, the disturbance can be accounted for only on the ground of modification of the natural tendencies. Thus, it is to be noted that the basin was formed along just that portion where the west jetty crossed the natural channel, as can be seen by a reference to the chart of 1875, and where, in consequence, a basin ought to have appeared, as actually appeared on the St. Johns River, and on the Seine, before mentioned. The scour also was worse on the west, the former channel side, since it is stated that in 1877 it undermined the foundation mattresses of the west jetty to a depth below water of 30 ft., while no trouble appears to have been caused on the east.* When no steps are taken to break up such eddies, they appear to remain permanently—a mute protest of the river against the infringement of its natural laws. The writer is aware of one case where a training dike was prolonged so that it impinged on the natural flow at about the same angle as did the west jetty of the South Pass, and a survey made 22 years after its construction showed the usual basin scoured out and still existing at just that point where the flow had been trained from its natural direction.

A still more instructive example of the laws of flow is to be seen in Galveston Harbor, Texas. The improvement of this bay

* "The Mississippi Jetties," p. 151.

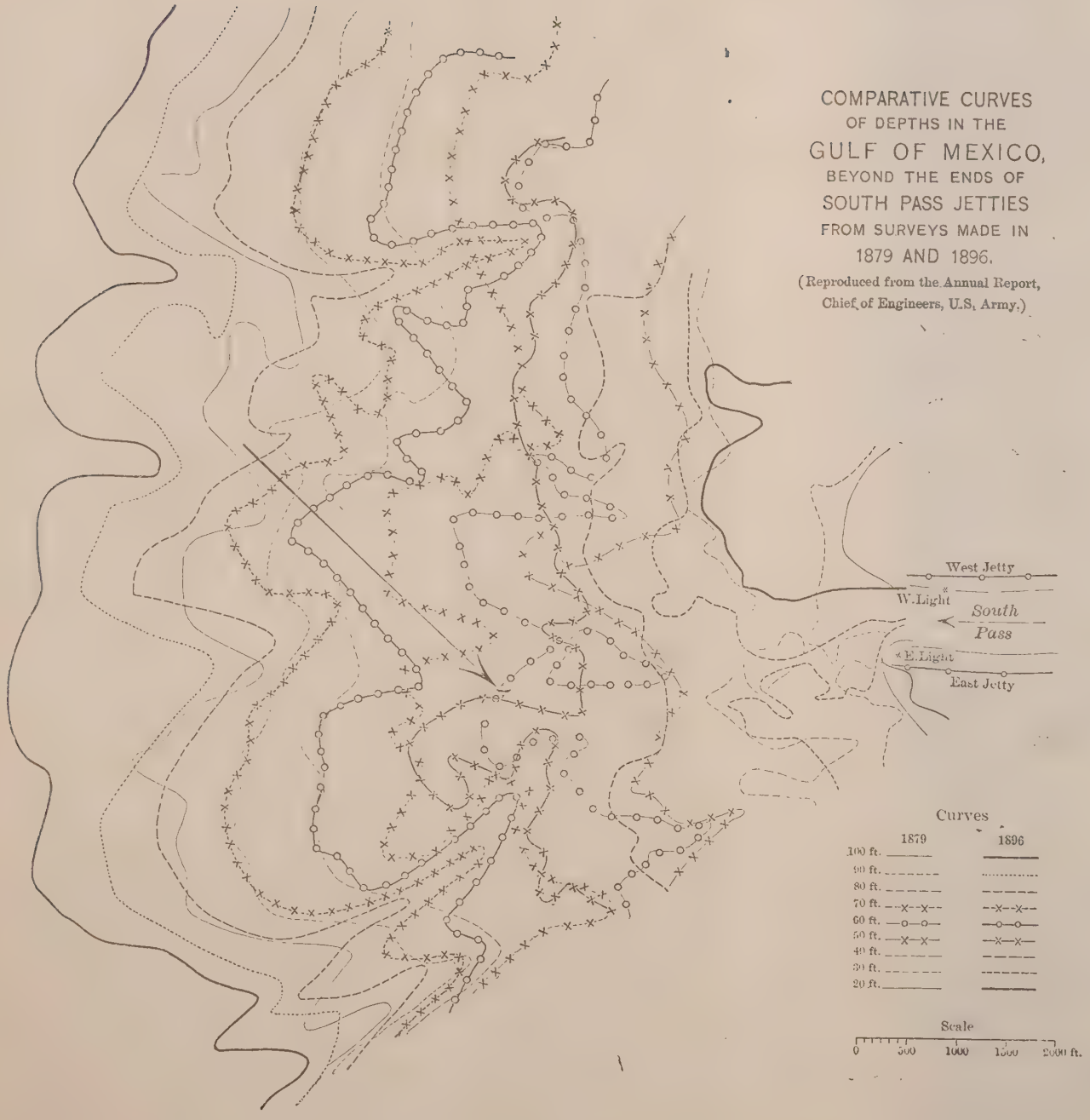
was dependent entirely on the tidal currents, which have a mean range of only 1.1 ft. As will be seen from Plate LIII, the main channel, when the final improvement was commenced, curved around the Bolivar Peninsula, reversed its course, and passed over the bar in a southeasterly direction. Before any improvement was begun, the channel to the bar, in accordance with the conditions found at nearly all large outlets, varied within a fixed quadrant from nearly east to nearly south, but never passing beyond those limits.

The first work was done in 1874, when a gabionnade, replaced later by a structure of brush and stone, was commenced on the location of the present south jetty. When completed, however, no betterment of channel resulted, except that the bar-crossing, being somewhat protected from storms, became much less shifting, and it was therefore decided to build a north jetty.

It will be seen from Plate LIII that during and after the completion of the jetties, approximately the same elements of flow were preserved and the same radii. As the works advanced, the outlet of the channel followed, curving in its course and opening to the southeast, and when they had been completed, it ran in a curve of long radius from the tangent between the points of reversion, impinged upon the north jetty, and swung to the southeast again, passing as close as possible to the end of the south jetty. The survey of 1902 indicates a radius for the gorge of about 13 000 ft., and an average radius, for the channel from the gorge to the bar, of about 13 500 ft. The latter, during its extension, appears to have reached a maximum of between 17 000 and 18 000 ft., which lessened as the water settled to a permanent course. The eastward extension of the bank on the north flank of the south jetty broke to some extent the regularity of curvature at the bar as the bank worked seaward, but, from present indications, it seems possible that the growth of this point has reached a stage where the currents will be able to hold it in check. This bank is supposed to be due to the scoured material being deposited in the eddy of the curve. Attempts to straighten the entrance by dredging parallel to and about midway between the ends of the jetties have proved fruitless, as the water always replaces the material removed and retains its curved channel.

COMPARATIVE CURVES
OF DEPTHS IN THE
GULF OF MEXICO,
BEYOND THE ENDS OF
SOUTH PASS JETTIES
FROM SURVEYS MADE IN
1879 AND 1896.

(Reproduced from the Annual Report, Chief of Engineers, U.S. Army.)



The success of this work, which increased the depth on the bar from 12 to 27 ft. in about 12 years, and made Galveston one of the leading ports of the United States, appears to have been due to the wide basin left between the jetties near their shore ends. This space allowed the current to continue its natural course, and gave it room enough to create its new channel without disturbing the balance of its forces. Had the jetties been built close together, so as to force the water in a straight line, there can be little doubt that the natural tendencies would have been gradually dissipated, and a shallow and unsatisfactory channel would have been the result. As before noted, in creating the new channel the current adhered approximately to the dimensions and curvature it before held in its natural state when approaching the bar, and, although the water was given unusual room in which to create new channels, it has slowly and surely continued to fill in all the spaces lying outside of its required course. The conditions here, as at the mouth of the Mississippi, are modified but little by sand-drift and littoral currents.

From the resulting conditions it would appear that in this case a geometrical solution of the problem, as to the course the new channel would take, would have been obtained by drawing in a southeasterly direction a line through the 30-ft. contour nearest to the original outlet, and joining this by a curve of radius equal to that of the channel to the bar before improvement, to an extension of the curve of the Bolivar Gorge, the two curves meeting on a common tangent, thus preserving the elements established by Nature.

6.—Experience seems to show that the question of the advance of the bar, which was formerly a much-discussed point, is not a serious factor where the works have been well designed and rapidly carried out. The pockets formed by the jetties on their outsides offer receptacles in which the sediment or sand-drift gathers, sometimes behind one jetty only, the other one showing erosion, sometimes behind both. With a good depth of water, also, the sediment has a wide field for deposit, and, as a consequence, the bottom builds up slowly.

This question gave rise to apprehension of serious trouble at the mouth of the South Pass of the Mississippi, where it was claimed that the jetties would have to be advanced into the Gulf every few

years. In point of fact, however, no extension has ever been made, nor is any contemplated. The vast amount of sediment brought down has been deposited behind them to the east and west, as the charts show, and the river has never failed to maintain a channel to the open sea (Plate LII).

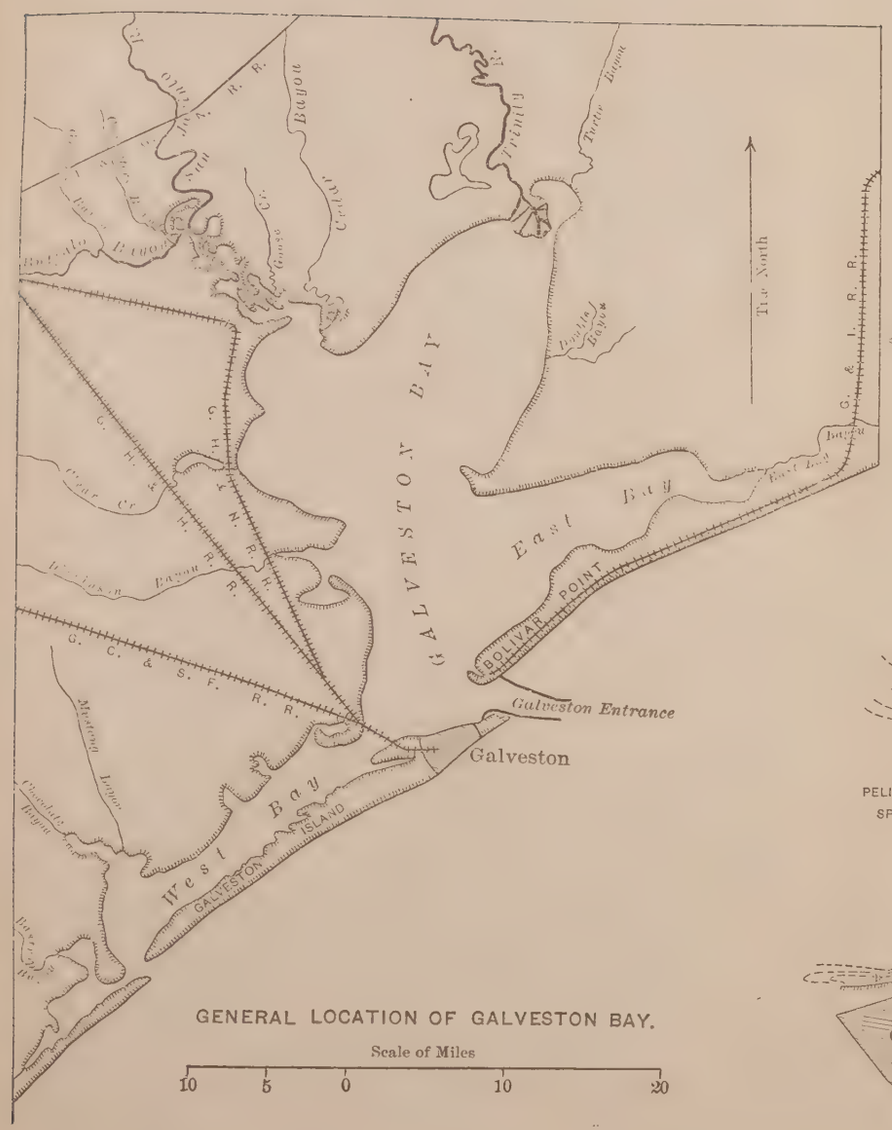
Mr. Vernon-Harcourt states that at the Harbors of Sunderland, Ymuiden, Dublin, and Port Said, the advance of the foreshore proved to be much less than was anticipated; and, at the Sulina Mouth of the Danube, which discharges a great quantity of sediment, Sir Charles Hartley stated that the rate of advance of the bar, according to observations taken for 10 years after the completion of the works, had been reduced by the improvement more than one-half.

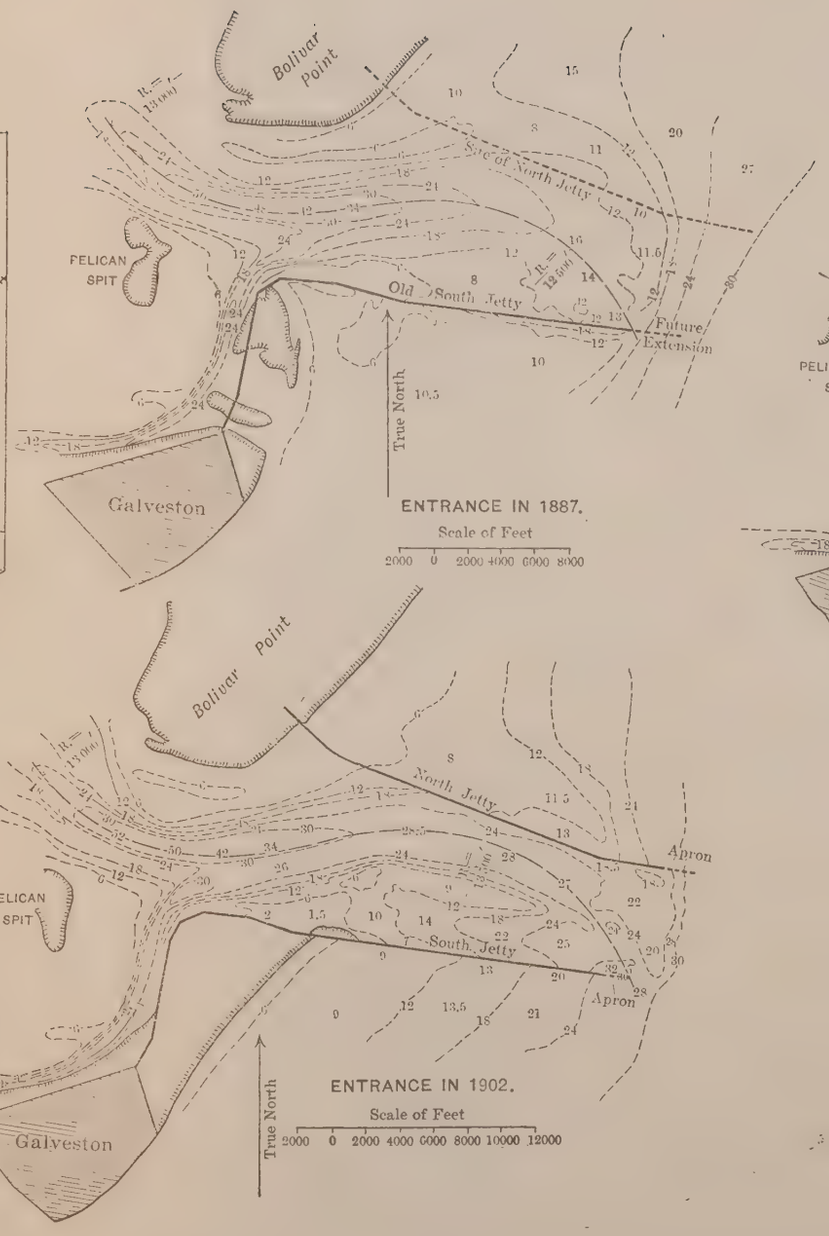
With the St. Johns River, and at other localities, the improvement of which was continually delayed by lack of funds, the eroded material and the sand-drift appear to have followed closely the gradual extension of the jetties. This feature is very noticeable on the maps of the St. Johns, and constant dredging is anticipated if a good depth of channel is to be maintained at that entrance.

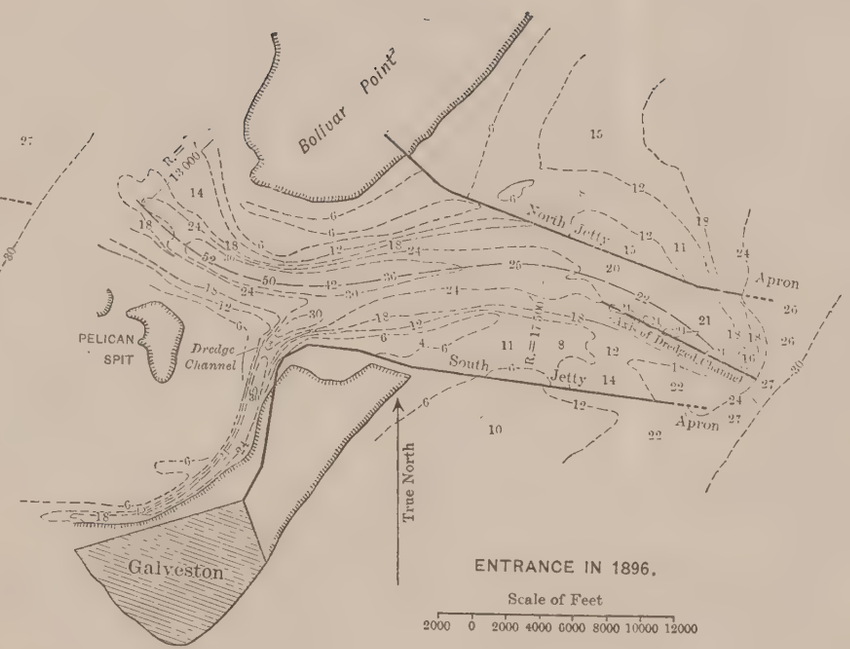
Where waves break in a considerable depth of water, or where the outside currents flow in a similar depth, the bottom appears to be slightly, if at all, disturbed, but where the depths are shallow, the waves and currents may bring into an entrance or a dredge-cut a constant supply of fresh material. Tests recently made at Cumberland Sound showed that coarse sand and shell, when stirred up by breakers, were carried to a considerable distance by light currents, and were not deposited until smooth water was reached. The same materials in quiet water lay undisturbed by currents flowing as rapidly as 4 ft. per second, although fine sand was moved by comparatively light currents.* Major W. M. Black, Corps of Engineers, in his paper before the International Engineering Congress of 1893,† quotes a case of a bank of fine sand which withstood a current of 3 ft. per second, but which was quickly eroded when the current was accompanied by a slight wave action.

With the Galveston Jetties, it was found that the bar was pushed seaward with the advance of the works, keeping pace with them as

* Annual Report, Chief of Engineers, U. S. Army, 1902, p. 2503.
† *Transactions*, Am. Soc. C. E., Vol. XXIX, p. 223.







COMPARATIVE SURVEYS
OF ENTRANCE TO
GALVESTON, TEXAS

long as the depth of water remained shallow, the material eroded by the current being replaced by the local forces of the waves, etc., which had caused the original formation. It was only when the jetties reached deep water that the scour began, and then the effect was rapid, and has remained permanent and self-sustaining.

7.—The advisability of advancing both jetties at once is based on the universal tendency of outlets exposed to sea influences to undergo a periodic alteration of channel, and, unless this can be forestalled, serious trouble may ensue, as instanced further on.

Regarding low jetties, Charleston Harbor, Cumberland Sound, the St. Johns River, and the Columbia River (a single jetty), may be cited as examples in the United States where such jetties have been found unsatisfactory, or have indicated that better results would have been obtained by higher ones, and the changes have been, or are to be, carried out, accordingly, by raising all or part of the works. As these outlets are typical ones, and experience with them has covered a period of many years, it appears to be safe to assume that the principle of low jetties is not usually an advantageous one.

8.—No engineer who has had experience with jetty work protracted for a long period of years, or who has studied the effects upon the channel during year after year of construction thus carried on, can fail to have been impressed with the seriousness of the results which may ensue from such delay. With channels of constant location the problem would be reduced principally to repairing damage from storms and to taking care of sand-drift or erosion, but with the shifting channels, the presence of which seems to be almost universal, the original location of the jetties, in the end, seems likely to be found to be of no value to commerce. An instance of such a risk is to be seen in the oft-quoted improvement of Cumberland Sound, on the southern coast of Georgia. (Plate LIV.) This work was first projected in 1879, and has been carried on since then to successful completion in the face of unusual obstacles. Always hampered by lack of funds, the construction was continued year by year as finances would permit, the channel during the interval changing until the entrance was at one time blockaded by sand, and the ship-channel passed outside the jetties, and the improvement, which was estimated to cost more than $2 000 000, was looked upon by many as an example of hopeless failure.

Work was begun in 1880, at which time the main or northerly channel ran to the bar about east-northeast or northeast, between the projected location of the jetties. The bar had from 11 to 13 ft. on it, with a mean tide range of about 6 ft., and the improved depth was to be 19 ft. This channel, shortly after work was begun, commenced a period of change and began to move southward, the map of 1886 showing two channels, one running about east and the other about southeast, and the report drew attention to the need of prompt extensions of the jetties in order to stop the shifting of the flow. By 1892, however, the sailing channel had swung to the south, crossing the end of the south jetty about 7 000 ft. from its commencement (work there having to be suspended in consequence), and thence running on the outside of it almost due east to the bar.

This condition of affairs continued for some years, the space between the seaward portions of the jetties, which were then considerably below low tide, gradually shoaling up, and the outlet of the sailing channel swinging further to the south, until about 1896 it reached its limit and began to shoal, a northerly channel having shown signs of opening along the north jetty during the previous year.* Apparently, however, this was inadequate for ships, which had to continue to use the southerly channel. The map of 1898 shows the latter as having returned to an easterly course, almost parallel to, and still outside of, the jetties, while the channel inside them was a shallow outlet passing to the north of a sandbank which stretched across two-thirds of the enclosed space.

So serious was the condition of affairs that the port was almost blockaded, and a portion of the south jetty at the channel crossing had to be removed, and dredging resorted to to relieve the urgency of the case. In 1900, however, a new channel between the jetties began to open, the ship channel still lying across the south jetty, but slowly shoaling. Dredging was commenced to assist this, and proved of material benefit, and by 1902 it had deepened sufficiently to permit of the south channel being abandoned and the gap in the jetty being closed. The new channel is reported to be of good depth and width, although, owing to the comparatively shallow depths between the ends of the jetties (19 or 20 ft.), the bar crossing is shifting. As the work is now practically completed, the jetties will

* Annual Report, Chief of Engineers, U. S. Army, 1897, p. 1534.

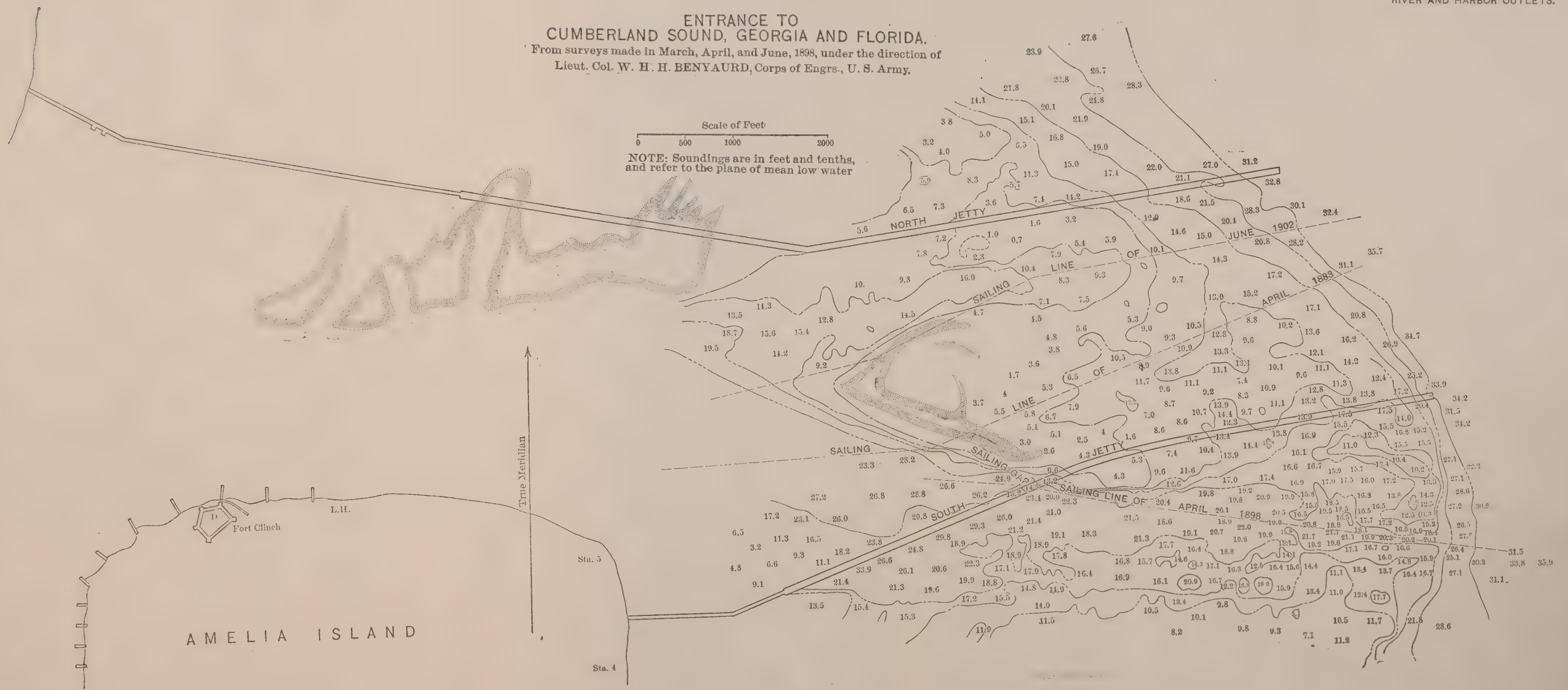
PLATE LIV.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1009.
WATT ON
RIVER AND HARBOR OUTLETS.

ENTRANCE TO CUMBERLAND SOUND, GEORGIA AND FLORIDA.

From surveys made in March, April, and June, 1898, under the direction of Lieut. Col. W. H. H. BENYAURD, Corps of Engrs., U. S. Army.

Scale of Feet
0 500 1000 2000

NOTE: Soundings are in feet and tenths, and refer to the plane of mean low water



guard the outlet from further serious change, but it is doubtful if the channel will ever reach the condition it might have attained had financial reasons permitted rapid construction.

Where funds do not permit sufficient work to control an outlet satisfactorily, experience seems to indicate the advisability of concentrating expenditures on one jetty only, which, it is believed, should be located on that side of the channel toward which the movement is tending, with the object of arresting any change. While the entrance will probably still continue to shift, it will be within narrower limits, and with less danger of escaping entirely from control, as did the channel at Cumberland Sound. The extreme positions of the field of change would, of course, prove inadvisable for the location of such a jetty, and the writer is inclined to believe that under usual conditions the best time for its construction would be when the channel has swung to about midway between its furthest limits.

9.—The method of assisting the scour of jetties by dredging is to be used on the new work at the mouths of the Southwest Pass of the Mississippi, of the Columbia River in Oregon, and elsewhere. It is already in use in many localities, and in all countries, and has generally proved satisfactory, since results can be obtained more rapidly and certainly. In some cases dredging has been relied on entirely, where the construction of jetties would have been too expensive; and, where the cuts have been located along the natural axis of the flow, the results have usually been very satisfactory. Where the cuts have not been thus located, they have filled up more or less, in the same way as has been found to be the case in dredging bars in rivers.

DISCUSSION.

Mr. Le Conte.

L. J. Le Conte, M. Am. Soc. C. E.* (by letter).—This paper is very suggestive, and brings out some important points too often overlooked. The great divergence of opinions of engineers of ability on any given harbor project, referred to by the author, may be safely ascribed to differences in diagnosis of the case in hand. This may be explained, in a measure, by assuming that each engineer naturally magnifies the importance of certain facts and features of the problem with which he has had much experience, and deliberately slurs over equally important facts with which he has had very limited experience. The whole question brings to mind very forcibly the fact that it is hard to find an engineer with a well-balanced mind, or one who can calmly consider all the well-known facts and give each one fairly its proper weight in the problem. This is an exceedingly rare trait, nowadays, when specialists are in demand, and will become still rarer in the future.

As to the nine categories submitted by the author: There seems to be no doubt whatever about the truth of the first statement. Any stream which has strength of character and stability of regimen must have some curvature. A straight channel is always uncertain and vacillating in its nature. The least obstruction either throws the axis of the channel to one side, or, worse yet, splits it in two, both of which are undesirable and hurtful in their effects.

The third statement is not expressed very clearly, but the author probably means the gorge section at or near the entrance, which certainly has a most remarkable relation to the tidal prism going in and out with each tide. That is to say, if we take the mean-tide sectional area at the gorge, in square feet, and divide this by the millions of cubic feet in the tidal prism going in and out at each spring tide, we will get as a result a very important relation peculiar to that tidal harbor, which will compare most favorably with other harbors in the same neighborhood. On the Pacific Coast this relation is 1: 33, or, in other words, Nature requires 33 sq. ft. of mean-tide section for each and every million cubic feet of tidal waters passing in or out at spring tides. This is true of San Diego, San Pedro, San Francisco and Humboldt. San Francisco Harbor has a tidal prism area of 480 sq. miles and yet this ratio is maintained, although the shores at the entrance are quite rocky. All these harbors face the Pacific Ocean, and the natural forces of the discharging ebb-tides are in constant battle with the heavy seas and ground swells which are ceaselessly beating in and trying to choke the entrance. In the case of inner harbors, the entrances to which are

* This discussion was presented before the San Francisco Association of Members, Am. Soc. C. E., at its meeting of August 19th, 1905.

very largely under shelter, the ratio above mentioned becomes 1: 43. This difference is no doubt due to the greater exposure to which the entrances of coast harbors are subjected. Mr. Le Conte.

Another most important relation, amply verified in almost every instance, has a very material bearing on the possibilities in any given case of improvement by jetties. It has been found by long experience that if we examine carefully the best defined channel in the inner harbor, not too far from the entrance, and note the width and mid-channel depth in feet and call it D, then, as a general rule, experience shows that by a system of high-water jetties, we cannot hope to get and maintain a bar-channel depth greater than one-half of D. But, in any case, it may be said that this is a function of the exposure at that particular site.

In reference to the author's statements regarding the wisdom of following always in the footsteps of Nature in establishing the jetty channel location from the entrance out across the bar, the writer has this to say: The difficulty in all cases is to decide when is the proper time to follow Nature. Take the case of Humboldt, for instance: The best channel is always found when the location is squarely abreast of the gorge. The shore drift from the south pushes the channel to the north from year to year, the navigable depth deteriorating all the time, and the difficulties of navigation getting worse and worse as it travels northward. Finally, the ebb-tide discharge from the gorge bursts out through the sand bar squarely abreast of the entrance, and, as a result, we have a fine channel once more. This cycle of events simply repeats itself in the course of a few years. It would seem from this that Nature does tell us in plain language that the proper place for the channel is undoubtedly squarely abreast of the gorge, and the existing jetties are practically on these lines.

The author says he is inclined to believe that under usual conditions the best time for its construction would be when the channel has swung to about midway between its furthest limits. The writer begs leave to differ with him, for the reasons above stated.

The author mentions the fact that in some cases dredging alone has been relied upon entirely and the results have been very satisfactory. This brings to mind a subject which has been much discussed of late by harbor engineers. Dredging operations in recent years have been so much cheapened by new devices and improved machinery that the whole financial problem of "jetties or no jetties" has undergone a vast change, in favor of dredging alone as a first trial. It is manifest to anyone who has given the matter much thought that the question whether a given entrance is to be improved by a system of jetties or by constant dredging alone is purely a financial one. If the annual cost of constant dredging to maintain

Mr. Le Conte.

a good navigable channel proves to be less than the annual interest on the cost of a system of high-water jetties extending out to the full depth called for in the jetty channel, then, of course, dredging is the proper solution of the problem. This calls to mind the case of Liverpool, where the Queen's Channel is now being maintained by dredging alone, without the assistance of any jetties, and the results —so often reported against—have proved to be highly satisfactory from every point of view.

The facts of the whole matter are just these: In any given case the quantity of traveling drift to be contended with is the factor to be determined. This can best be ascertained by actual trial in a dredged channel, and will at once lay aside all mere conjecture. If this trial shows conclusively that the annual cost of dredging operations alone is more than the annual interest on the cost of high-water jetties, then it is time enough to begin their construction. If, on the other hand, the cost of dredging should be less than the annual interest, then give up the idea of jetties and continue annual dredging operations.

There are two very important features connected with this subject, both of which go to strengthen materially the dredging side of the problem. One is that we have good reasons to know that the cost of dredging work when prolonged for some years with improved machinery will grow less and less. The second is that one cannot generally hope to get and maintain more than a certain limited depth on the bar by a system of high-water jetties, and this depth is generally only about half the mid-channel depth inside the gorge. This depth may not be enough for the interests of navigation. On the other hand, where the navigable channel is being maintained by dredging alone one is at liberty to make any depth he may see fit; 40 to 45 ft. at low water, if necessary. These practical features, coupled with the further fact that the interests of navigation get the results so much sooner, give great weight to dredging operations as being the most suitable and best solution to the problem in a great many cases now coming up.

Capt. Harts.

WILLIAM W. HARTS, M. AM. SOC. C. E.* (by letter).—The extent of experience in the United States in the improvement of rivers and harbors for the benefit of navigation is but little realized among engineers and less by the general public. For more than a generation—commencing, in fact, before the War of the Rebellion—the Government of the United States has been engaged in vast works for the benefit of water-borne commerce.

But few countries in the world can boast of the variety and extent of their waterways when compared with those of the United States, and in but few are the problems of improvement as wide in

*This discussion was presented before the San Francisco Association of Members, Am. Soc. C. E., at its meeting of August 19th, 1905.

scope and as large in extent. The construction of jetty harbors, such as those of the Pacific, Southern Atlantic and Gulf Coasts, and the harbors of refuge like those of New England; the protection and development of the fresh-water lake ports, such as Buffalo and Duluth; the deepening of rivers in alluvial basins, like the Lower Ohio and Tennessee; canalization work, like that on the Monongahela; regulation of rivers like the Missouri; constructing locks and canals, like the Sault Ste. Marie, show, briefly, that the scope of the work is large. Capt. Harts.

These works, almost exclusively, have been in the hands of the Corps of Engineers of the United States Army, and the vast measure of benefit to commerce derived from them cannot be closely estimated.

It is stated by the author that not all these improvements have fully met the sanguine expectations of the engineers. This can scarcely be gainsaid, but where can a body of engineers be found the average of whose success is higher than that of the Corps of Engineers of the Army, or the widespread benefit of whose work bears any more economical ratio to the money invested?

Perhaps it is a wholesome thing that, in all branches of art, letters and science, there will be found few successful persons who are thoroughly satisfied with the way their creations have met their hopes.

In those branches of engineering, such as water-works and railroads, where the conditions and forces at work are more easily analyzed and ascertained, and where a great diversity of opinion is not ordinarily to be looked for, there are found, even here, frequent differences of mind and method, and principles thought to apply to the general case are put forward with considerable hesitation.

On the other hand, in a branch of engineering where the combinations of natural agencies are so many and complex, and where their results are so often hard to foresee definitely—as occurs in river and harbor engineering, admittedly one of the most difficult branches of the science—the establishment of correct principles, of anything like a general nature, is a task of no small magnitude and responsibility.

In all branches of science the statement of correct rules explaining natural phenomena is usually made only after a thorough and prolonged study, and after sufficient examples are observed to insure that the principles as stated do not apply rather to the exceptions than to the general case. One of the most dangerous things a scientific man can do is to attempt to establish wide principles on a few isolated facts.

Accepting the author's invitation for criticism, and taking up the principles enunciated by him in order, we notice first the state-

Capt. Harts. ment that when the conditions of depth and width are constant in flowing water, the water appears to be moving in a curve.

The converse of this, that the most constant and best channels of rivers are usually found in the bends, with shoaler depths in the crossings, was undoubtedly known by river pilots for many years before rivers were studied to any great extent by engineers. There are very few straight lines of any considerable length in natural river channels, and the thread of the stream, or the "thalweg," usually winds from side to side within its banks, so that practically all quietly flowing water in Nature "appears as moving in a curve." This probably results from many conditions, among which may be recognized the non-homogeneous character of the banks and bed, and the tendency of water to travel along the shortest line to a lower level.

Practically straight reaches in rivers do exist occasionally, however, where depths and widths are fairly permanent, as in the Sacramento below Sutter Island, recently observed by the writer. May not the principle of curved channels, as stated by the author, be found, on closer examination, to result rather from the non-homogeneity of the containing banks than from any inherent and inscrutable purpose of the currents to follow a curved and fixed line when not "bewildered" by the engineer?

Greatest depth is ordinarily found in rivers having movable beds where the filaments of the currents are most concentrated both in location and direction. This concentration, of course, is found in Nature most generally in concave bends, but it is not at all impossible in straight reaches where the inter-relation of the width, depth, velocity and character of banks and bottom is a suitable one. Scour in the bed depends on the bottom velocity of the water, and, where there is sufficient to pick up and transport the material of the bed, an increase of depth may be expected. Likewise, when velocities are reduced beyond what is necessary for transportation, suspended material is deposited. A stable channel lies somewhere between.

Would not the author's first principle, as stated, deny that a straight canal could be constructed where the depths and widths would be constant?

In dredging channels for navigation the straight line is usually necessary on account of the difficulty otherwise encountered in laying out the work and in setting ranges so that vessels may easily follow the deepened channel. When properly designed and located with reference to currents and sediment, these channels usually remain open satisfactorily. No experienced river engineer lays out dredged channels in curved lines, particularly if far from the banks and landmarks, unless there is no alternative.

Capt. Harts.

It is not believed that the author intended his statement as written.

In his second principle he refers to the fixed relation between the curvature and best attainable condition of the channel. This is not very clear, as stated. It will scarcely be denied that there is some relation between these two, but there seems to be grave doubt as to its fixedness, except for exactly similar conditions.

In the very nature of things, it is plain that this relation would change for different rivers, for different stages in the river, for different velocities, with the character of the banks and bottom, and with the quantity of sediment already in motion.

The author's third principle can scarcely be admitted as a general truth, that there exists, or has existed, under natural conditions, a channel at each river mouth that limits the possibilities of artificial improvement as to depth, width and curvature.

It is quite within the range of possibilities, of course, that the position of the headlands, the location and shape of the bar, extreme flood conditions of tide, together with the action of storms, might fortuitously combine to cause a bar channel to scour out to even a greater depth than could be secured by jetties or training walls acting under normal conditions, but that this happens in all cases can scarcely be maintained.

Unless the exact moment of such a maximum depth, in case it should exist, could be secured for making a survey before normal conditions reassert themselves, how is the information to be obtained as to these special channels so that it can be applied?

Even if obtainable, can it be shown that there is any known relation between this unusual and accidental depth and the depths and widths possible with improvement under the normal and usual conditions?

Furthermore, it can scarcely be claimed that the rule, even if true, would be a good guide, as lack of sufficient information, often impossible to secure, might easily cause an engineer following it to be misled.

The depth at the Sulina Mouth of the Danube varied from 9 to 11 ft., and, from descriptions available, was never found to be more than 11 ft. before improvement. After the completion of the training works a permanent depth of 21 ft. was secured. At Humboldt Bay, California, the maximum depth of channel before improvement was reported as 25 ft., whereas, after the improvement, channel depths of 31 ft. are found. Further examples could probably be found, with further search, which would doubtless add to the conviction that it is entirely futile to attempt to establish the relation between the natural and artificial depths mentioned by the author.

In the light of recent observation and practice, the author's idea

Capt. Harts. of the formation of bars also seems to be erroneous. In the United States it is usually conceded that the bars at river mouths in tidal seas are the result far more of the external forces in the sea, such as sand-bearing, littoral currents and sand-moving storms, than the result of the deposit of sediment from sediment-bearing streams.

Such channels as exist are undoubtedly the resultant of the action of the deepening forces of the tidal currents flowing in and out of the bay twice each day, and the bar-forming forces mentioned above. But for these tidal currents, the mouths of all the harbors of the Pacific Coast and the South Atlantic Coast would in all probability be sooner or later ruined for navigation by being filled with sand.

Except perhaps at the mouth of the Mississippi, the bars at the entrances of sea harbors are usually of sea sand and not of river sediment. The author probably confuses the formation of ocean bars with that of river deltas in tideless seas—two very different processes.

In his fourth principle the author remarks upon the winding course of river currents at their mouths in the open sea. As long as the river currents are directed by their banks, it is, as was mentioned above, only natural that the channels of deepest water should curve from side to side. The river is only fulfilling its well-observed laws in this. It would be more worthy of notice if this were not so. As soon, however, as the current passes beyond the control of the banks or underwater shoals, its force is often quickly dissipated in the sea, and the channel becomes subject to decided change in location and depth as well as in curvature, due to the action of the forces of the sea and storms. In many cases these changes have wide limits. That there is any direct and permanent relation between the many different curves of the channel found at different times across the bar with any of its other curves seems to be very improbable. Such a fact would certainly be very hard to establish. Certainly, one harbor and one survey of this harbor can scarcely be convincing.

As long as the dynamic force of the current is conserved and directed by the banks, the best depths are found. Artificial improvement may be regarded as simply continuing this controlling effect by means of jetties, so that the channel in its newly fixed location may maintain the best depths possible over the obstructing bar. It is more and more becoming a matter of general belief that the trace of these jetties is of much less importance than their proper location. Even concave, "reaction," or other theoretical jetties may often do good work if well located. Their main effect being independent of their shape, would in most such cases be accomplished equally well with a cheaper and simpler trace.

Capt. Harts. With regard to the cycles of change in location of the channels, it is believed that the author's observation is again at fault. At Coos Bay, before improvement, at Nehalem, at Tillamook, and at other harbors on the Pacific Coast, the cycle of change has been closely observed and found to agree in the main. The channel as it enters the sea is diverted one way or the other—at Coos Bay to the north, at Nehalem to the south, and at Tillamook to the north. This action of channel flexion might continue indefinitely in the same direction but for the fact that the surface slope of the river seaward is thereby so prolonged that the outgoing tide produces eventually a hydraulic head against the diverting sand spit, which is more than the spit can withstand. Then the tide cuts out a new and more direct channel for itself. In other words, the spit across the entrance thus acts as a diverting dike, prolonging itself by the sand-bearing currents across the mouth and holding back the outflow of the tide more and more by compelling it to take a longer course to sea. When the difference of level between the water surface in the channel and the ocean is sufficient to break through the dike when aided by storms at sea, a shorter line of discharge is at once cut out. A similar action is found on the South Atlantic Coast.

Deltas are not formed in river mouths and estuaries having a pronounced tide.

The statement that middle grounds in tidal river mouths are formed by the deposit of river-borne sediment in the deepest channel, building it up into a shoal, thus dividing the former channel into two parts, seems to be singularly erroneous. In the deepest channels the velocities are greatest and the capacity for carrying sediment is greatest, and the tendency to deposit, therefore, is least. If this statement were generally applicable, would there not inevitably be found in the center of the Lower Mississippi River a long, narrow, middle-ground shoal with a double channel? Such a condition has never been reported, as far as known. All indications point to other causes for middle-ground shoals.

In the author's fifth principle he says it is essential that the natural conditions of location and curvature, especially as regards the bar crossing, be interfered with as little as possible. He has already explained, under his fourth principle, that after leaving the shore, the channels vary in location over very wide limits when "bewildered" by the ocean.

Which one of these many locations would he adopt under his fifth principle?

The demands of navigation must be met, attention must be paid to the direction of the worst storms, and the other natural conditions of the entrance must be studied for each locality, and given

Capt. Harts. their due weight in selecting a location for an improved channel. How does the author's fifth principle help? Does he not again confuse river conditions with those of the sea?

The author's idea of the inherent tendency of the water to follow its chosen path should also be examined. Perhaps some simpler explanation can be found for what he considers a mysterious and incalculable tendency, not to be tampered with, except at the engineer's peril.

He refers to several examples of harbors where the location of the jetties has not been in accord with what he considers a proper one, so that the main channel was diverted too much from what he accepts as its chosen location. Two of the cases mentioned—St. Johns River and the entrance to Cumberland Sound—are on the South Atlantic Coast. In both of these cases the littoral drift of the sand is plainly southward across their entrances, and at both places, before improvement, the entrance was decidedly flexed to the south, slowly shifting further to the southward in more or less regular cycles until moved to a more direct location more nearly easterly, somewhat as above described for Pacific harbors.

A glance at the map of St. Johns River will show the southern bank of the river projecting beyond the northern bank. The southerly drift of the sand, interrupted by this south bank, has turned the main channel well to the southward around this bank and filled in the open space north of it with a shoal having slight depths. It is probable that the flood-tide coming from the sea without any preponderant directing influences has found less resistance in breaking across this shoal, thus forming a swash channel, than in following the longer and deeper channel made mainly by the ebb-tide, which is strongly directed by the river banks and headlands.

In building the south jetty first, the location of the main channel was limited in its southerly location. Encroachment on the channel, due to the supply of sand from the north, was not cut off at that time. It seems more probable that in this natural way the channel was crowded against the south jetty rather than by any inherent tendency of the channel to follow chosen paths. As the north jetty was built out, the channel has shifted to the north, so that in 1905 the channel enters the harbor across the prolongation of the north jetty.* As the supply of sand from the north is cut off more and more by building the north jetty to full high-tide level, and as the shoal between the jetties, no longer supplied with sand, is carried seaward, the channel may be expected to shift still farther to the northward and perhaps move away entirely from its present location against the south jetty. This sort of action has already taken place at Coquille River, in Oregon, and has also taken place

*See map, p. 1686, Annual Report, Chief of Engineers, U. S. Army, 1904.

at the entrance to Cumberland Sound. As fast as the north jetty at the latter place has been extended and raised, the sailing channel has moved to a more protected location farther north, as shown on Plate LIV. Capt. Harts.

The author also mentions the mouth of the Mississippi River as an example of the inherent tendency of currents to follow their chosen paths, showing on the map a deep hole which had scoured as a mute protest against encroachment by the jetties upon its chosen path. He plats the chosen path arbitrarily, making it cross the line of the west jetty near station Kipp. If the line of deepest water be plotted on the map it will be found that it lies within the line of the west jetty, far beyond the point where he makes it cross, and even beyond the deep hole. By what method does he obtain the "resultant line of flow," and how sure can he be that the contours of the maps of 1875 would give the same line as earlier surveys, and that change was not taking place at this mouth from time to time? Unfortunately for his theory, also, the map of 1904 not only shows no unusual scour in this location, but shows short spurs, built later in this vicinity, to prevent fill.* Apparently the "mute protest" had failed.

A few words as to the early history of the Mississippi River improvement may not be out of place.

The Mississippi mouth is one of the few works of river improvement not originally constructed under the direction of the Corps of Engineers of the Army.

As early as 1852 a Board of Engineer Officers recommended that the improvement of the mouth of the Mississippi be made by constructing parallel jetties, provided the operations of harrowing or raking the bar then being tried were not successful. No action on this recommendation was taken by Congress.

In 1871 Congress directed an examination to be made of the mouth of the Mississippi, under the direction of the Secretary of War, and an estimate of cost to be submitted for a canal with locks connecting the deep water of the river with deep water in the Gulf of Mexico.

This examination was made, and a project was submitted by Captain Howell, Corps of Engineers, in 1873. A Board of Engineers was called early in 1874 to report on this project. Considerable dissatisfaction was manifested by several members of the Board over any plan not contemplating an open river.

The scope of the Board was then widened, and a project was called for and submitted for an improvement by means of jetties.

This project was made and submitted in 1874, and contemplated the construction of parallel jetties made of brush fascines built into

* Annual Report, Chief of Engineers, 1904.

Capt. Harts. mattresses, weighted with rip-rap; the jetties to be about 4½ miles long and lying 2 200 ft. apart. This width was to be reduced by spurs if found necessary. The cost was estimated at nearly $5 000 000, and the time required for construction was estimated at four years. Members of the Board disagreed as to their recommendation of this plan, some preferring the system of canal and locks, as a similar experiment, some years before, had been successfully put into service at the mouth of the Rhone.*

Congress failed to act, on either the jetty or canal and locks project, until the late J. B. Eads, M. Am. Soc. C. E., apparently believing more in the jetty project of the engineer officers than did Congress, came forward in 1875 with a proposal by which he was to take the risk of failure, guaranteeing to secure by parallel jetties a channel 300 ft. wide and 30 ft. deep within five years for $5 252 000, and to maintain it for twenty years at an additional cost of $100 000 per year.

The long record of sediment and current observations of Generals Humphreys and Abbot were available for his guidance, and the surveys and experiments, covering many years, made by the army engineers were all presumably of much value to him. After securing the co-operation of the river pilots and other similar associations, and the indorsement of the citizens of the region affected, he was rewarded by Congress with the acceptance of his proposal by the Act of March 3d, 1875.

This work was pushed to completion within five years, and has since maintained excellent depths for navigation. Being carried out on practically the proposed plans of the army engineers, it has vindicated the good judgment of the early engineer officers who advocated this plan of improvement. The full width of 300 ft. having 30 ft. depth, however, was not secured, so that Congress, later, modified its original act, and made a concession in its terms that authorized payment to Mr. Eads, if he could maintain a channel 200 ft. wide and 26 ft. deep, with a central channel 30 ft. deep without regard to width. By resorting to dredging from time to time he was enabled to maintain this channel.

Another startling statement of the author assumes that the excellent results obtained at Galveston were due, not to any wisdom in the selection of a suitable location for the jetties, not to the tidal prism of the bay, not to any well-studied design as to the height and length or width apart of the jetties at their outer ends, but rather to the "wide basin left between the jetties near their shore ends," permitting the channel to continue in its natural course.

If this principle can be supported by the author, he has indeed made a discovery.

* Annual Report, Chief of Engineers, 1874.

Capt. Harts.

In the author's eighth principle he advocates rapidity of construction. Harbor works are no exception to the general rule that there is an economical rate of speed in all construction, depending on many considerations. It often occurs that funds are supplied for public harbor and river works by Congress, not in accordance with the needs of the work, but for other good reasons, such as the state of the public treasury or the more urgent need of other localities, and the like; so that some improvements are carried on, in spite of the engineers, in a halting and interrupted manner, not only injurious to the efficiency of the work, but necessarily uneconomically.

For economical management, the progress of every such work, of course, should never be hampered by lack of funds, but a work once commenced should have sufficient funds in sight for its completion.

With this exception the writer cannot agree with the author that rapidity is always a desirable feature or an essential to success. Jetties may sometimes be built too rapidly, and cases are known where "more haste makes less speed."

At the mouth of the Columbia River the rapid extension of the jetty during construction caused underscour, at one time threatening the existence of the jetty itself. Spurs had to be built to protect the base from the current. These spurs were effective, but after some time were found to be left far from the channel after the entrance had accommodated itself to its new conditions and a new equilibrium had been more nearly restored. Time is often necessary to enable the engineer to know just what effect his works will have.

Nor is his ninth principle of universal application. At the Columbia River the dredging operations after a full season's trial have been found unsuccessful. All the smaller Pacific Coast ports change considerably from time to time, especially in depth. Many of them are perceptibly shoaled in storms, but, with well-designed works, after a short period of good weather, this shoaling is again scoured out. At Coos Bay, on one occasion, a shoaling of 4 ft. during a single severe southwest storm was reported, which was completely scoured out again within a few days. Dredging under such conditions must necessarily be of doubtful value.

There can be no doubt, however, that improvement by dredging is receiving much more attention than formerly. Channels in some localities which were thought to be impossible to maintain without permanent works are now found to be kept open successfully by dredging. New York Harbor's 40-ft. channel, now being dredged, is believed to be a permanent improvement, results in neighboring channels being sufficient to decide the engineers in recommending this form of improvement.

Capt. Harts. The author's paper, on the whole, emphasizes the fact that the statement of general principles, to be convincing, must be made with considerable care, especially with regard to river and harbor engineering, and that no "rule-of-thumb" or similar formula will ever replace the careful study of the conditions surrounding the work at each place to be improved. It emphasizes, further, the wisdom of studying and digesting particulars thoroughly before proceeding to establish wide general principles.

Mr. Watt. D. A. WATT, M. AM. SOC. C. E. (by letter).—In matters connected with improvements of the nature referred to in the paper, where the conditions at each locality are more or less varied, and where the construction of the works must necessarily occupy some years, it is scarcely necessary to state that the plans best suited can only be determined by a close study and comparison of local causes and effects. At the same time, it seems possible that certain points of resemblance between the results obtained at different places may exist and be shown in the development of the new conditions, and that, if such do exist, they might throw some light on the physics of moving water.

From their experience with the diking of their rivers, German engineers have formulated certain general principles which are applied in the studies of projected works, and from conversations held by the writer at various times with those connected with coast improvements in the United States, the question arose as to whether any similar principles might exist which influenced the flow in the channels both of outlets and of rivers. The conditions of flow seem to be usually much more complicated in the former than in the latter, but the opinion of those who had watched the development of improvements in certain localities appeared to agree on several fundamental points, and a somewhat wider criticism of the principles suggested in the paper had been hoped for, which might indicate whether any general laws could be found applicable, or whether experience had shown that conditions at outlets were quite too complicated to admit of any generalization. If the former, the relative conditions would have afforded some interesting side-lights on certain of the phenomena of flow and channel formation, both in outlets and in rivers. It is a matter of regret, therefore, that the paper should have been construed as a reflection on past work, as it was certainly very far from the writer's intention to do this in any way, or to assume the dogmatic role with which he seems to have been credited.

In regard to the statement in the discussion as to the original depth in the Sulina Mouth of the Danube, the writer desires to call attention to Sir Charles Hartley's map of the outlet as it existed in 1857, prior to any improvement, which shows that the depth of

channel where the river entered the sea, and for some distance above, was from 18 to 20 ft. When the jetties were subsequently constructed, the channel between them was scoured out until it had reached a depth of 20 to 21 ft., with a corresponding width, not greatly different from the depth and width existing originally at the point where the river entered the sea. Comparative small-scale plans of this improvement, beginning with 1857, can be found in the *Proceedings* of the International Engineering Congress, held at Glasgow in 1901. The depth is now maintained at about 24 ft. by dredging. Mr. Watt.

Regarding the statement that middle grounds in tidal river mouths are due to the deposit of sediment, the paper states them to be:

"The natural result of a sediment-bearing flow meeting still water. The greatest amount of heavy sediment, being carried along the center or deepest portion of the water, is deposited gradually as the flow slackens, and begins to form a bank along the middle of the channel."

The meaning of the last words might have been made clearer by saying that the sediment begins to form a bank at the point where the flow first slackens, and that the head of this bank is usually found opposite the center of the approaching channel. The process of forming this deposit of sediment by a river when its flow is checked has been found, by the observations on the Mississippi and Missouri Rivers, to be as follows: As long as the flow can keep to a deep or swift channel it can bear along the sediment, but when it comes to a wide stretch where its velocity slackens, the heaviest particles are let fall, and a bar begins to form, extending down stream, its head being approximately opposite the center of the channel approaching it. These bars are often found dividing the river into two channels, as in the case of the middle grounds referred to. This process of sediment-deposit has been similarly traced and described by Francis J. E. Spring, M. Am. Soc. C. E., in his work on "River Training and Control" in India. The conditions at the mouths of rivers where middle grounds occur appear to resemble in general features the conditions of flow just referred to, since these middle grounds usually exist where the channel spreads out and the flow is checked, and where, accordingly, the first deposit of sediment might be looked for.

The reference to the jetties at the mouth of the St. Johns River and at Cumberland Sound was in no wise intended as a criticism of these works. The St. Johns River was cited as an instance of the tendency of a stream to adhere to certain paths. At this outlet there were formerly two channels, and as the northeast one was the better for improvement, the other was accordingly closed, and the

Mr. Watt. water was concentrated into a single channel as before described. The work at Cumberland Sound was quoted as an example of how delays from failure to obtain expected funds are likely to result in serious complications in an entrance. There appears to have been no doubt at any time that had funds permitted the early completion of both these works, as was the expectation of their designers, the ensuing trouble and expense would not have occurred. The inevitable consequences of delay were frequently pointed out by the Engineer Officers, but the work once commenced could not be altered, and the results which ensued, such as the shifting of the channels, the growth of the shoals, etc., have fulfilled their predictions.

The case of the South Pass of the Mississippi was referred to as indicating that where a natural channel has to be encroached upon, a proportionate disturbance is likely to result. If it be granted, as stated in the discussion, that the line of flow marked on Plate L, survey of 1875, crosses the west jetty incorrectly, the contours still show that this jetty was built across the deep water of the channel. As the jetties were well under construction at this point at the end of 1876, a year and a half after the survey was made, it is hardly probable that much change had occurred in the channel location, as this coast is said to alter very slowly. The history of the eddy which appeared along the west jetty, shown on Plate L, can be traced on the various maps of this outlet published in the Reports of the Chief of Engineers, United States Army. As will be found described in the paper, it continued to exist for several years, as long as the river was wide enough to give it free play, and its gradual displacement and final breaking up by 1888 seem to have been due to the narrowing of the channel by the building of the spurs and inner jetties, shown on Plate LI. Similar cases of checking eddies by cramping them can be found on the Mississippi, and notably at Memphis, where spurs were built to check a large one which was working into the bank. The writer has met with several instances, such as the one quoted on the lower Seine, where dikes had encroached on a channel, as in the case in question, and in each there was to be found a scour-hole near the point where the dike approached the deepest flow. The action is surely not an impossible or unnatural one. Is it not similar to the impinging of a jet of water on a surface, where the effect of impact is in proportion to the angle of inclination? If the jet strikes the surface at a very flat angle, its effect will be much less than if at a steep angle. If such holes are not due to some such cause, what produces them?

Regarding the Galveston entrance, it is stated in the discussion that:

"As long as the dynamic force of the current is conserved and directed by the banks, the best depths are found. Artificial im-

provement may be regarded as simply continuing this controlling effect by means of jetties, so that the channel in its newly fixed location may maintain the best depths possible over the obstructing bar. It is more and more becoming a matter of general belief that the trace of these jetties is of much less importance than their proper location." Mr. Watt.

This quotation appears to support the contention that the good results at Galveston were due to the fact that the Engineer Officers in charge of them realized the desirability of continuing the natural conditions, and of obtaining a proper location for the jetties which would not interfere with the flow. That they held this view is shown by the various documents relating to the work; and, having determined on that basis the width apart and the location of the jetties on the bar, they created a wide basin near the shore ends which would permit the water to follow without obstruction the path which they judged, from the local indications, it would take, and which in fact it actually has taken.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1010.

THE FERRY BRIDGE ACROSS THE SHIP CANAL AT DULUTH, MINNESOTA.*

By C. A. P. Turner, M. Am. Soc. C. E.

This novel structure was designed to secure a satisfactory means of transportation of street traffic across the ship canal, on Lake Avenue, Duluth, to Minnesota Point. This point is the natural breakwater separating Lake Superior from the Harbor of Duluth, extending in a southeasterly direction 6⅔ miles from the Minnesota shore, with a width varying from 300 to 800 ft. At its outer end it is separated from Wisconsin Point by the natural entry through which formerly flowed all the waters of the Nemadji and Navajoe Rivers, and which was the only means of access to the Harbor of Duluth.

As no deep channel connected the natural entry with the Harbor of Duluth in 1870, the city commenced excavating a canal across the point ½ mile from the mainland. This canal was 250 ft. wide, with sides protected by timber cribs. (Fig. 1.)

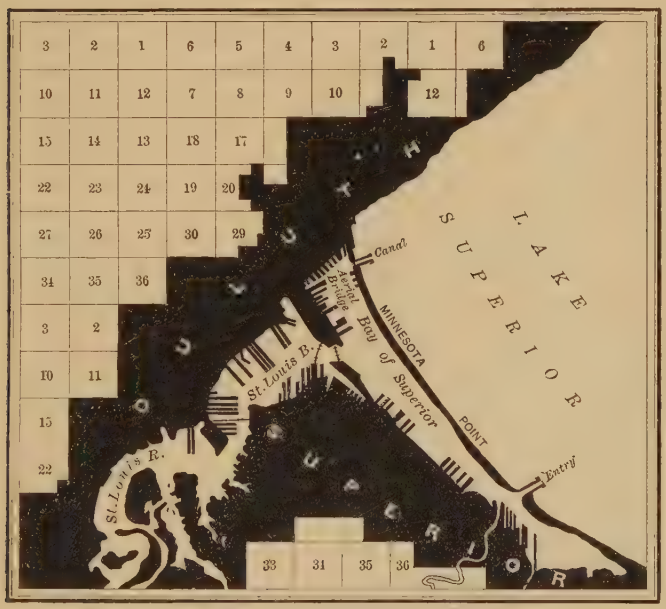
The 6 miles of Minnesota Point thus severed from the mainland continued to be a place of residence for a small population which traveled across the canal in the open season in rowboats, and in winter on the ice, or, for a number of years, on an improvised suspension bridge, to cross which, according to Mr. Patton,† required more than ordinary courage.

* Presented at the meeting of November 15th, 1905.
† *Engineering News*, March 20th, 1902.

FIG. 1.—DULUTH SHIP CANAL IN A STORM.

FIG. 2.—DULUTH FERRY BRIDGE. TOWERS AND FIRST PANELS BUILT OUT, AND NORTH FALSEWORK STARTED.

The City of Duluth, ever since the cutting of the canal, has looked with regret at the large amount of valuable dock frontage—the bay side of the Point—which it is impossible to use through lack of means of access. For years efforts have been made to devise practical means of transportation across the canal which would provide for the business development of the property on the Point and for the convenience of its residents and the rapidly increasing number of people of Duluth who are using its beautiful, wooded grounds for camping and picnicking.



MAP SHOWING LOCATION OF FERRY BRIDGE, DULUTH, MINN.

FIG. 1.

The first attempt to provide a permanent crossing was made in 1890, when the services of A. P. Boller, M. Am. Soc. C. E., were secured to prepare plans for the bridge. His report, submitted in March, 1890, recommended a draw-bridge with a pivot pier on the south side of the canal, and with a clear span of 200 ft. between fenders, the clearance above high water being 20 ft.

The estimated cost of the design, exclusive of right of way and

damages, was $400 000. These plans were approved by the city, but, owing to strenuous opposition from steamship companies, the project was abandoned.

The next scheme was for a tunnel, and plans were prepared for the city by William Sooy Smith, M. Am. Soc. C. E., with different capacities, and estimated costs from $880 000 to $1 300 000, figures so large that the city could not undertake the work, and agitation was continued for some form of bridge.

In November, 1891, a prize of $1 000 was offered for the best design of a draw-bridge,* and twenty plans were submitted, among others, a lift-bridge, by J. A. L. Waddell, M. Am. Soc. C. E., which was considered favorably by the city. On submitting the matter to the Board of United States Engineers, appointed to consider the request of the city for permission to erect the bridge, approval was refused for any type of bridge submitted in this competition.

In the winter of 1893 the City of Duluth offered a prize of $1 000 for competitive designs for the construction of a tunnel under the canal. S. G. Artingstall, M. Am. Soc. C. E., was engaged by the city to pass upon the plans submitted, and reported in favor of a design consisting of concrete tunnels to be built in open cofferdams. This design had been submitted by C. C. Conkling, M. Am. Soc. C. E., and he received the prize.

For the convenience of the people living on the Point, a free rowboat ferry was established, which in 1898 was replaced by a steam-tug ferry, costing the city $9 000 per year for operation. The cost of this service, and its unsatisfactory nature, kept alive the desire and agitation for something better, but in the meantime the difficulties had been increased by the fact that the United States Government had condemned property on both sides of the canal and widened the canal at Lake Avenue to 350 ft., with concrete walls on each side, making necessary a span of nearly 400 ft. for any form of bridge erected at this location.

In 1889, Thomas F. McGilvray, Assoc. M. Am. Soc. C. E., City Engineer of Duluth, suggested and prepared sketches of a structure resembling the Arnodin Bridge at Rouen, France, which aroused considerable interest. On the writer's suggestion that a girder construction of span in place of a suspension bridge, and a stiff traveler

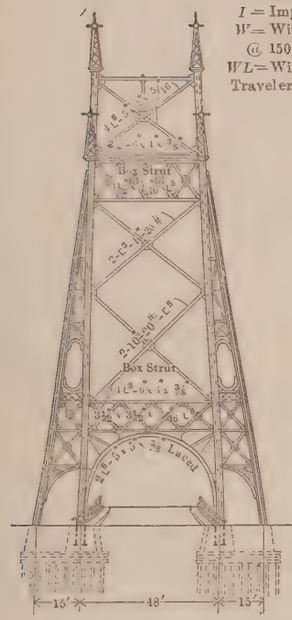
* *Engineering News*, October 27th, 1892.

PLATE LVI.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1010.
TURNER ON
DULUTH FERRY BRIDGE.

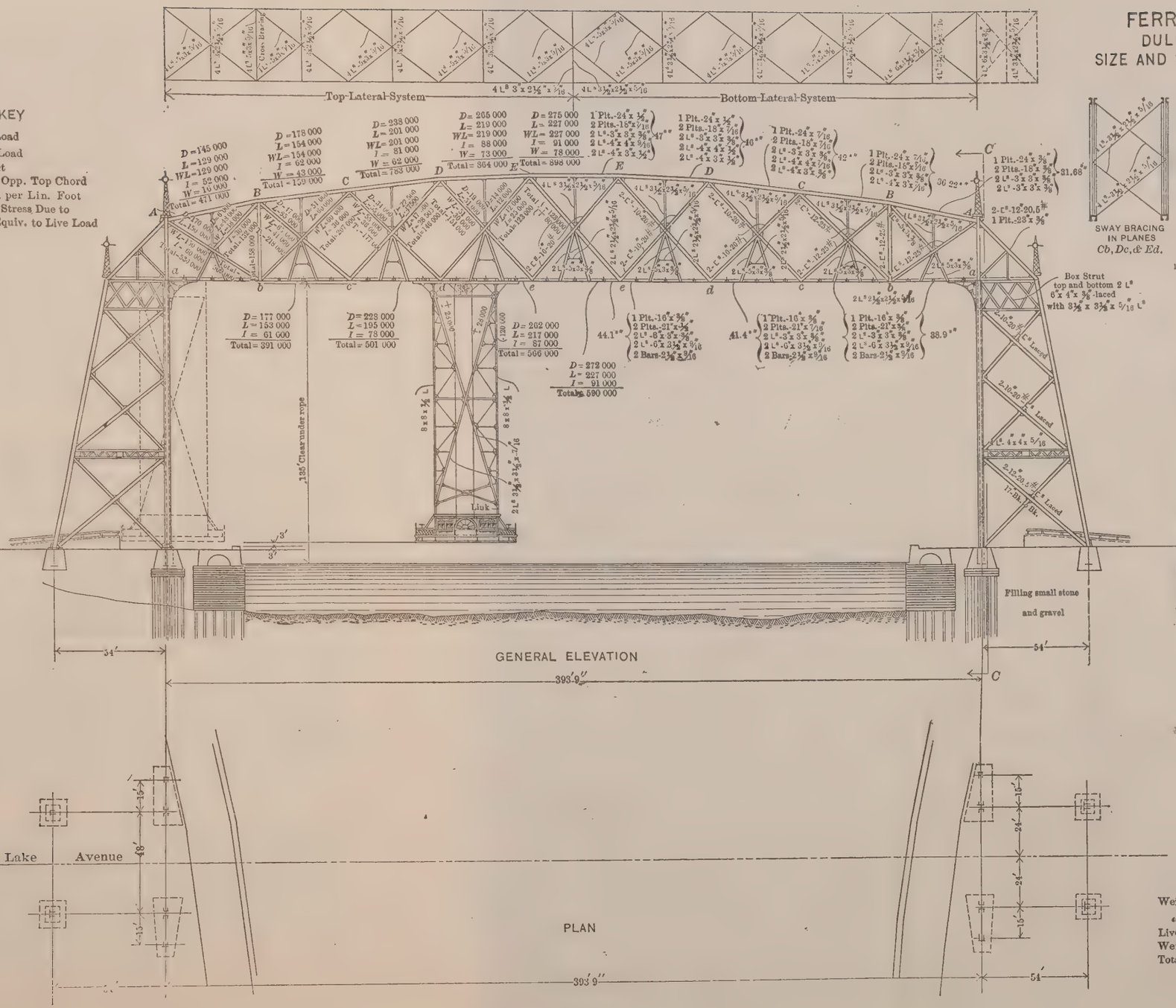
FERRY BRIDGE
DULUTH, MINN.
SIZE AND SECTION SHEET

KEY

L = Live Load
D = Dead Load
I = Impact
W = Wind Opp. Top Chord @ 150 lb. per Lin. Foot
WL = Wind Stress Due to Traveler, Equiv. to Live Load

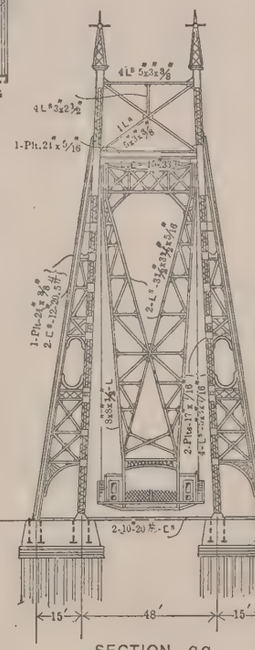


END VIEW

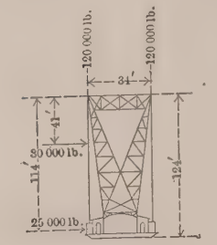

Top-Lateral-System
Bottom-Lateral-System
Box Strut top and bottom 2 L⁸ 6″ x 4″ x 5/8″ laced with 3½″ x 3½″ x 5/16 L⁸
Filling small stone and gravel
Lake
Avenue
54′
393′ 9″

GENERAL ELEVATION

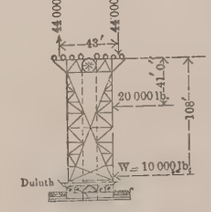
PLAN



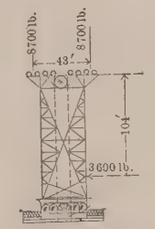
SECTION *cc*



LATERAL WIND FORCES ON TRAVELER



WIND LONGITUDINALLY



BRAKING EFFECT

WORK OF STOPPING CAR

Speed, 4 Miles per Hour, 5.85′ per Sec.
Stopping in 35′ Work Required

$$\frac{1}{2} \times \frac{240\,000}{32.16} \times (5.85)^2 = 127\,000 \text{ Foot-Pounds}$$

$127\,000 \div 35 = 3600 = \text{Braking Effect}$

LOADS

Weight of Main Trusses 1400 lb. per Lin. Foot	
" " Carrier Car	80 000 lb.
Live Load on " "	125 000 "
Weight of Traveler and Trucks	35 000 "
Total Weight, Carrier and Car	240 000 lb.

in place of cable suspenders for the car would be stiffer and cheaper, he drew up additional sketches for this construction. The idea found ready acceptance, and steps were taken to overcome the many difficulties in the way; for, in addition to the former obstacles, the city could not finance the proposition without special legislation, and, further, the permission of the United States Government had to be obtained, not only to erect and maintain the bridge over the canal, but to occupy Government property with the piers.

The special legislation was secured, and assurances as to overcoming the other obstacles were obtained, so that in February, 1901, advertisements were published calling for bids for the erection of a bridge in accordance with the general specifications prepared by Mr. McGilvray. On March 25th, 1901, only one bid, that of the American Bridge Company, was received on the general plans for a stiff girder, prepared under the direction of the writer, then Engineer of the Western Contracting Department of the American Bridge Company.

Under this proposal the city was to pay $7 000 per year for 20 years. The city immediately applied to the Secretary of War for a permit to erect the bridge, but this was not granted until September 6th, 1901; and, in the meantime, the American Bridge Company had withdrawn its proposal. A proposition was then submitted by the Duluth Canal Bridge Company, a corporation organized to build the bridge, and a contract was closed with the city.

These plans, in their incomplete state, have been described by Mr. W. B. Patton.* The structural work was sublet to the American Bridge Company, the steel was ordered from the mill, and 500 tons were delivered at the Minneapolis plant, when, in June, 1902, the Duluth Canal Bridge Company failed to make the first payment provided for in the contract with the American Bridge Company, and the work stopped. The foundations had been put in, but not paid for, at this time.

In May, 1903, the City Council of Duluth passed a resolution canceling the contract with the Duluth Canal Bridge Company, and directing the City Engineer to advertise for new proposals. The writer, in the meantime, had spent considerable time in getting plans in shape to secure propositions on an improved design.

* *Engineering News*, March 20th, 1902.

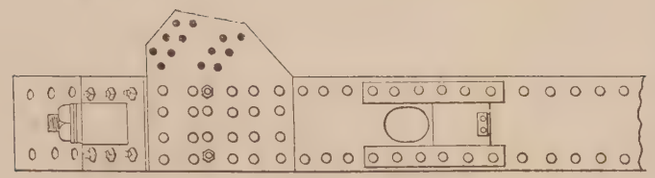
The time set for receiving bids was getting short, and the chances of getting any responsible firm to submit a proposition appeared to be more and more remote, when the Modern Steel Structural Company, of Waukesha, after rejecting the proposition once, reconsidered the matter and agreed to submit a bid somewhat different from that required by the advertisement.

This proposal was the only one, and was accepted by the Council subject to the contract being held valid by the Court. A test suit was brought by the city and carried to the Supreme Court, which rendered a decision holding that, while the city had authority to make a binding contract, the proposition of the Structural Company, being in the nature of a counter proposition to that called for in the advertised specification, could not form the basis of a valid agreement.

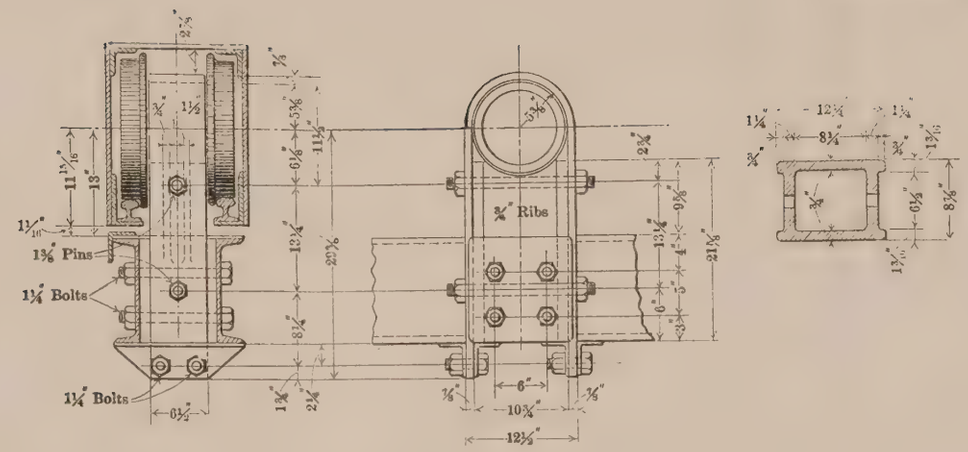
The proposal being thus rejected, the Structural Company was not inclined to renew it, as the terms of payment involved completing the bridge and operating it a month before any payment whatever was to be made on the work.

The contractor was to be held responsible for the design, for the satisfactory execution of the work, and for the operation of the bridge, by a surety bond equal to the full amount of the contract, to operate the structure continuously for 30 days before acceptance, and make good any defects in the machinery developing in the course of the first 6 months' operation. The total cost of the bridge was limited to $100 000 by the enabling act.

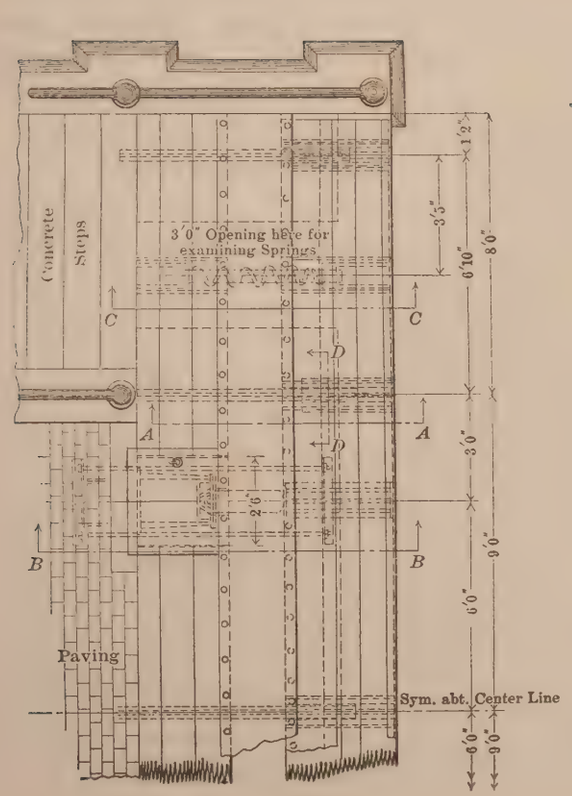
The bridge company and the city committee were so far apart in their views, after a month's correspondence, that the writer concluded he would of necessity have to start over again, and went the rounds of the various manufacturers, personally or by letter, and reported the results to the city bridge committee. In the meantime the Structural Company was gradually receding from its position, while the bridge committee was becoming more disposed to meet it half way; and, just as the writer had about induced another concern to take up the proposition, the committee and the Modern Steel Structural Company came to terms, and the second concern was advised by wire to drop the matter. Accordingly, bids were called for on January 22d, 1904, and only one bid, accompanied by the requisite check, was received, that of the Modern Steel Struc-

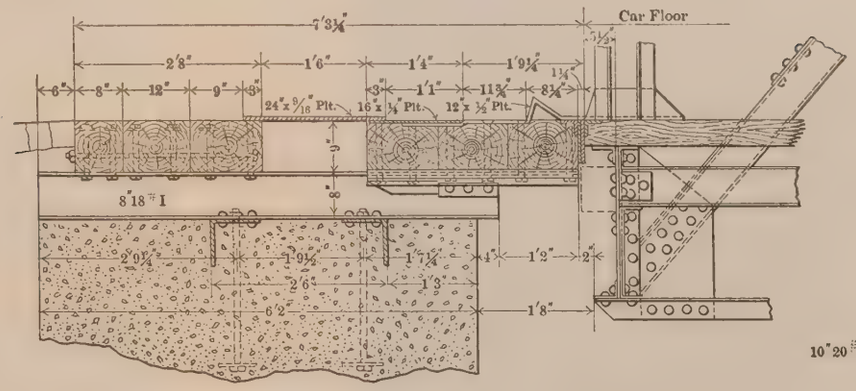


SECTION SHOWING KILGORE'S PNEUMATIC CUSHION, AND CABLE CONNECTIONS AT END OF TRUSS.

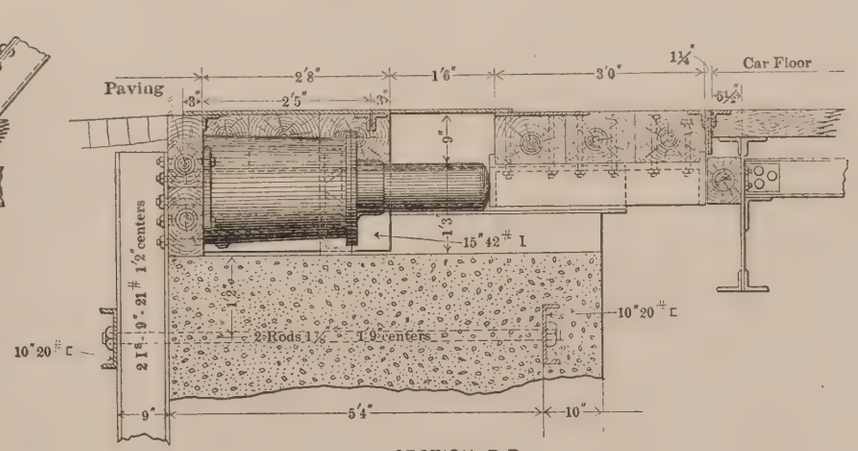


DETAIL OF TRUCK HANGER.

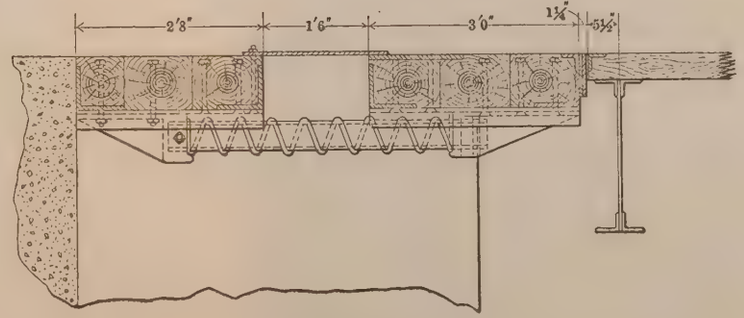




SECTION A-A.

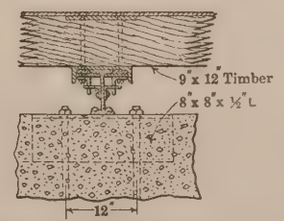


SECTION B-B.



SECTION C-C.

DETAIL OF BUFFER AND PLATFORM.



SECTION D-D OF SECTION A-A.

tural Company, on the writer's plans. This was accepted by the city, and work was promptly taken in hand.

The somewhat peculiar and difficult conditions to be met were as follows: The canal points directly in the teeth of the prevailing winds and most severe storms of the locality, and hence vessels using it require an absolutely unimpeded passageway. Fig. 1, Plate LV, is a view of the canal in a storm, and gives a fair idea of the conditions. Any structure designed to move across the canal should have certainty of action and be under complete control at all times, owing to the large number of vessels arriving and departing; 8 309 vessels, besides tugs and small craft, carrying 6 852 000 tons of freight and 88 800 passengers, having passed in and out during the season of 1901. Any design for spanning the canal permanently must have a clear headroom of 135 ft., as masts 125 ft. high are carried by some of the vessels.

The car is required to clear the side of the canal when in the landing, and, for safety of operation, the method of suspension should be such as to limit to a minimum the lateral swing of the car under the heaviest wind.

The Arnodin patent suspended car transfer is arranged with a submerged cable or chain brought up to the bottom of the car and over a wheel. In the Duluth Bridge the ice and the requisite clear channel for vessels would prevent the use of such a device, and resort to a stiff traveler was the only alternative.

Plate LVI is a general view of the structure. In order to reduce the span of the overhead truss to a minimum, the side of the tower facing the canal is open, forming a huge portal, 130 ft. in height, through which the car runs into the tower, as shown in the photographs. The side posts forming this portal consist each of twin columns, joined at the top, and spread 15 ft. apart at the base, connected by suitable bracing, and anchored down by four 2½-in. bolts. Upon noting the width of the braced post and comparing the height with the width of the post, it will be seen that this ratio is not far from that common in through bridges, while the fact that the posts are given a considerable batter and are well anchored at the base, places it in a more favorable light in considering its efficiency in resisting the lateral force of the wind. The transverse bracing of the rear pair of legs, however, is ample to carry this stress without counting the front portal.

The desirable requirements to be met in the design of the truss are evidently stiffness, ease of erection, neatness of appearance and economic form, combining concentric application of the vertical load to the truss, or uplift of the traveler from wind; application of the lateral force of the wind on the traveler concentrically with the lateral bracing, and a rigid system of sway bracing to enable the four plans of bracing, *viz.*, the two trusses and the top and bottom lateral systems, to act together in resisting the great twisting moment of the traveler caused by the wind.

The double-system, Warren type of stiff-riveted construction throughout, was selected as best filling the above requirements, the bottom chord being of box form, open on the bottom, and carrying two lower rails and two upper rails to provide for uplift.

Sections of the truss towers are shown on the strain sheet, Plate LVI. The loads provided for are noted on this sheet, and it remains only to add that the impact allowance used for live load was as follows: 40% for chord members, 60% for posts and diagonals, and 70% for hangers. The working stresses were those given in the Specification for Railroad Bridges by C. C. Schneider, President, Am. Soc. C. E. The wind pressure allowed for 50 lb. per sq. ft.

The further apart the trusses are placed the less the moment from the traveler affects the truss, but, beyond certain limits, the greater the weight of the bracing; and 34 ft. was decided to be the desirable mean.

The load of the traveler is practically a concentrated one, hence stiff web bracing capable of taking tension or compression is evidently the economic type.

The arrangement of the tracks in the bottom chord secures concentric application of the load, both to the truss and to the bottom lateral system. The chord is prevented from spreading laterally by cast-steel yokes, or truss connections designed to act as yokes, at approximately 6-ft. intervals. From the intersection points of the web members two stiff hangers are dropped to the chord, supporting it at 13-ft. intervals.

While the trusses are designed as simple trusses, they have an additional factor of strength and stiffness due to the rigid connection of both chords to the towers, causing them to act as trusses restrained at the ends.

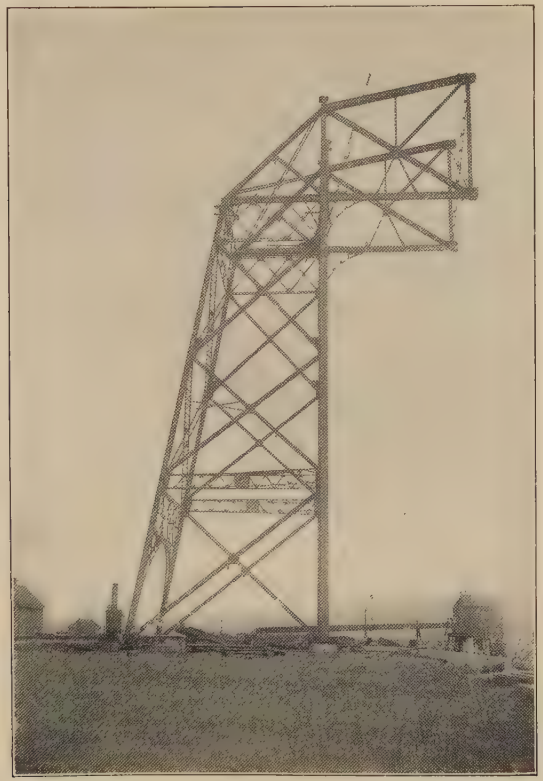


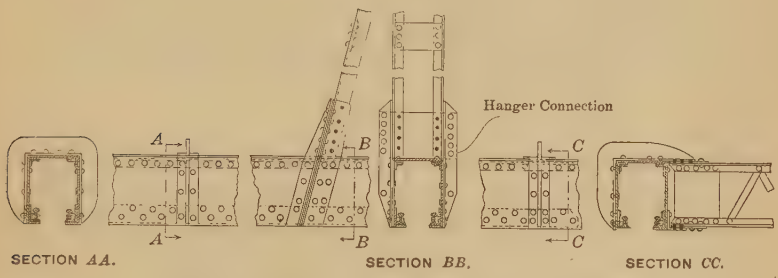
FIG. 1.—DULUTH FERRY BRIDGE, SOUTH TOWER.

FIG. 2.—DULUTH FERRY BRIDGE, NORTH TOWER.

Temperature stresses are provided for by placing one tower on rollers. An unusual roller nest is required under the front legs, as shown in detail in Fig. 3. One pair of rollers extends from the main column to the batter brace, acting as driving rollers, to insure the two nests under the outer and inner booms of the twin columns moving together.

As noted earlier in the paper, a stiff traveler was decided upon, to prevent any lateral sway of the car.

To obviate shock to the traveler in bringing the car into the landing, the car is suspended from the foot of the traveler by short links, with pins at 20-in. centers. Further provision is made at the top by a buffer block on the traveler and a 12 by 16-in. pneumatic cushion placed in each chord. (See Plate LVII.)



STIFFENING YOKES AND CONNECTIONS OF WEB MEMBERS, AND LATERALS TO BOTTOM CHORD OF TRUSS.

FIG. 2.

At the bottom, also, a buffer in the shape of a sliding platform is provided. As the car comes into the landing it latches automatically to the sliding platform by gravity latches, and is brought to rest by two 15-in. Kilgore pneumatic cushions and by coiled springs behind the buffer platform.

The computations for these cushions were made from the following data from saw-mill practice. The saw-mill carriage, with log, etc., weighing about 5 tons, driven by the steam piston at a speed equal to 20 miles per hour, has been brought to rest by a 20-in. cushion with an 18-in. stroke.

By mechanics, the kinetic energy of a moving body varies directly as its weight and the square of its velocity. The resistance of the cushions would vary as the area or square of the diameter;

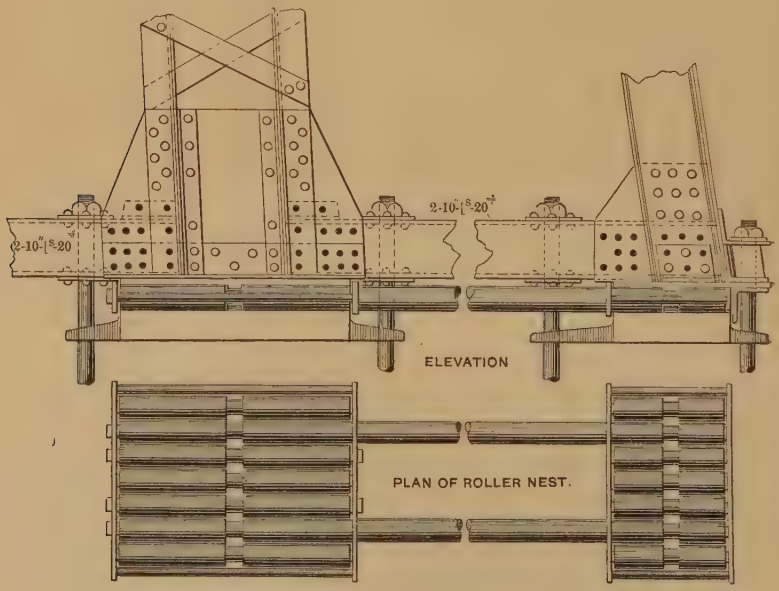
hence, comparing the kinetic energy of the saw-mill carriage and the car, in terms of tons and miles per hour as a unit of speed, with the relative cushion areas (d^2) we have:

For the Saw-Mill Carriage:

Weight, in tons, $\times$ speed, in miles per hour, $= 5 \times (20)^2 =$ 2 000 units for the relative cushion area (d^2) $= 400$.

For the Car:

Weight, 120 tons, $\times$ maximum speed, 4 miles, $= 120 \times (4)^2 =$ 1 920 units of energy to the relative cushion area of $2 \times (12)^2 +$ $2 \times (15)^2 = 738$, or nearly twice the area requisite from the data.



ROLLER EXPANSION BEARINGS UNDER TOWER LEGS

FIG. 3.

From this it would appear that the car might be run into the platform at full speed without the destruction of the cushions, but the jolt it would give the motorman, in being brought to rest in so short a space, would undoubtedly insure the avoidance of such an occurrence a second time. In addition to the cushions, the car must overcome the inertia of the buffer platform, and, as the springs

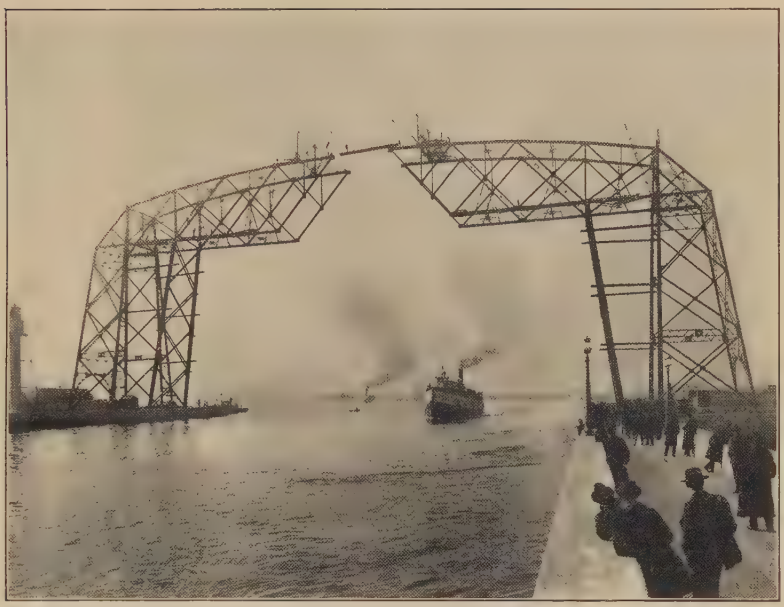


FIG. 1.—DULUTH FERRY BRIDGE. CLOSING THE GAP BETWEEN THE TWO ARMS.

FIG. 2.—DULUTH FERRY BRIDGE FINISHED.

for the recoil in the Kilgore cushions are designed only to push out the piston to its extreme position, without any material external resistance, recoil springs are provided for the platform, and these aid in absorbing the impact of the car, though their primary object is to keep the platform in its extreme position. The traveler presents a novel problem in construction. It embodies an effort to secure sufficient flexibility in the upper frame to distribute the load on the trucks with an approach to uniformity; to be sufficiently limber so that the frame, by flexure, can absorb any impact in landing that may be imparted to it through the links suspending the car, without over-straining the metal, combined with sufficient rigidity for the safe and satisfactory handling of the car.

To accomplish the approximately uniform distribution of load on the trucks, the bottom chord of the top truss of the traveler is connected to the legs by bolts, the bolts passing through holes slotted $\frac{3}{16}$ in.

The form of the traveler leg, as may be seen on Plate LVI, will allow some little bending longitudinally without straining the metal severely, owing to the form.

Transversely, the legs of the traveler are battered inward, and the bracing is made as light as the wind pressure would admit, with the view that should the car be struck by a vessel, through the recklessness of the motorman, the traveler only, and not the truss overhead, would be wrecked. How perfectly the designer has solved the problem, the operation of the structure will shortly prove.

The machinery is very simple, and consists of a rope-drive, the drums being rotated, through suitable gearing, by standard street railway motors. The ropes are 1-in. steel cables, made fast at the extreme end of the chords over the towers, carried in the chords over double-grooved sheaves on the top of the traveler, and down to spirally-grooved drums on the car. The drums are rotated by two General Electric motors (G. E. 67) of 40 h. p. The rotation of the drums winds the rope up on one side and pays it off on the other. Guide sheaves are provided on top of the traveler to carry the rope over the trucks; and, to prevent unreasonable sag of the rope, it is supported at intervals of 44 ft. by automatic swing hangers, shown in Fig. 4. The end of the traveler is made wedge-shaped to engage the roller in the end of the swing hanger, which is merely pushed aside laterally as the traveler passes.

To guard against the possibility of being stalled midway in the channel, through stoppage of the current, two independent sources of supply are provided and two independent trolleys are used. The car is wired in such a way that the motorman need only throw a switch to change from one power supply source to the other.

The motors are fitted with General Electric standard electric brake-shoes and discs, and their B–13 controllers are used.

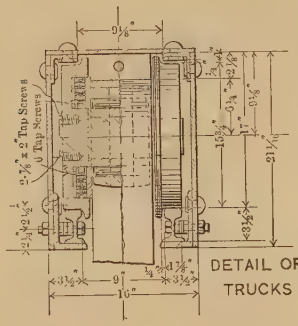
In place of the usual feed wire and trolley, which might be affected by sleet and ice, the conductor designed by the writer was a 4 by 4-in. angle, with the apex at the top, and properly spliced with a square-root angle and copper rivets. The special trolley and conductor were not used by the company, as they were somewhat more expensive than the common trolley and their advantages were not thought to be as important as the writer rated them. The truck wheels and bearing designed are shown in Fig. 4. The Modern Steel Structural Company, however, has modified the thrust bearings, to save expense, and has used a ball bearing in place of short cylinder rolls.

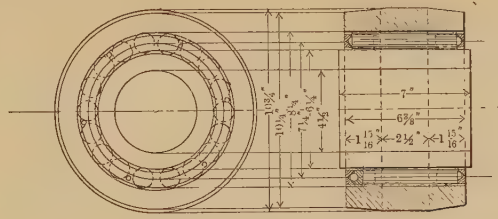
A hand-power drive is also provided, though the expectation is that it will rarely, if ever, be necessary to call it into play, through failure of both sources of power. The car has a roadway 18 ft. wide and 50 ft. long, and is arranged with extra heavy stringers for one line of street railway track.

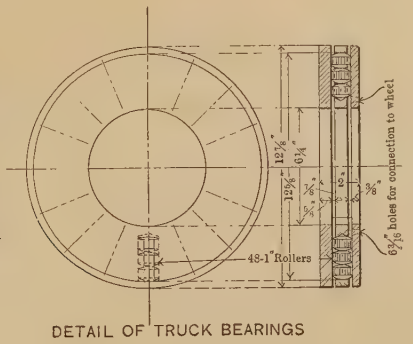
In a structure of this magnitude it seemed proper to finish the car in a manner that would make it unique, from the architectural, as well as from the engineering, standpoint. The contractors took this view at first, and the interior was designed by the writer with mahogany crotch veneer panels and ornamental name-plate panels of composition, copper plated. Owing to the fact that it was necessary to have a truss in the cabin wall next to the roadway, the treatment of that side of the cabin, in this design, differed from the other, in that beveled plate mirrors were used in place of windows, thus the passenger might look down the canal when seated at either side of the cabin.

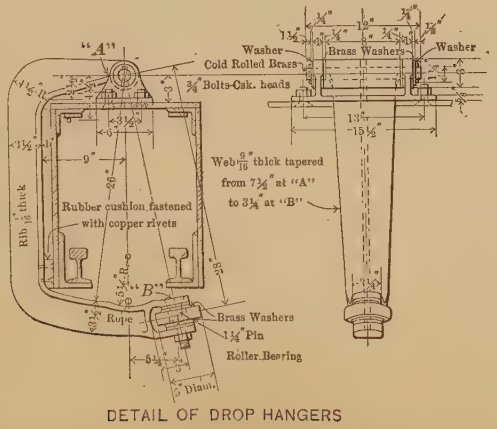
Unfortunately, the slight increase in cost caused the contractors to change their ideas, and the finish adopted by them was disappointing.

The erection of the bridge involved unusual conditions: Any interference with navigation in the summer would not be allowed;


DETAIL OF TRUCKS


DETAIL OF TRUCK BEARINGS


48-1" Rollers
holes for connection to wheel
DETAIL OF TRUCK BEARINGS


"A"
"B"
Washer
Cold Rolled Brass
Brass Washers
Bolts-Csk. heads
Web thick tapered from 7½" at "A" to 3¼" at "B"
Rubber cushion fastened with copper rivets
Rib thick
Rods
Brass Washers
Pin
Roller Bearing
Diam.
DETAIL OF DROP HANGERS

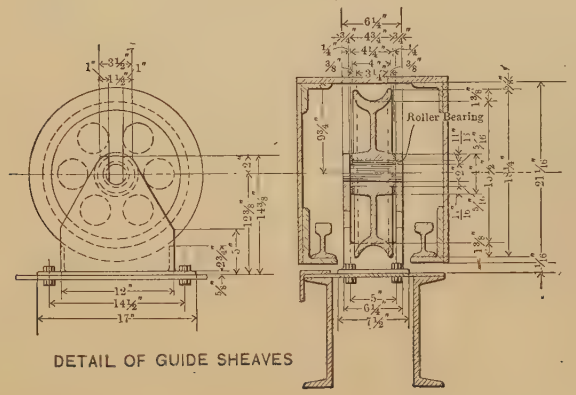

Roller Bearing
DETAIL OF GUIDE SHEAVES

Fig. 4.

in the winter (the closed season of navigation) erection would be very expensive, from the labor standpoint, on account of the extreme cold and the exposed position of the structure, while falsework, if placed in the canal, would be in danger in case the span was not swung before the spring storms and the breaking up of the ice.

For these reasons the writer decided upon a cantilever system of erection, using a single falsework bent, supported by the sea wall on each side of the canal and braced back to the steel towers.

The falsework booms were each made up of three pieces of 12 by 18-in. Oregon fir, with bracing of the Howe type, of two pieces of 10 by 12-in. for the diagonals and two 1½-in. round rods for the ties in each panel.

A steel cap and thrust-block was arranged for the top of this bent, which was bolted rigidly to the bottom chord of the trusses.

Fig. 1, Plate LIX, gives a very fair idea of this falsework, with the traveler used above for handling members as the work was built out. The bottom lateral system served as track supports for carrying members from the derrick at the top of the tower to a portion under the traveler. The lowest section of the towers was erected with gin-poles, the upper sections with a stiff-leg derrick supported by a temporary platform of truss members utilized as falsework. When the top section of the tower was erected this derrick mast and boom were moved up and supported by the 15-in. channel strut connecting the front legs of the tower, and in this position it was utilized to erect the first panel of the truss and then the falsework bents, and to raise material until the span was connected at the center. These derricks then served in taking down the falsework, the derrick on the Duluth side being used in erecting the traveler and car. It will be noted that the end panel of the top laterals was left out until the last, in order to leave room for swinging the boom. In the end panel of the bottom bracing one of the half diagonals was left out until the last, in order to give clearance in raising material.

The traveler and derrick details were worked out by the Structural Company. The traveler was light, and only about an hour was required to move it from one panel to the next one.

For ordinary cantilever erection, the overhanging frame is controlled from two points of support with adjustable wedges; here

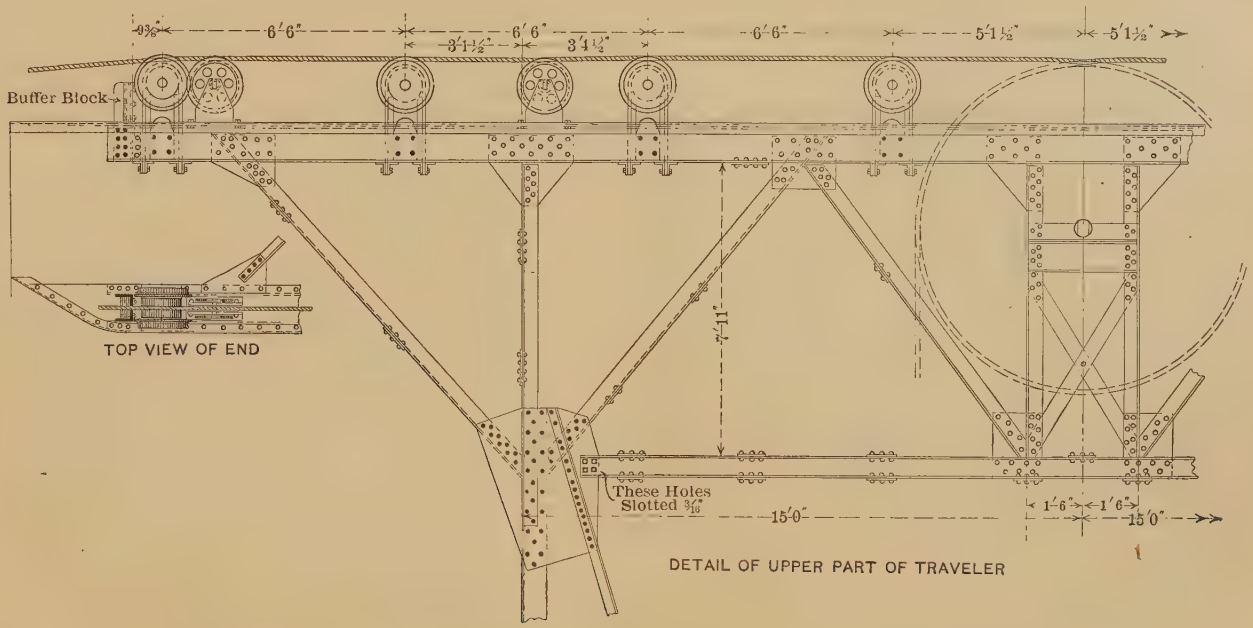

0⅜"
6′6″
3′1½″
6′6″
3′1½″
6′6″
5′1½″
5′1½″
Buffer Block
TOP VIEW OF END
7′11″
These Holes
Slotted ¾″
15′0″
1′6″
1′6″
15′0″
DETAIL OF UPPER PART OF TRAVELER

Fig. 5.

there is the interesting complication of a three-point, or three-level, support from which to adjust the ends in making connection with the two arms. Each of the falsework bents was supported on an adjustable wedge-shoe. The rear legs of the tower were also supported on wedge-shoes.

The survey for the foundations was made by Mr. McGilvray, and reduced to measurements at 60° fahr. This survey was checked by the writer and found to be exact. For this reason, he was somewhat surprised to find the ends of the top chord some 9 in. too close together, on building out the panels on each side of the center, ready to enter the center section of the top chord. A little computation indicated that this could readily be accounted for by the compression of the timber falsework, the points coming to a bearing as they took the load; together with the shrinkage of the timber as it dried out; and the deflection of the tower and the overhanging truss.

As the base for the rear-leg wedge-shoe was placed at too high an elevation to secure the necessary rotation of the end to enter the center section, the roller nests under the rear legs on the Duluth side were run out and the opening widened in this way $10\frac{1}{4}$ in.

In wedging up on the falsework and pulling down on the rear leg the men were advised that the two adjustments must be worked together. Overlooking this, they heaved away on the nuts of the rear bolts until one of the four 2-in. round bolts was given a considerable set, and another was broken. While the front leg anchorage was ample to secure the span with 22-ft. leverage between the center of this and the falsework, the 1-in. driving cables were utilized as back-stays, thus giving additional purchase from the top of this tower only, and the weakened anchorage was supplemented by two $1\frac{1}{2}$-in. round bolts set in the pier and sulphured in.

On the south, or Minnesota Point side, the wedges under the falsework bent had been driven in until there was little, if any, strain on the rear-leg bolts, and the foreman on that side, not clearly comprehending the conditions, was a little dubious about the safety of his end, until he found that the writer intended to be aloft with him while the top chords were placed connecting the two overhanging arms. These were entered without the slightest difficulty. The north side being higher, the diagonals connecting the south top

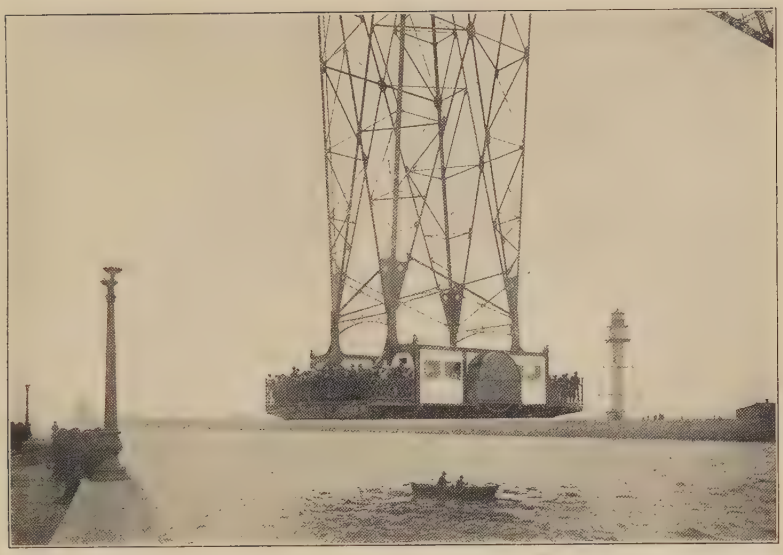


FIG. 1.—CAR AND TRAVELER, DULUTH FERRY BRIDGE.

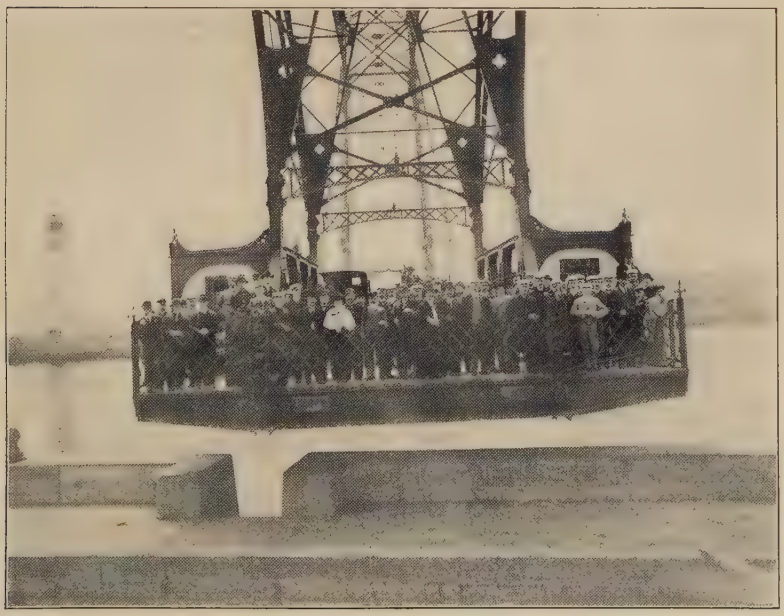


FIG. 2.—END VIEW OF CAR COMING INTO LANDING.

chord with the north bottom chord were readily placed in position and connection made. The condition of the structure was then practically that of an arch hinged at the center. The bottom chord and other members were then placed in position and it remained to make the closing joint of the bottom chords, which was open some 3 in. The writer's idea was to use a toggle of channels to pull this together. The company decided to use the equivalent in a number of strands of ½-in. rope, attaching a fall to the center, which latter method was followed. It took some time to arrange this, and a cold snap coming on increased the difficulties materially, and necessitated further wedging up of the falsework and moving the tower over with jacks.

The bridge has been in continuous operation from March 10th to the middle of July, and reports state that it has transferred daily as many as 34 000 people, besides street traffic.

Aside from the difficulty of bringing the traveler into the landing without shock to the frame, the problem of satisfactory truck bearings gave the designer the most concern. These bearings, for structural reasons, are not large, and have a projected area of shaft of 36 sq. in. The maximum pressure, under the wind load figured, is 950 lb. per sq. in. of projected area, assuming the load to be distributed uniformly on the bearings and between the various trucks, which may not be the case. Under ordinary conditions the working pressure would be about one-third of this amount. These bearings are to take vertical loading or upward thrust under wind, and end thrust in both directions, and the design adopted by the writer was the Ball Bearing Company's type, using eighteen steel rolls, 1 in. in diameter, hardened and ground, and for the thrust bearing forty-eight cylinders, 1 in. in diameter and ¾ in. long, in a cage as shown. The writer recommended that these bearings be tested under a load of 30 tons at 300 rev. per min., that the test be continuous for 6 hours, and that, under this test, the bearings should show no appreciable wear or damage. The thrust bearing was to be tested under the same speed and at half the load.

Unfortunately, the manufacturers did not choose to consider the writer's recommendations in this particular, and modified the design without referring the details for the writer's approval in the following important particulars, as he learned later: First, they

neglected to fasten the wheels to the axle with the set-screws detailed; second, they changed the thrust bearing from forty-eight cylinders, 1 in. in diameter, to ¾-in. balls; third, they and the City Engineer accepted these bearings apparently without testing them, as recommended.

The Structural Company was frankly advised that in trying to save four or five hundred dollars on these bearings they would be very likely to run into an expense of as many thousands, in the stoppage of the operation of the bridge to renew them, and the expense in connection therewith; that, in the writer's judgment, the ball thrust-bearing was capable of sustaining the pressure, but that the balls would be likely to wear grooves in the plates, and that he would not be willing to assume any responsibility therefor.

About the middle of July the attendant noted that the flanges of some of the truck wheels were wearing away rapidly, though the traveler was still running apparently as smoothly as ever. The writer, happening to be in Duluth, visited the Clyde Iron Works, where the bearings were being overhauled, and found the following astonishing conditions:

The Structural Company had economized by leaving out the sixty-four ⅞ by 2-in. tap-bolts which fastened the wheels to the axle; considering, perhaps, that railroad practice in merely forcing the wheel on with hydraulic pressure was a warrant for this procedure in this case, without stopping to consider the difference in conditions, the former with the bearing outside and the pressure forcing the wheel toward the shoulder, while, in this case, the bearing is inside and the pressure tends to force the wheel off.

The natural result was that, in a number of cases, the wheels were merely prevented from coming off by the flanges grinding against the upper and lower rails. The hardened plates for the race-way against the wheel were not fastened, as required in the writer's details, and in some cases this plate rotated about an axis normal to that of the wheel, became jammed in a position out of the vertical plane, and, when the balls were jammed against it, it was bent and scores were ploughed in it by the balls, in one case $\frac{3}{16}$ in. deep. In one instance a ball was crushed, and many of them were flattened ⅛ in.

Evidently, working under such conditions is hardly a fair test of the bearing proper, though the conclusion arrived at from an

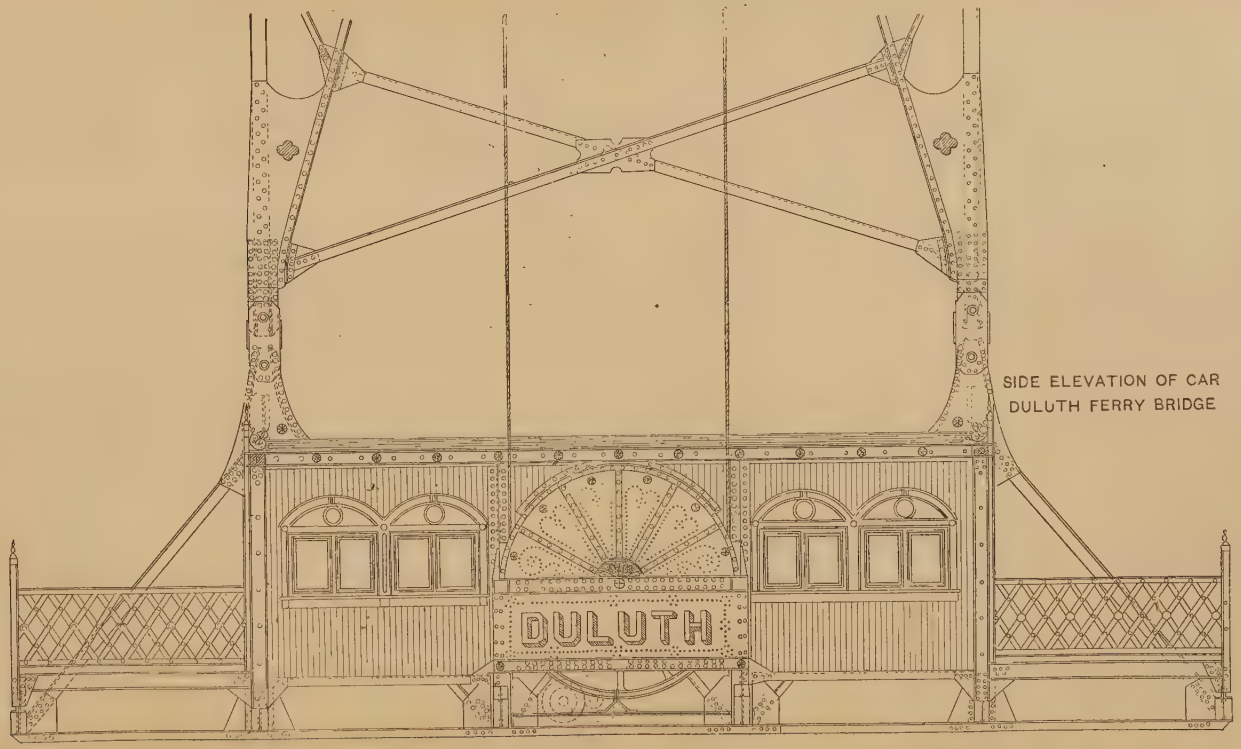


Fig. 6.

examination of both the main and thrust bearings was that neither would have stood the test recommended and which all the manufacturers with whom the writer corresponded claimed could readily be met in these bearings, were the load even several times greater.

While the difficulty developed in the trucks has caused an annoying stoppage of operation, it is a matter which is easily remedied, and is only another of the innumerable examples in the machinery line where the cheapest job in first cost is the dearest in the long run.

The writer is indebted to the United States Engineer Office, Duluth, Clarence Coleman, M. Am. Soc. C. E., Assistant Engineer, for some of the progress photographs. He also desires to acknowledge his indebtedness to the courtesy of his fellow engineers for data and information regarding special points in the preparation of the design; Professor J. J. Flather, M. E., of the Minnesota State University; M. A. Beck, Chief Engineer, Pawling and Harnischfeger Company, Milwaukee; Mr. E. P. Burch, E. E., Street Railway Expert; Mr. C. H. Chalmers, of the Electrical Machinery Company, Minneapolis; and Mr. E. L. Lockwood, of the Union Iron Works, Minneapolis.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1011.

THE WALWORTH SEWER, CLEVELAND, OHIO.*

By Walter Camp Parmley, M. Am. Soc. C. E.

With Discussion by Messrs. C. G. Force, C. E. Gregory, L. J. Le Conte and Walter Camp Parmley.

The Walworth sewer, which, up to the present time, is the largest sewer in the City of Cleveland, has only recently been completed, and, as yet, all the final settlements have not been made. In round figures, the cost has been about $850 000.

A considerable portion of the designing, mainly the determination of sizes, grades, section of sewer required, kind of materials to be used, etc., was done in 1896. The actual construction was in progress from the spring of 1897 to the summer of 1903. In consequence, therefore, the work extended through several political administrations, and came successively under three chief engineers, *viz.*, Mr. M. E. Rawson, from the beginning to April, 1899; James Ritchie, M. Am. Soc. C. E., from April, 1899, to April, 1901; and William J. Carter, M. Am. Soc. C. E., from April, 1901, to the present time. Until April, 1899, C. G. Force, M. Am. Soc. C. E., was Assistant Chief Engineer, and had general charge of all sewer work. During the entire period of designing and constructing, the work was under the direct charge of the writer.

* Presented at the meeting of October 4th, 1905.

Apportionment of Cost.

The estimated cost of the Walworth sewer was $888 188.17. Part of the cost was divided between a frontage tax and an apportionment against the sewer districts benefited, and the remainder was paid by the city at large out of the intercepting-sewer fund, which was created by Act of Legislature, April 16th, 1896. In obtaining the right of way for Walworth Street, a large portion of the expense was assessed upon the adjoining properties, in consequence of which the frontage tax was made a nominal amount in order to comply with the provisions of the law. A frontage tax of 50 cents per linear foot was placed against the abutting property, but this was afterward reduced to 25 cents per linear foot. One-half of the remainder, after deducting this frontage tax from the total estimated cost of the work, was assessed against the intercepting-sewer fund of the city at large, and the other half was distributed among the several sewer districts. The distribution among the districts was based upon the theory that the more valuable properties in the dense portions of the city are benefited in proportion to their valuation. The total assessed valuation of all real and personal property in each sewer district draining into the Walworth sewer was divided by the number of acres in the district; this gave an assessed valuation per acre for each sewer district. The amount of tax levied against each district is the proportion which this assessed valuation per acre, multiplied by the number of acres from each sewer district draining into the Walworth sewer, bears to the sum of the products thus found for all the several districts. It may be objected, to this method of procedure, that ultimately, when the entire area is improved, those portions which now bear the least tax per acre will receive equal benefits with the more heavily taxed acreage. On the other hand, however, it is clear that the more heavily taxed portions will have enjoyed the benefits of the sewer for a considerable number of years before those portions which are now unimproved will receive the increased benefit therefrom. In consequence, therefore, it seems that the indicated method was as equitable to all the properties affected as any that could be devised.

General Description.

Walworth sewer drains approximately 3 000 acres in the southwest part of the city. This area lies upon the table-land west of the

PLATE LXI.
TRANS. AM. SOC. CIV. ENGRS.
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PARMLEY ON
WALWORTH SEWER.

Cuyahoga River, and has an elevation, for the most part, of from 100 to 160 ft. above Cleveland base.* The general slope of the region is toward the northeast, and the surface waters flow in the stream known as Walworth Run. Walworth Run is only one of several streams in deep ravines which have been eroded through the alluvial deposits of the Cuyahoga Valley. Beginning near the west end of the area, the ravine is only slightly depressed below the general surface, but as it extends eastward it deepens rapidly until it becomes a narrow precipitous gulch, cutting its way down nearly to the level of Lake Erie. It is so important a feature in the topography of the city that it forms the boundary between what is locally known as the "West Side" and the "South Side" portions of the city. It is crossed at several places by streets upon embankments, as at Burton and Junction Streets; or by large steel viaducts, as at Rhodes Avenue, Pearl Street and Abbey Street.

In the development of the city the stream had gradually changed from a country brook to a filthy watercourse carrying refuse from numerous slaughter houses, breweries and other industrial plants.

For more than twenty years the stream had been foul, and a menace to the health of the community, and many attempts had been made to obtain a main sewer in this ravine. It was not until 1890, however, that anything tangible was effected. In that year a 60-ft. street, extending along the bottom of the valley, from Scranton Avenue, near the outlet, to the intersection of Gordon and Clark Avenues, was obtained by appropriation at a cost of about $100 000. Various plans for the main sewer were proposed by the engineering department, but it was not until 1896 that it became possible to carry any of these plans into effect. In 1896 an Act was passed by the State Legislature under which it became possible to build the Walworth Run sewer as a part of the main intercepting sewer system of the city. Plans were then perfected, and the necessary municipal legislation was completed. Construction was actually begun on March 27th, 1897.

Plate LXI shows the general location of the water-shed and the various subdivisions of the Walworth Run drainage area, with the main sewers, as already constructed or proposed. Before de-

* The Cleveland base is the elevation of high water in Lake Erie in 1838, or 575.2 ft. above mean tide at New York. The present elevation of Lake Erie is about 2.5 to 3.5 ft. below base.

scribing the construction of the Walworth sewer it is well to consider first some of the peculiarities of the region, and the principles used in locating and designing the sewer.

From the map it will be seen that the largest districts lie on the south side of the run, and the longest lines of drainage lie nearest the outlet. It was found, also, owing to the steep grade upon which the sewer was to be constructed, that the time required for flood waters to reach the upper portions from the tributary sub-area, plus the time required for these flood waters to pass down the main sewer to a given point near the outlet, was about the same as that required for the flood waters to pass down one of the more easterly sewer areas discharging at the same point in the main sewer. In other words, the time interval required for concentrating the flood waters of the entire 3 000 acres is approximately the length of time required to concentrate them from one of the larger down-stream sub-areas, as, for example, the area tributary to Mill Street. The effect, therefore, is to shorten the interval of time required to concentrate the water from the entire 3 000 acres, and, consequently, increase the intensity of rains to be provided for.

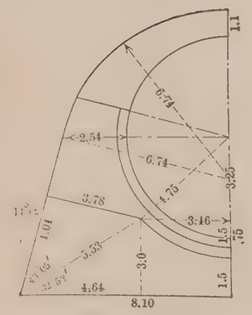
It has been observed, furthermore, that there is a peculiar tendency for rains to follow down this ravine. This peculiarity has been noticed for a number of years by different observers, and the rains occurring over these areas have been particularly intense. The general direction of heavy summer rains is toward the northeast, which fact tends further to increase the volume of flood in a sewer flowing in the same general direction, as was pointed out by Mr. Force.

Inasmuch as the proposed sewer was to occupy the lowest portion of the valley, entirely obliterating the natural channel of the brook, it became necessary to provide an artificial channel capable of carrying the heaviest floods which are likely to occur, even after the territory is completely built up and sewered. Otherwise the volume in excess of the capacity of the sewer would sweep down the valley, carrying everything before it, as well as possibly rupturing or destroying portions of the sewer itself.

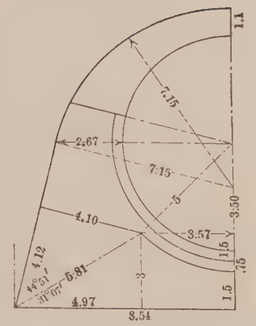
Observation of flood-water heights in a great number of main and lateral sewers in the city gave convincing proof that, as a rule, the sewers, and particularly the main sewers, had been designed for

PLATE LXII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1011.
PARMLEY ON
WALWORTH SEWER.

CROSS-SECTIONS OF WALWORTH SEWER.



8 FT. DIAMETER



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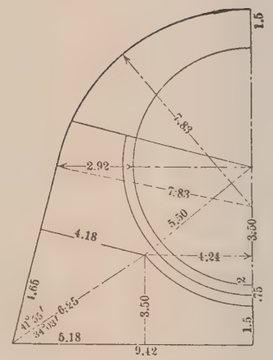

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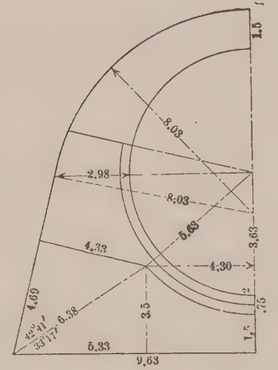
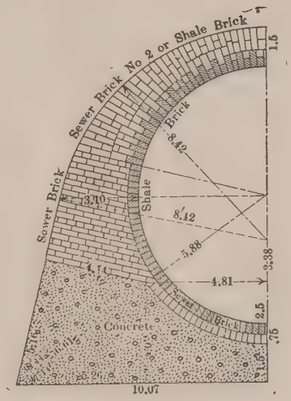


9 FT. 9 IN. DIAMETER



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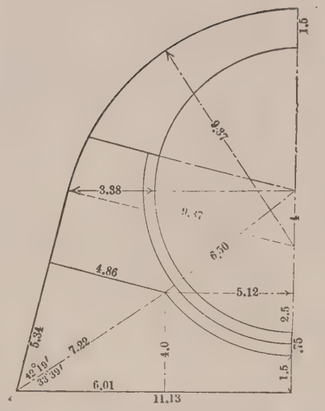
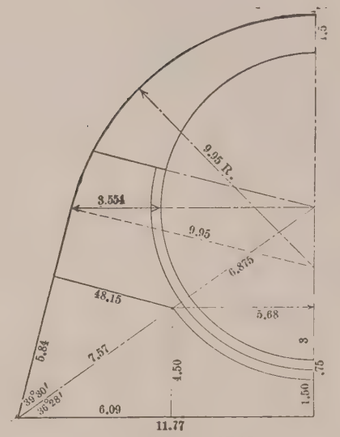


11 FT. 6 IN. DIAMETER



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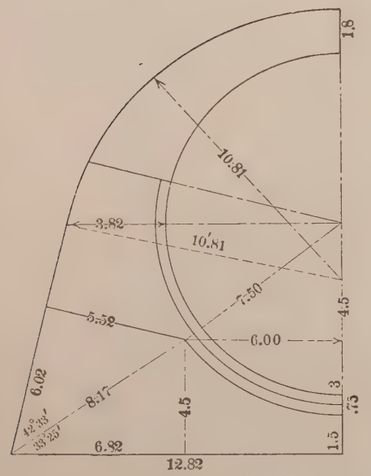

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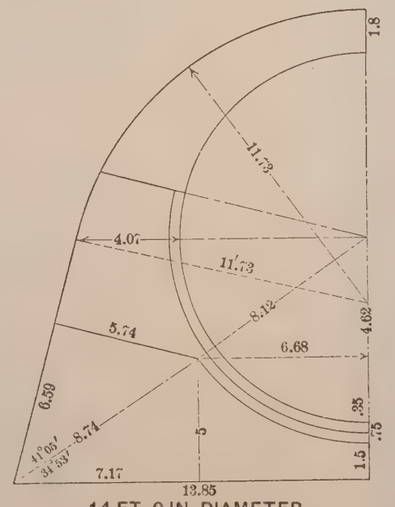
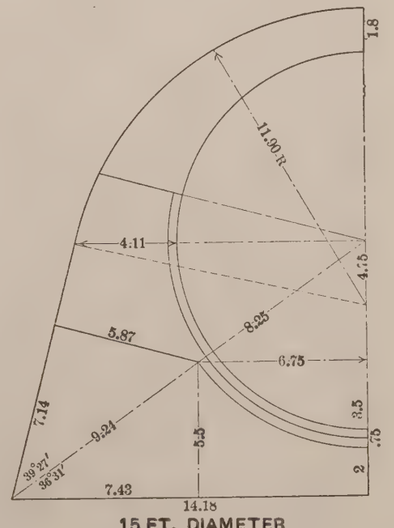
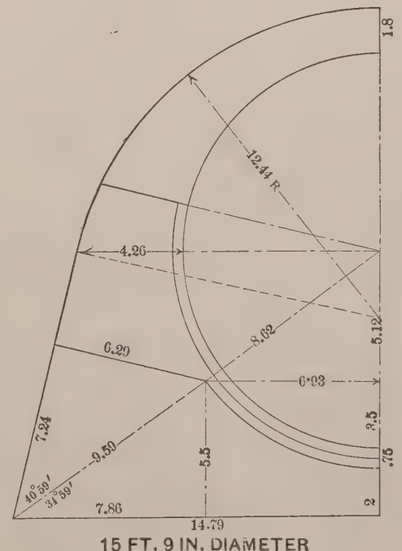


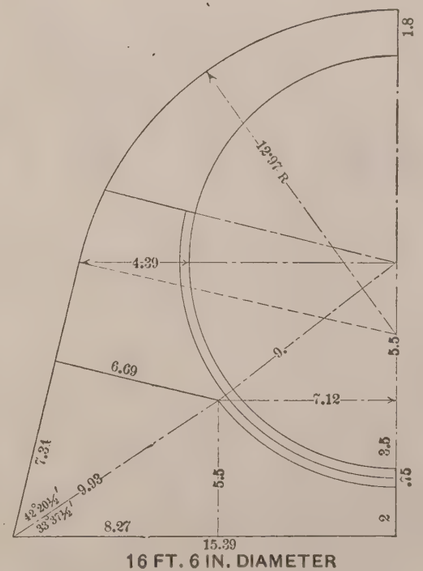
14 FT. 9 IN. DIAMETER



15 FT. DIAMETER



15 FT. 9 IN. DIAMETER



16 FT. 6 IN. DIAMETER

too small a flood allowance. The lateral systems, in general, had been constructed too small to carry off immediately the flood water reaching them, and this had produced two results: First, in backing water into the adjacent cellars; and, second, in holding back the floods so that the main sewers were not required to carry as large a volume of run-off as they would if the sewers of the lateral systems had been of adequate dimensions.

In spite of this holding back tendency of the lateral systems, many of the main sewers ran under such a head that the water rose to the surface of the ground in the manholes, lifting off the iron manhole covers.

A careful study of these conditions, in connection with the rain records, as far as they were available at that time, was made by the writer. The results of this investigation, together with some observations of the surcharge of sewers, have been discussed in another paper* and will not be repeated here, but, for convenience, some of the results regarding the provision to be made for flood water may here be stated.

Inasmuch as most of the sewers which had not been designed by rule-of-thumb had been proportioned by using the Burkli-Ziegler formula, with various values for the constants, it became evident that, for large areas, a greater run-off, in proportion to the smaller areas, should be provided for than was given by that formula.

The general formula adopted was:

$$Q = f \sqrt[4]{s}\, R\, A^{\frac{3}{4}},$$

in which

Q = volume of run-off, in cubic feet per second;

f = factor of run-off, depending upon the impervious nature of the soil and surface;

R = an intensity of rainfall for a rain lasting about 8 or 10 min.;

s = the general slope or fall of the sewers, in feet per 1 000 ft.;

A = the number of acres drained.

It is at once evident that raising the exponent of A gives the desired increase in volume of run-off for large areas in proportion to that given for small areas. The equation is rapidly solved for any case, and thus becomes a practical working formula.

* "Rainfall and Run-off," by W. C. Parmley, *Journal*, Association of Engineering Societies, March, 1898.

To illustrate the proportionate increase in volume of run-off by various formulas, the comparisons in Table 1 may be made.

TABLE 1.—Comparison of Run-off Formulas.

(1) Number of acres. A	(2) Burkli-Ziegler. $A^{\frac{3}{4}}$	(3) McMath. $A^{\frac{4}{5}}$	(4) $A^{\frac{5}{6}}$	(5) Column 4 / Column 2	(6) Column 4 / Column 3
10	5.6	6.3	6.8	1.21	1.08
100	31.6	39.8	46.4	1.47	1.17
1 000	177.8	251.2	316.2	1.78	1.26
3 000	405.4	604.9	790.0	1.95	1.31

Columns 5 and 6 show the relative volumes of run-off for given areas by various formulas, providing the constants are the same in each case. There is a slight error in Column 6, as the McMath formula uses the fifth root of the slope instead of the fourth. Had this correction been made, however, the values given in Column 6 for ordinary slopes would have been only slightly increased. A value of $R = 4$ was finally adopted as a basis of calculation for the Walworth sewer, in order to provide for the most violent storms, and, also, for the further danger caused by the prevailing direction of the storms. The other constants were chosen by judgment to represent the conditions in each sub-area when the same shall have become fully developed.

For brevity, let $K = f R \sqrt[4]{s}$, then $Q = K A^{\frac{5}{6}}$.

In Table 2 the data pertaining to each sub-area tributary to the Walworth sewer are given, together with the accumulated quantities of flood water below each sub-main, by adding together the volumes for the several sub-areas.

In Column 2 is given the number of feet from the remotest lateral to the point of discharge into Walworth sewer. Column 3 gives the number of minutes required for the water to flow in the sewer, over the distance, l, plus 8 min., assumed as the time required for the water to concentrate on the surface and reach the sewer. Column 4 gives the estimated number of minutes for the water to reach a given point in Walworth sewer from the remotest lateral tributary to that point. The remaining columns are self-explanatory.

TABLE 2.—Data and Volume of Flood Flow for Walworth Sewer, Cleveland, Ohio.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Sub-main.	Length, in feet. l	Time, in minutes. t	Time, in minutes. T	Area. in acres.	Total area, in acres.	f	s	K	$Q = K A^{\frac{5}{6}}$	Total cubic feet per second. Q
									cu. ft.	
Gordon, N.	1 600	12	21	12	12	0.62½	5	3.76	30	30
Gordon, S.	5 800	21	21	123	135	0.62½	10	4.44	245	275
Clark, W.	6 600	19	21	350	485	0.62½	7	4.08	540	815
Alum, N.	1 300	11	22	8	493	0.62½	6	3.92	22	837
Alum, S.	4 800	21	22	58	546	0.62½	10	4.44	121	958
Purdy.	1 300	11	22	9	555	0.62½	6	3.92	25	983
Waverly.	1 300	11	23	9	564	0.62½	6	3.92	25	1 008
Guernsey	700	10	24	8	572	0.62½	6	3.92	22	1 030
Swiss, N	800	11	24	75	647	0.62½	6	3.92	144	1 174
Swiss, S.	600	10	24	6	653	0.62½	8	4.20	19	1 193
Junction, N.	2 900	16	25	71	724	0.62½	4	3.52	124	1 317
Junction, S.	7 200	25	25	187	911	0.62½	7	4.08	320	1 637
Burton, N	2 600	17	27	57	968	0.75	4	4.24	123	1 760
Burton, S.	9 800	32	32	466	1 434	0.62½	5	3.76	630	2 390
Pollock	1 200	12	.....	19	1 453	0.62½	5	3.76	44	2 434
Mill, N.	700	10	35½	42	1 495	0.75	4	4.24	96	2 530
Mill, S.	13 900	45	45	797	2 292	0.62½	3	3.28	858	3 388
Pearl, N.	4 100	19	45½	154	2 446	0.75	4	4.24	282	3 670
Pearl, S.	2 400	15	45½	58	2 504	0.75	3	3.96	117	3 787
Brevier, N.	Pearl St. from the north enters at Brevier St.									
Brevier, S.	600	9	46	4	2 508	0.62½	15	4.92	14	3 801
Kenilworth.	11 150	39	.....	416	2 924	0.62½	3	3.28	500	4 301
Scranton.	2 000	10	.....	26	2 950	0.75	40	7.56	115	4 416
Cliff.	3 400	19	48	32	2 982	0.62½	3	3.28	59	4 475

By comparing Columns 3 and 4 of Table 2, it will be seen that the time required to concentrate the water in Walworth sewer at Gordon Avenue is about 21 min., *i. e.*, the time required to concentrate the water from the remotest lateral tributary at that point. The three branches joining to form Walworth sewer at this point are Gordon Avenue from the north, Clark Avenue from the west and Gordon Avenue from the south. Evidently, it would be wrong to apply a rain formula directly to the sum of these three areas, since a rain of 21 min. duration would cause the Gordon Avenue sewer from the north to discharge the run-off from its entire area for 9 min., and the Clark Avenue sewer to discharge its entire run-off for 2 min. before that from the entire area tributary to Gordon Avenue from the south would arrive at the junction of the three sewers.

In a case of this kind it is necessary to apply the rain formula to each area separately and add together the three volumes thus obtained, for the reason that it is quite possible that the most intense portion of the rain may occur near the end of the 21-min. period, in which case all the short-time areas may be affected by the increased intensity, whereas only a portion of the long-time area could be thus affected. The discharges, calculated separately for the three areas, are added and give 815 cu. ft. per sec. as the necessary capacity of the Walworth sewer at the upper end.

In like manner, proceeding down the line of the main sewer, where the velocity is very great and the time of flow correspondingly short, the main sewer may be required to carry, in addition to what is already flowing therein, the contribution under short intense rain conditions from some lateral. Thus, at Alum Street, at the end of 22 min., the sewer may be required to carry the run-off from an intense rain for the immediately preceding 11 min. required to give the full discharge from the 8 acres tributary from the north. At the same time the discharge from the 53 acres coming in at Alum Street from the south will meet the volume coming down the main sewer.

The fact that the prevailing direction of the heavy rains is the same as the direction of flow in the main sewer makes this method of treatment the more necessary, as already mentioned.

The conditions at Mill Street, where the largest sub-main contributes its flow to the main sewer, need special mention. Here the time element of the sub-area is apparently 10 min. greater than that of the main sewer at the point of junction. This appears to be a possible exception to the method adopted, and it would appear that the flood volume would be receding when the maximum discharge of the Mill Street sewer fro mthe south occurred; but it will be observed that the area tributary to Walworth sewer above Mill Street is about 1 450 acres. Now, the flood wave from such an area occupies an appreciable time in passing a given point. Furthermore, it is impossible to determine the exact time elements of the several areas, in fact the time will vary with different rains. Therefore, in this case it would not be safe to assume that the flow in Walworth sewer at Mill Street was appreciably less than its maximum at the time Mill Street discharges its maximum flood. The additive

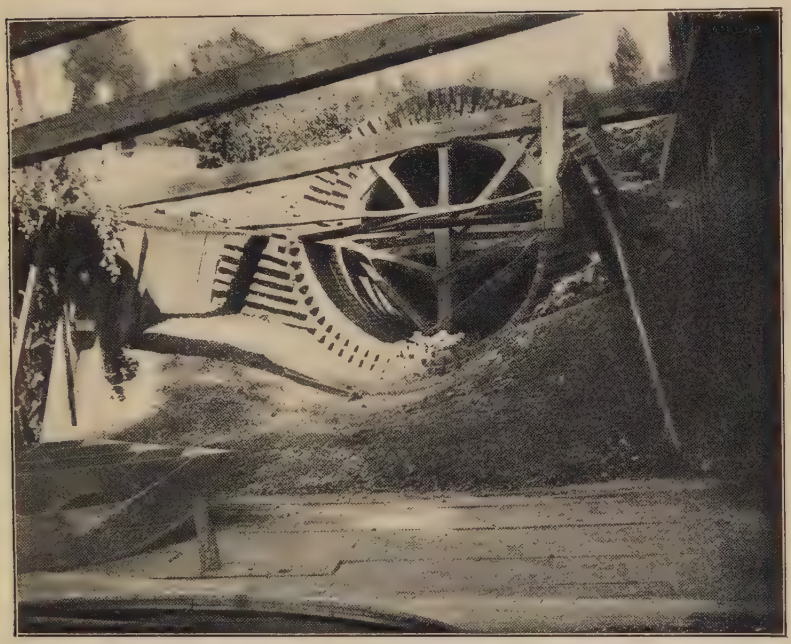


FIG. 1.—SEWER AT GUERNSEY ST., 9 FT. 9 IN. IN DIAMETER.
INVERT FORM IN LEFT FOREGROUND.

FIG. 2.—WALWORTH OUTLET. GENERAL VIEW OF COFFER-DAM, AND VIEW LOOKING UP CUYAHOGA RIVER.

method, therefore, is observed here, the same as in the more obvious cases.

Each sub-area, therefore, is treated independently, and the volume assumed to be flowing in the sewer at any given point is equal to the sum of the separate volumes obtained from all the tributary sub-areas above that point.

While discussing the subject of run-off, it may be well to test the correctness of the method of treatment which was followed in 1896 in fixing the sizes of the various portions of this sewer.

At a later date, in connection with another subject, the writer made a more exhaustive study of the rain records of the city than was possible at the time the sizes of the Walworth sewer were fixed. The rain records of the United States Weather Station at Cleveland for 10 years—1890 to 1899, inclusive—were analyzed, as well as the records of an automatic rain gauge, at St. Ignatius College, for the years 1897 to 1899, inclusive. Records from three automatic gauges installed by the writer during the spring of 1900 were also studied.

In general, it has been found that the Government records, for the violent rains, have shown a less intensity than recorded by the gauges in other parts of the city. This is caused, possibly, by the fact that the Government gauge is located near the lake front and upon the roof of a building some 200 ft. above the ground. At this altitude, a gauge will show less precipitation than if located nearer the ground. It is possible, also, that the violent wind which usually accompanies such storms strikes against the windward face of the building, and causes an upward blast of air which carries a portion of the rain over and beyond the gauge. In using the Government records for this city, therefore, it must be assumed that, as a rule, they under-estimate the intensity of the more violent storms.

Fig. 1 shows graphically a probable frequency, for any given year, for rains of various intensities, as based upon the Government records for the 10 years, 1890 to 1899, inclusive. It is produced by platting all the rains during this period. The probable number occurring in any given year is shown on the left, for the durations indicated at the bottom. Thus, for example, two rains of 1 in. intensity and lasting 25 min. are likely to occur in any given year; or, again, the probability of the 3-in. rate lasting 20 min. is about

0.2, or about once in 5 years. In similar manner, the frequency for rains of any other intensities and durations may easily be obtained. Applying the observed frequency of rains, as determined by Fig. 1, to the problem of run-off for Walworth sewer, the following comparisons with the volume of run-off determined by the formula used in 1896 may be made. Taking first the area tributary to Walworth sewer and Gordon Avenue from the north, the time element is 12 min. Since it is necessary, as before stated, for the Walworth sewer to carry the flood from the most intense rains, Fig. 1 shows that about once in 2 years a rain lasting 12 min. and of at least 3-in. intensity is likely to occur. For the Gordon Avenue tributary from the south, the time element is 21 min., for 123 acres, and for the Clark Avenue tributary from the west the time element is 19 min. for 350 acres. Again, the diagram, Fig. 1, shows that once in 5 years there may be a 3-in. rate lasting 20 min. The sewer, therefore, should have a carrying capacity at least equal to the run-off produced by such a rain. The factor of run-off assumed for this area is $62\frac{1}{2}\%$, therefore the discharge is $0.625 \times 3 = 1.875$ cu. ft. per sec. per acre, and, for the 485 acres tributary, the total discharge would be 909 cu. ft. per sec. The volume provided for, as shown by Table 2, being only 815 cu. ft. per sec., there is, in the carrying capacity, an apparent deficiency of 94 cu. ft. per sec. Had the more correct method been used, of applying more intense rains to the area tributary to Gordon Avenue from the north than that applied to the other two areas, the deficiency would have been slightly increased.

Testing the capacity of the sewer below Junction Street, where the time element is 25 min., Fig. 1 shows that the 3-in. rate is likely to occur about once in 10 years. As it would not be permissible to have the sewer surcharged as often as once in 10 years, it must carry the volume of run-off from such a rain, and since a 3-in. rain intensity was used for shorter intervals than 25 min., the rain rate may be applied directly to the total area tributary at this point. This area is 911 acres, and the factor of run-off is 0.625. The volume of run-off, therefore, is 1.875 cu. ft. per sec. per acre, as before stated, or a total of 1 708 cu. ft. per sec. The deficiency in Walworth sewer as designed, therefore, is 71 cu. ft. per sec. Another test may be made, at a point below Mill Street, where the total area tributary is 2 292 acres and the time element is about 45 min.

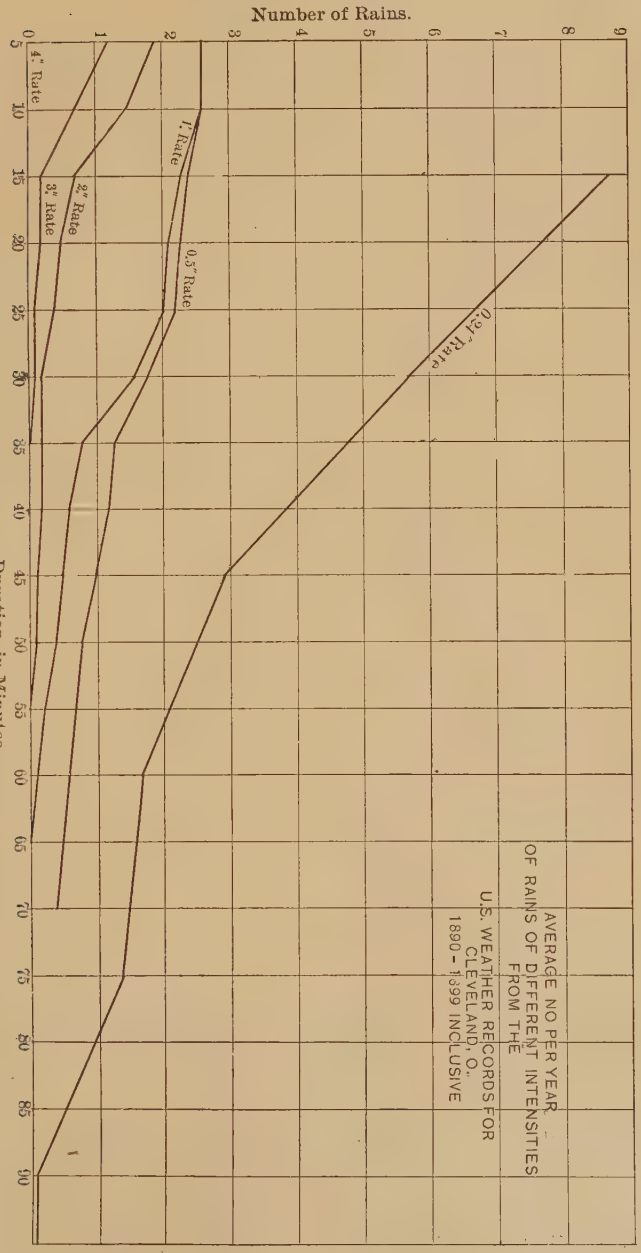

Number of Rains.
0 1 2 3 4 5 6 7 8 9
0.21" Rate
0.5" Rate
1" Rate
2" Rate
3" Rate
4" Rate
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
Duration, in Minutes.
AVERAGE NO PER YEAR OF RAINS OF DIFFERENT INTENSITIES FROM THE U.S. WEATHER RECORDS FOR CLEVELAND, O. 1890 - 1899 INCLUSIVE

Fig. 1.

Fig. 1 shows that a rain of 2 in. intensity and lasting 45 min. is likely to occur about once in 10 years. Since it is observed by the diagram that 3-in. rates lasting 30 min. are also likely to occur, a 3-in. rate should be applied to all the areas having a time element of 30 min. or less, and the 2-in. rate on the Mill Street area, which has a time element of 45 min. As a factor of run-off, 0.625 may be used, as only two small areas have a factor of 0.75. The area tributary to a 3-in. rate, therefore, is 1 495 acres, and the area tributary to a 2-in. rate is 797 acres. The total volume of run-off to be provided for in the sewer is,

$$(1\,495 \times 0.625 \times 3) + (797 \times 0.625 \times 2) = 3\,799 \text{ cu. ft.},$$

or an apparent deficiency of 411 cu. ft. in carrying capacity. However, as this method of treatment may be considered unreasonable, the 2-in. rate may be applied to the total area of 2 292 acres, in which case the total run-off is 2 865 cu. ft. per sec., or an excess of 523 cu. ft. in carrying capacity. The actual carrying capacity of the sewer is about intermediate between these two extreme determinations.

Again, testing the capacity of the sewer at Cliff Street, where the total area is 2 982 acres and the time element for the whole area is about 48 min., leads to the following calculations. The prevailing factor of run-off, without serious error, may still be considered as 0.625. Taking the total area, as shown by Table 2, which has a time element of about 30 min., and applying to it a 3-in. rate, and for all the area having a time element exceeding 35 min. applying a 2-in. rate, gives 4 831 cu. ft. per sec. as the total run-off, or, as shown by Table 2, a deficiency of 356 cu. ft. per sec. in carrying capacity. If the 2-in. intensity is applied over the entire area of 2 982 acres the total run-off is 3 727 cu. ft., or an excess of 748 cu. ft. per sec. in carrying capacity.

It is possible that neither of the two methods just pursued is entirely correct. The probability, however, is that the method giving the larger discharge is the nearer correct, since the time required for the flood volume to pass through 13 300 ft. of sewer from Gordon Avenue to Cliff Street is only 13 min., which makes it possible for the rain affecting one of the smaller sub-areas to be discharged into the main sewer at the time of maximum high water produced by the areas tributary and lying farther up stream.

Some have contended that the sizes determined for the Walworth sewer were excessive, but, in the light of the rain records for the decade covered by Fig. 1, taken in connection with the fact that the sewer must carry all the flood waters ever occurring in the valley, or subject the property interests of the valley and possibly the structure of the sewer itself to great injury or destruction, and in view of the fact that the records under-estimate the frequency of the most intense rains, as already shown, and the further fact that rains greater than those occurring in any given decade are likely to occur every 20 to 40 years, it seems apparent that these objections are not well sustained.

The inclination of Walworth sewer, from 0.3 to 0.8%, is such that the velocity of flow is from 17 to 21 ft. per sec., and in order to withstand the force of the water it was necessary to have a most substantial construction. It was thought possible to build a sewer of sufficient stability and to line it with material having sufficient abrasion resistance to carry the water safely upon these grades. If a less maximum velocity of flow were determined upon, it would be necessary to decrease the grade of the sewer and to increase correspondingly its diameter and cost. It was believed by the engineers of the work that a special burned shale brick, laid in Portland-cement mortar of the best quality, would furnish a surface resistance sufficient to withstand a 20-ft. velocity. The sewer, therefore, is designed to have approximately the same inclination as that of Walworth Valley.

The increased carrying capacity due to a smooth lining is sufficient to warrant the care required to obtain it. The coefficient of friction, n, in Kutter's formula, 0.015, applies to brick masonry of fairly good quality. It was believed that it would easily be possible to give the interior of the sewer such a smoothness that the coefficient of friction would not be greater than about 0.014, and this coefficient, therefore, was adopted in the calculations. The sewers are assumed to have their maximum capacity when nine-tenths full, which, for practical purposes, is sufficiently near the theoretical maximum depth. The grade line is calculated to be a water-plane grade for a sewer nine-tenths full, and the inverts of sewers of the larger sizes are depressed sufficiently below the inverts of the smaller connecting sizes to give a continuous water-plane grade at the nine-tenths point for each sewer.

The depression in the grade of the invert is made in a funnel-shaped section, about 30 ft. long, connecting the two sewers. The drop in any given case, therefore, is nine-tenths of the difference in diameters of the two sewers, and the invert grade of the sewer is the same as the water-plane grade for the sewer flowing nine-tenths full. The water-plane grade for any less depth is also the same, except for a short distance at the up-stream and down-stream ends of a given section. The various diameters were determined by using the volume given in Column 11 of Table 2, with a friction coefficient of 0.014.

The diameter of the sewer being determined, its external form and the thickness of the masonry would depend upon the topography of the valley and the character of the material in which the excavation was to be made. It was apparent that, if Walworth Street were opened as a public highway, it would be necessary to establish the street grade several feet above the extreme bottom of the valley in order to make proper connections with intersecting streets. It was further desirable to give the street such an elevation that the basements of business houses fronting thereon would not have to be excavated below the bottom of the present valley. In general, therefore, the grade of Walworth Street will be practically on a level with the first floor of buildings fronting thereon. As a consequence, the sewer is constructed in a shallow cutting, making the invert or flow line only deep enough to drain the lowest part of the depression. For much of its length, therefore, at least one-half the diameter of the sewer is above the natural surface of the ground. This fact, together with its great size and the yielding character of the clay upon which the masonry rests, made it necessary to have a substantial reinforced section.

The greater part of the sewer foundation rests upon a heavy bed of plastic blue clay. Two direct tests were made to determine its supporting power, one being made near Cliff Street and the other in Walworth Street west of Scranton Avenue. In each case a pit was sunk to the desired depth. A 12 by 12-in. timber, sufficiently long to reach several feet above the level of the surface, was encased in plank boxing fitting closely about the timber, but not so close as to cause the post to bind in the casing. Each post and its boxing was set vertically in the pit, with the squared

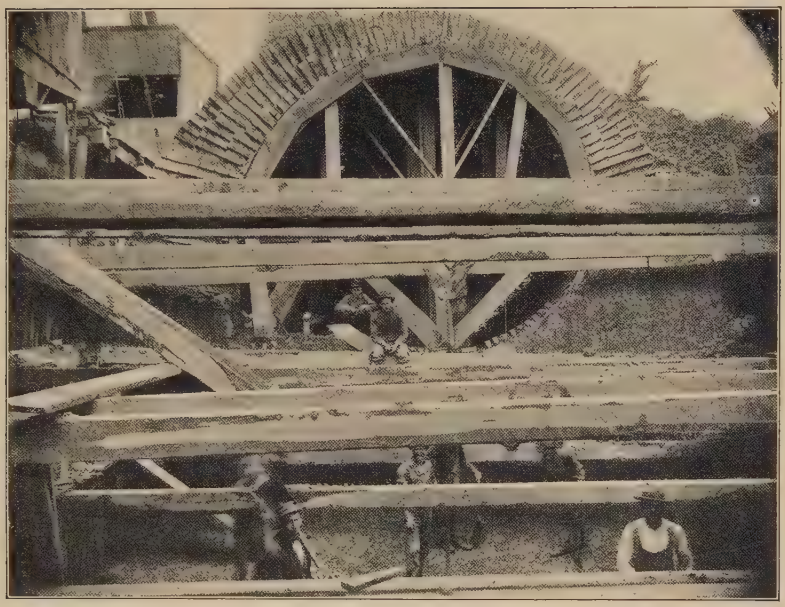


FIG. 1.—LOOKING DOWN STREAM INTO UPPER END OF OVERFLOW CHAMBER, SHOWING EXTRA THICKNESS OF CONCRETE FOUNDATIONS, ETC.

FIG. 2.—UP-STREAM VIEW OF CENTERING REQUIRED FOR OVERFLOW CHAMBER. SEWAGE CHANNEL ON THE LEFT, AND STORM-WATER OVERFLOW CHANNEL ON THE RIGHT.

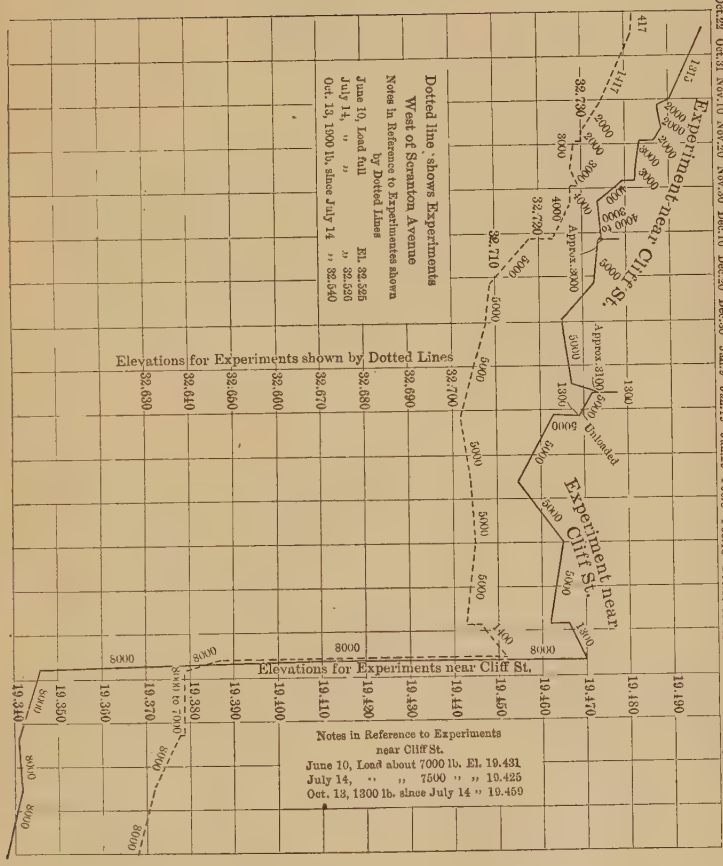

TESTS OF BEARING POWER OF EARTH FOUNDATIONS FOR WALWORTH SEWER IN 1896-1897, SHOWING SETTLEMENT UNDER DIFFERENT LOADS ON 1 SQ. FT. OF SURFACE
Oct. 22 Oct. 31 Nov. 10 Nov. 20 Nov. 30 Dec. 10 Dec. 20 Dec. 30 Jan. 9 Jan. 19 Jan. 29 Feb. 8 Feb. 18 Feb. 28 Mar. 10 Mar. 20 Mar. 30 Apr. 9 Apr. 19 Apr. 29
Experiment near Cliff St.
Experiment near Cliff St.
Dotted line shows Experiments West of Scranton Avenue
Notes in Reference to Experiments shown by Dotted Lines
June 10, Load full El. 32.525
July 14, " " " 32.520
Oct. 13, 1000 lb. since July 14 " 32.540
Elevations for Experiments shown by Dotted Lines
Elevations for Experiments near Cliff St.
Notes in Reference to Experiments near Cliff St.
June 10, Load about 7000 lb. El. 19.431
July 14, " " 7500 " " 19.425
Oct. 13, 1300 lb. since July 14 " 19.459

Fig. 2.

bottom of the post, resting upon the soil, the resistance of which was to be determined. The earth was then filled in around the casing until the original surface of the ground was restored. The post was allowed to stand without loading until the ground had had some time to settle. Cross-arms were spiked to the upper end of each post and each was then loaded with different quantities of pig iron and left standing for various periods, observations of the sinking of the post being made from fixed bench-marks with a wye-level. The behavior of these posts, under various loads per square foot and by various changes from light to heavy loads, is shown by the full and dotted lines in the diagram, Fig. 2.

The general conclusion derived from these tests, taken with the unit loads upon similar soils supporting various bridges and viaducts in the vicinity, indicated that not more than 2 tons per sq. ft. could be applied to the clay without danger of serious settlement.

Various horse-shoe-shaped sewers with sewage channels in the central portion of the bottom were considered, and, while it was found that some money could be saved by adopting such a section for the larger sizes, it was deemed expedient to build a circular sewer on account of the excessively filthy nature of the flow and the danger of filth stranding upon the bottom.

Because much of the sewer was to be above the natural surface of the valley, and also because the surface soil had very little resisting power, only active earth pressure or earth thrust was considered in computing the lines of resistance in the several arches. For the most part, the grade of the finished street will be about 3 ft. above the crown of the sewer, and, since greater depth in most cases will increase rather than decrease the stability of the structure, the arches were calculated for earth 3 ft. above the crown of the sewer. The vertical pressure of the earth was assumed to be 100 lb. per cu. ft., and the horizontal active earth pressure equivalent to that of a fluid weighing 16 lb. per cu. ft. This value is about one-half of the theoretical horizontal earth pressure for earth having a natural slope of 1½ horizontal to 1 vertical, but experience has shown that the horizontal thrust of freshly made fill is not greater, and is often less, than one-half of the ultimate theoretical thrust. The masonry of the sewer would be subjected to the vertical loading of earth before it had acquired its final horizontal pressure, therefore these

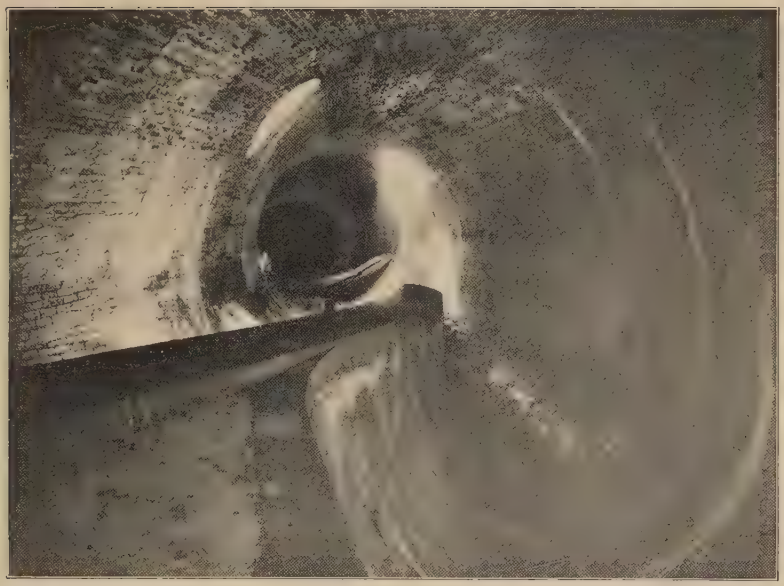


FIG. 1.—UP-STREAM VIEW IN OVERFLOW-CHAMBER ; 14-FT. 9-IN. SEWER IN THE DISTANCE. SEWAGE CHANNEL AT THE LEFT AND OVERFLOW-CHANNEL AT THE RIGHT.

FIG. 2.—DOWN-STREAM VIEW IN OVERFLOW-CHAMBER ; SEWAGE CHANNEL ON THE RIGHT AND 13-FT. 6-IN. STORM-WATER OVERFLOW SEWER IN THE DISTANCE AT THE LEFT.

values were taken to provide for the worst possible conditions. The line of resistance in sewers varying in diameter from 6 ft. to 16 ft. 9 in. was calculated for arches of various depths of key and horizontal thickness of arch. Calculations were made for two different cases, one for the sewer nine-tenths full of water, the other for the sewer empty. Several cases were calculated, in order to determine the thickness of arch required to keep the line of resistance inside the middle third for both cases. The depth of key was determined somewhat arbitrarily, yet was made to conform in general with actual arches and with the usual rules. Inasmuch as it is probable that not many years will elapse before a street car line will be constructed along Walworth Street, in order to provide against the vibrations in such a case and its destructive effect upon masonry near the surface, one ring of brickwork was added to the depth of key as above indicated. From the numerous cases in which the strains were actually calculated, the following simple empirical formula was obtained for the horizontal thickness of the sewer arch on a level with the center of the sewer.

Let D = the diameter of the sewer, in feet;

Let T = the required thickness of the arch on a level with the center of the sewer, in feet;

then $$T = \frac{D}{\frac{D}{14} + 2.572}.$$

The crown thickness, or depth of key, is fixed arbitrarily according to the conditions in a given case, and when the depth of key is thus fixed and the thickness at the springing line is determined by the formula, the required thickness at any other portion of the arch is determined by drawing the arc of a circle through these two points, this arc having its center directly below the center of the sewer. As thus determined, the line of resistance remains within the middle third of the arch ring, and the maximum pressures per square foot at the springing line for sewers of various diameters are approximately as follows:

8	ft.	diameter,	about	3 800 lb.
13½	"	"	"	4 200 "
16	"	"	"	4 600 "
16¾	"	"	"	4 700 "

Below the springing line the wall is given a batter of 1 horizontal to 4 vertical. The line of resistance then follows down to the earth nearly parallel to the outer face of the wall, and the maximum load upon the soil will be about 2 tons per sq. ft.

The foregoing method of determining the form of the extrados produces an arch increasing in thickness gradually, from the key to the springing line. It is customary in building large sewer arches to increase the thickness of the arch in passing from the crown to the springing line abruptly, by the addition of 4½-in. rings of brickwork. With this form of construction the bricks of the arch are usually arranged in concentric rowlock rings, occasionally tied by header courses. In the present instance it was desired to give the arch greater strength than would be obtained by rowlock construction, and it was desired also, that the extrados of the arch should present a uniform continuous surface for better drainage and in order that the pressure of the earth might be exerted more nearly radial to the arch. It was decided, therefore, to make the actual arch conform to the extrados, as above described; and, in order to do so, a considerable amount of brick cutting became necessary. The extrados surface was then plastered with a heavy coating of Portland-cement mortar, giving the arch a smooth, continuous exterior.

For greater strength, the bricks composing the arch were arranged in alternate headers and stretchers in Flemish bond, and, in order to avoid excessively thick mortar joints, as would be the case were the joints to continue radially from intrados to extrados, the masonry was broken up into cylindrical rings, as illustrated in cross-section in Fig. 1, Plate LXIV. The entire arch was also cut into a certain number of segments separated by radial planes. The bricks composing each segment included between these radial planes were laid up in rings, the inner and outer faces of which are parallel with the inner surface of the completed sewer. The radial distances between these parallel cylindrical surfaces are either 9, 13, or 17 in. The number of courses required to build one of these cylindrical rings for any particular segment is one more than the number contained in the next inner ring, and one less than the number contained in the next outer ring. By this means mortar joints of ordinary thickness are obtained in all portions of the arch, even to

PLATE LXVI.
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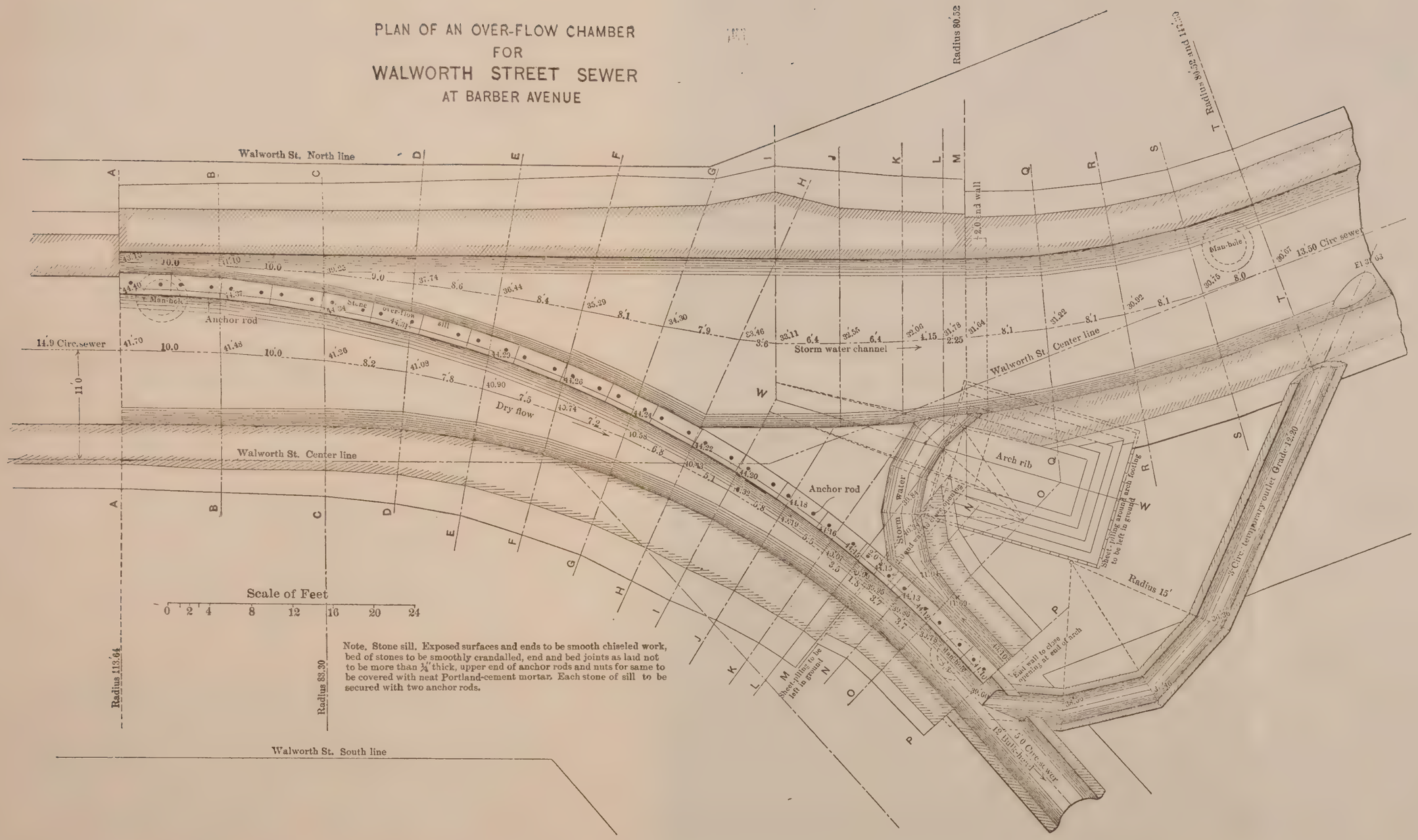
PARMLEY ON
WALWORTH SEWER.

PLAN OF AN OVER-FLOW CHAMBER FOR WALWORTH STREET SEWER AT BARBER AVENUE



Note. Stone sill. Exposed surfaces and ends to be smooth chiseled work, bed of stones to be smoothly crandalled, end and bed joints as laid not to be more than ¼" thick, upper end of anchor rods and nuts for same to be covered with neat Portland-cement mortar. Each stone of sill to be secured with two anchor rods.

the extrados. The cylindrical surfaces separating the inner and outer rings of the segments are plastered smooth with Portland cement. The radial thickness of each cylindrical ring composing the superimposed segments of masonry is adjusted so as to break joint in adjoining segments by at least 4 in. Thus the completed arch consists of bricks in Flemish masonry interlocked and bonded together by broken joints.

ARRANGEMENTS OF BRICK IN
FLEMISH BOND FOR SEWER WALLS.

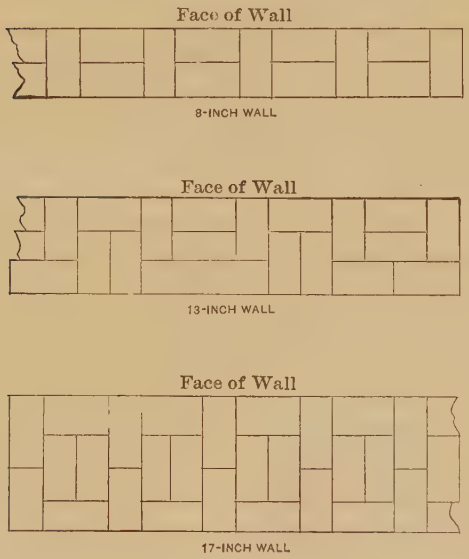


FIG. 3.

Apparently, this requirement would increase the cost of the masonry of the arch, but it was found by experience that as soon as the masons became familiar with the method of construction they placed in the wall nearly as many bricks per day as in a straight wall laid in English bond. Fig. 3 shows the arrangement of the Flemish bond in 9, 13 and 17-in. rings, respectively.

The earth supporting the sewer being in general a plastic blue clay containing considerable water, the sewer is built upon an oak

plank grillage. Lines of 3 by 12-in. oak sleepers, longitudinal to the sewer and separated not more than 4 ft. from center to center, are bedded in the clay, and upon them is spiked a tight flooring of 3-in. oak plank laid transversely to the sewer.

For economy, the entire lower portion of the sewer is built of natural-cement concrete. The height on each side to which the concrete is carried is such as can be put in place conveniently between suitable forms, and the top of the concrete is brought to a plane parallel to the axis of the sewer, inclining downward and inward at a rate of 4 horizontal to 1 vertical. The minimum thickness of concrete under the two rings of lining brick of the invert is 1.5 ft. for sewers from 8 ft. to 14 ft. 9 in. in diameter, inclusive, and 2 ft. for larger sizes. The side walls, composed of bricks laid in English bond in natural-cement mortar, are carried upward from the concrete with the courses pitching inward parallel to the upper surface of the concrete. The tops of the side walls are brought to the planes which slope inward and downward at a rate of 4 horizontal to 1 vertical, which contain the axis of the sewer. From these planes the masonry of the arch springs.

Inside of the foundation concrete, the invert of the sewer, and up to the top of the concrete, is composed of two rowlock rings of brick laid in Portland-cement mortar. Two rings are used in order to produce a smoother and more uniform inner surface than could be obtained by the use of only one. The lining for the side walls, above the concrete and below the arch, consists of one thickness of brick laid in Portland-cement mortar, the cylindrical inner surfaces of the brick being roughly beveled to conform to the outer surface of this ring of brickwork. The bricks forming the inner surface of the sewer are side-cut shale brick, burned especially hard for the purpose, the requirement being an absorption of not more than 2 per cent. This produces a harder brick than that required for brick pavement. It is more highly vitrified, and consequently too brittle to stand the abrasion of a street surface. Its brittleness, however, is not detrimental to its use in a sewer, and the increased hardness caused by excessive vitrifaction produces a more resisting surface in a sewer than would be obtained by ordinary paving brick.

Several typical sections are shown on Plate LXII.

On account of the great difference in size between the main

PLATE LXVII.
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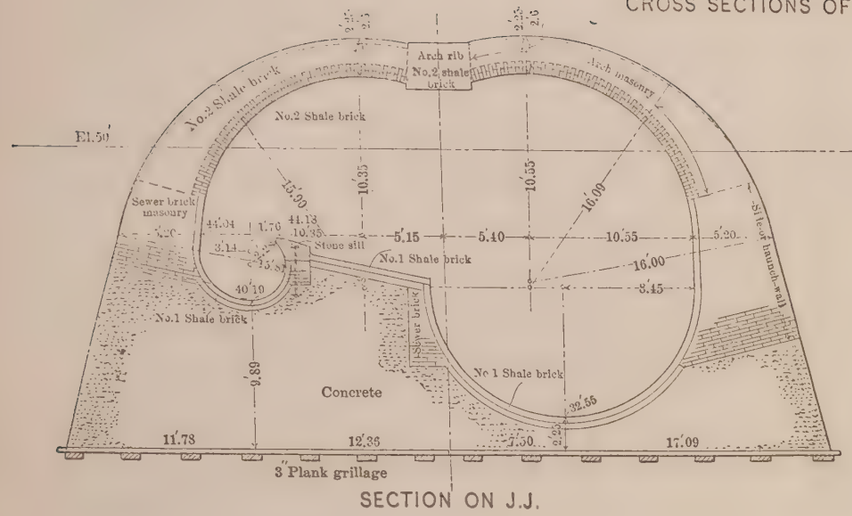
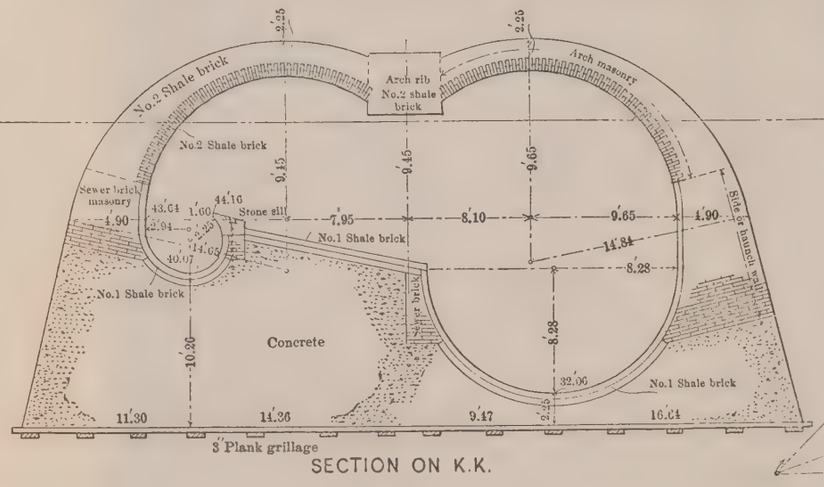
PARMLEY ON
WALWORTH SEWER.

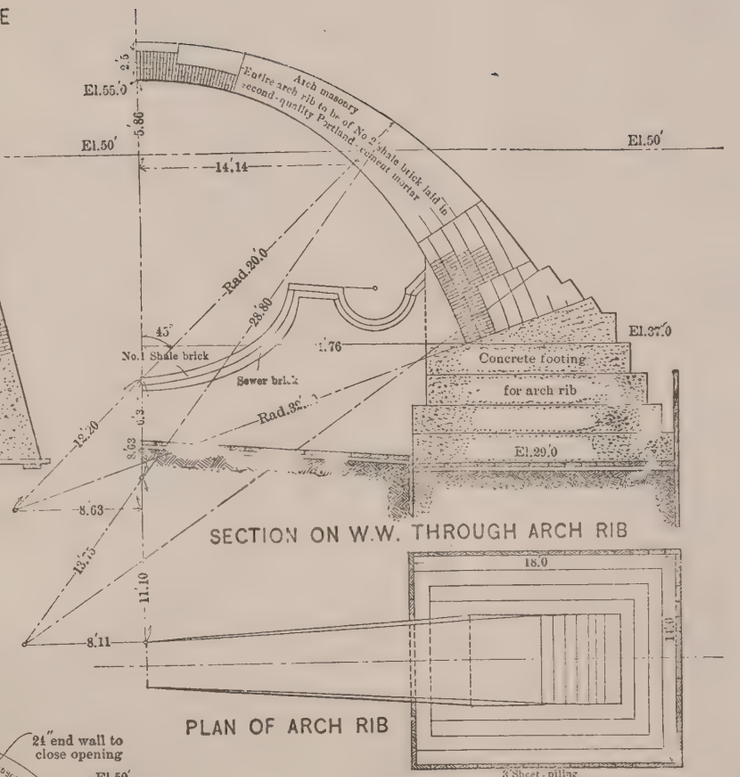
CROSS SECTIONS OF OVER-FLOW CHAMBER IN WALWORTH STREET SEWER NEAR BARBER AVENUE



SECTION ON J.J.



SECTION ON K.K.



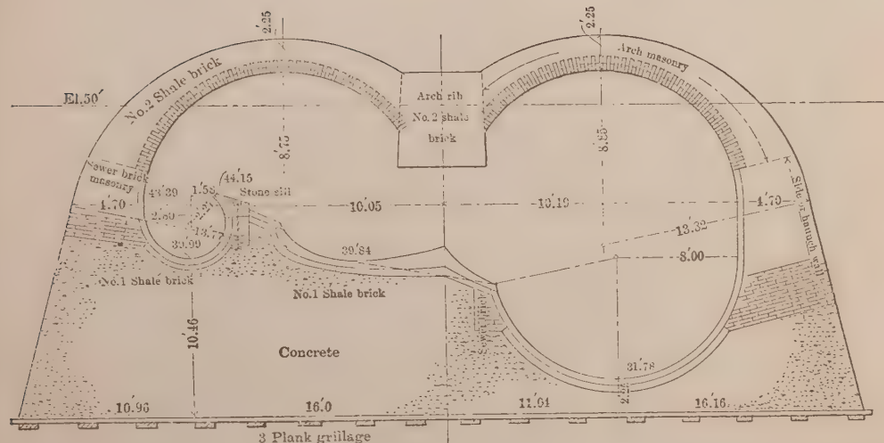
SECTION ON W.W. THROUGH ARCH RIB

PLAN OF ARCH RIB

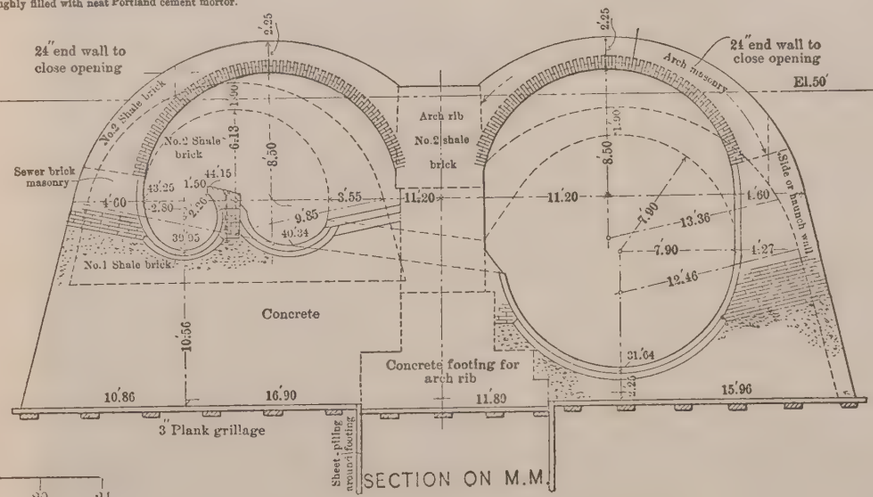
3′ Sheet-piling

Sheeting around arch rib footing to remain in ground

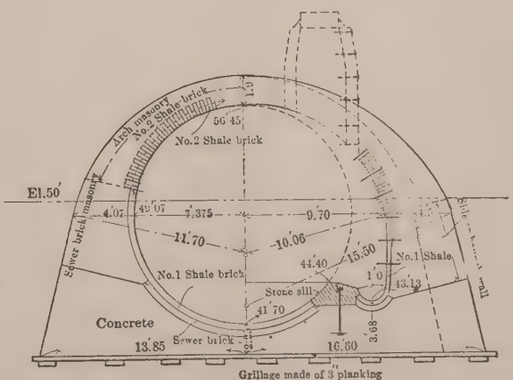
Note. Stone sill to be secured with ⅞″ anchor bolts, 3 feet long. Anchor plates 6″ x 6″ x ⅜″ to be placed at end of bolts. The nut of bolt to be let into the stone and the hole made by same to be thoroughly filled with neat Portland cement mortar.



SECTION ON L.L.



SECTION ON M.M.



SECTION ON A.A.

Scale of Feet

0 2 4 8 12 16 20 24

sewer and the connecting lateral sewers, it has not been necessary to use bell-chambered junctions. The largest junction is at Mill Street, where an 8-ft. branch connects with a 15-ft. sewer, and in this case, as well as in the cases of smaller laterals, a reinforced, groined-arch junction was preferred. In each case where a lateral connects with a main sewer, the foundation masonry of both the lateral and the main sewer is built up continuously as far as the side of the ditch of the main sewer. Any possibility of the masonry of the lateral breaking loose from that of the main sewer at the line of junction is thus avoided. The junctions connect with the main sewer at a uniform angle of 30°, and the bricks at the line of intersection are carefully mitered for the lower half of the curve. The mitered junction is carried above the center in ordinary cases to about 45°, and above this the brick rings of the lateral are brought through flush with the inner face of the completed main sewer, and beveled to make a smooth junction.

The manner of placing the manholes upon the main sewer is important. The walls of each manhole are circular, and 42 in. in diameter at the base. They form a vertical cylinder extending upward to a level with the back of the sewer, above which level they taper gradually to a diameter of 32 in. at the top, where they are surmounted by a cast-iron rim and grated cover weighing about 800 lb., the cover being 32 in. in diameter. The 42-in. cylinder forming the lower portion of the manhole rests directly upon the arch masonry, and is placed so that the lower side of the intersection of the interior of this cylinder and the interior surface of the main sewer is at an elevation eight-tenths of the diameter of the main sewer above the invert. This avoids any obstruction to the smooth flow of water in the main sewer in all cases except the most extraordinary flood, and then the interference caused by the manhole opening would be slight. Round ladder irons, $\frac{3}{4}$ in. in diameter, spaced 16 in. apart, are built into the side of the manhole, and continue downward along the inside of the main sewer to within 3 ft. of the bottom.

All changes in direction in the main sewer are made with as easy curves as possible. At the intersection of Walworth and Guernsey Streets, where the deflection is about 90°, two lots were purchased, and the sewer was built on a curve of 164 ft. radius. The sewer at this point is shown by Fig. 1, Plate LXIII.

For private connections, 8-in. slants are built through the side walls every 30 ft. on either side at about the level of the center of the sewer. At numerous places along the line of the sewer and at all street intersections, 12-in. slants are built in the walls to furnish inlets for various drainage and catch-basin waters.

The catch-basin slants are usually set somewhat higher than those for private connections, and all slants are capped and cemented in the usual manner. In some cases, as at Pearl Street, Willey Street and Brevier Street, the lateral sewers consist of 5-ft. cast-iron pipes. These pipes are connected with the main sewer by building an ordinary brick-groined junction and enclosing the end of the iron pipe with a suitable collar of brick masonry.

Storm-Water Overflow.

At a point about 400 ft. east of Rhodes Avenue, at the intersection of Barber and Walworth Streets, Walworth sewer passes below the water-plane elevation established for the high-level intercepting sewers. This is the point fixed for the upper terminal of the "South Side" branch of the intercepting system. It became necessary, therefore, to separate the excess of storm waters from the ordinary sewage flow, and, for this purpose, a suitable diversion structure was built. The problems involved in its design were similar to those which had been solved for a considerable number of large overflow chambers in the city, but in no other place upon such magnitude. The main sewer entering the chamber is 14 ft. 9 in. in diameter, with a maximum calculated flow of 2 500 cu. ft. per sec. The calculated maximum flow of dry-weather sewage is about 30 cu. ft. per sec. The intercepting sewers are designed to carry the dry-weather flow and an equal volume of storm water, in order to provide for the interception of the foul, first flow from the streets. The required carrying capacity of the intercepting sewer at this point, therefore, is about 60 cu. ft. per sec. The problem was to design a structure which would always divert 60 cu. ft. per sec. before any storm water was discharged to the main outlet, and one also that would not divert more than 60 cu. ft. per sec., under any condition of storm flow in the main sewer. The diversion is based upon the weir principle, and, on account of its unusual dimensions,

FIG. 1.—JANUARY 28TH, 1898.
VOLUME FLOWING, 2.74 CU. FT. PER SEC.

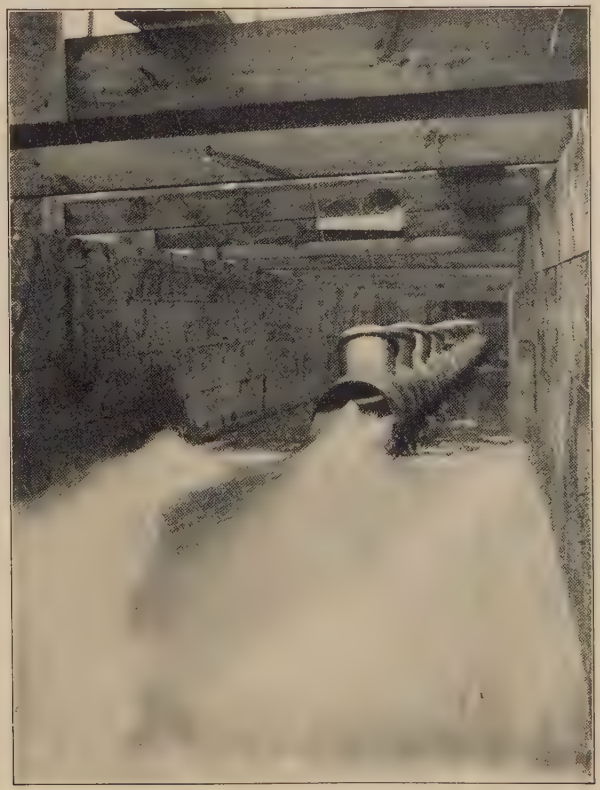


FIG. 2.—FEBRUARY 14TH, 1898.
DISCHARGE OF 10 CU. FT. PER SEC.

it will probably be of interest to explain the method adopted in solving the problem.

From experience with smaller storm-water overflows, it was evident that a weir of considerable length was needed in order to accomplish the object effectually, and it was the failure of the older structures, in this particular, which led to the following theoretical investigation.

Suppose one side of the sewer to be cut away and converted into an overflow weir such that the flow of the volume of water below the level of this weir is not obstructed, but that all the water above its level can discharge sideways over the weir. With a given depth upon the upper end of the weir the water will tend to be discharged sideways according to the ordinary weir formula. There is, however, the forward velocity of the water in the sewer behind the weir to be considered. In the first unit of length a given quantity of water per second will be discharged, thereby reducing the head upon the weir in the second unit of length; this reduction of head in the second unit of length, caused by the water discharged in the first unit of length, will make the rate of overflow in the second unit less per second than it was in the first. In a similar manner each succeeding unit of length of weir will discharge a less volume than the preceding unit, owing to the continual reduction of head as the water moves forward in the sewer. The forward velocity in the sewer tends also to slacken, due to the lessening volume carried. An analysis of the problem shows that, theoretically, a weir would have to be of infinite length in order to reduce the water to the level of the crest of the weir; therefore it is not attempted to discharge all the water above the level of the weir, but to reduce the head upon the weir to some small amount. The problem involved may be stated as follows:

Let Fig. 4 represent the cross-section of the overflow chamber at the upper end of the weir, at the point where the water emerges from the sewer.

Let X and Y represent the axes of co-ordinates, with the origin in the axis of the sewer. Consider this section to represent a unit length of sewer.

Let A be the crest of the weir, and let $a + y$ be the depth of water over the weir.

Let the radius of the sewer equal r.

The co-ordinates of the weir, therefore, are $x = x_1$ and $y = -a$.

How long will it require for the water flowing over the weir to reduce the head of water on the weir from $a + y$ to any given lesser head?

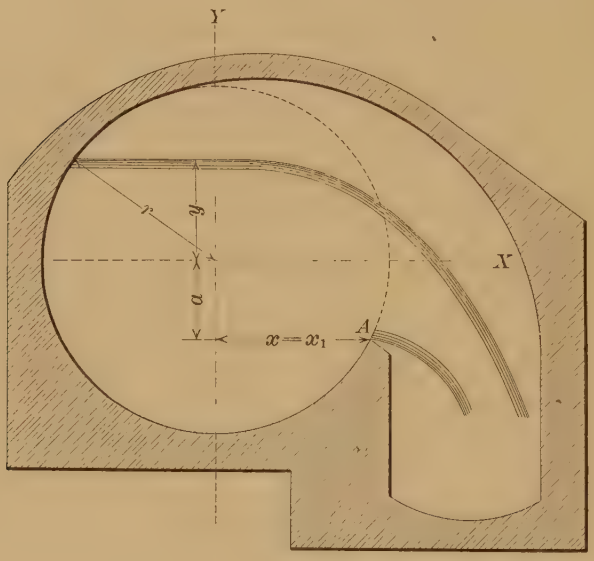


FIG. 4.

Let $d\,Q$ equal the volume of water discharge for a reduction of head, $d\,y$, and let $d\,t$ equal the time required for the discharge of the quantity, $d\,Q$. We then have the equations:

$$d\,Q = 2\,x\,d\,y = 2\,\sqrt{r^2 - y^2}\,d\,y.$$

For the head, $a + y$, the rate of discharge, q, equals approximately $3.33\ (a + y)^{\frac{3}{2}}$,

then $$d\,Q = q\,d\,t = 3.33\,(a + y)^{\frac{3}{2}}\,d\,t,$$

therefore,

$$d\,t = \frac{0.6\,\sqrt{r^2 - y^2}}{(a + y)^{\frac{3}{2}}}\,d\,y.$$

Integrating between the limits, y_1 and y_2, for any two heads upon the weir, gives the time required to reduce the head from y_1 to y_2.

PLATE LXIX.
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VOL. LV, No. 1011.
PARMLEY ON
WALWORTH SEWER.

PROFILE OF PLATFORM
FEB-8-1898

CROSS SECTION OF PLATFORM

CROSS SECTION OF PLATFORM
JAN-28-1898
Initial veloc. = 0.52 ft. per sec.
Q = 2.73 cu. ft. per sec.
(h = 0.346 ft.)

FLUME

Before Profile initial velocity 1.25 ft per sec. Q = 11.19 cu ft (h = 0.901)
After " " " 1.33 " " Q = 12.79 " " (h = 0.974)

Q = 2.74 cu. ft. per sec.

Dead Water

PROFILE OF PIPE LINE
JAN-28-1898
No. of Branches

PROFILE OF PIPE LINE
FEB-8-1898
No. of Branches

It has not been possible, however, to integrate this equation, and, therefore, it has been necessary to make use of it in the approximation form :

$$t = \sum \left[\frac{0.6 \sqrt{(r^2 - y^2)}}{(a + y)^{\frac{3}{2}}} \triangle y \right]_{y_2}^{y_1}$$

Obtaining the $\triangle t$ for successive differences in head, $\triangle y$, between the limits, y_1 and y_2, and taking the sum of all these $\triangle t$'s, will give the approximate time, t, required.

This being a tedious process, an approximation can be made by reducing the circular sewer to a rectangular one of the same average width. In this case, let Fig. 5 represent the cross-section of the

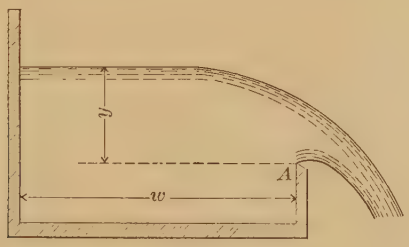


FIG. 5.

rectangular sewer, with the weir at A, and with an initial depth of water, y, over the weir. Let the width of channel, w, equal the average width of the circular sewer, shown in Fig. 4, to the left of the weir, A. In this case the water overhanging the weir on the right is assumed to fall away by the force of gravity without interfering with the weir discharge of the water over and back of the weir. In this case, then, we have

q = the rate of discharge for the head, y = $3.33\ y^{\frac{3}{2}}$; and

Q = the total quantity discharged.

For an infinitesimal reduction in head, $d\ y$, we have

$$d\ Q = w\ d\ y = q\ d\ t = 3.33\ y^{\frac{3}{2}}\ d\ t,$$

$$\text{therefore } d\ t = \frac{w}{3.33}\ y^{-\frac{3}{2}}\ d\ y.$$

Integrating between the limiting heads, y_1 and y_2, gives

$$t = \left(-\frac{w}{1.67\sqrt{y}} \right)_{y_2}^{y_1} = \frac{w}{1.67} \left(\frac{1}{\sqrt{y_2}} - \frac{1}{\sqrt{y_1}} \right).$$

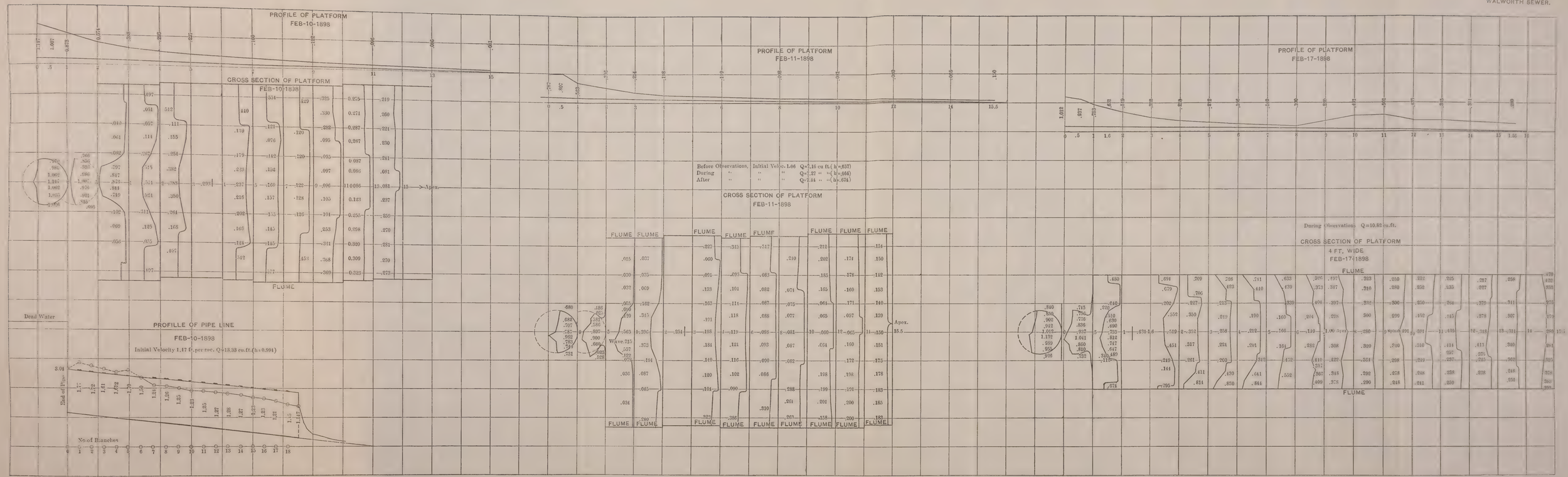
If $y_2 = 0$, $t = \infty$, which shows, as before stated, that theoretically it would require a weir of infinite length to reduce the water to a zero head. The last formula is simply and easily applied, and does not give results varying greatly from those obtained from the differential equation for the circular sewer.

If the velocity in the sewer were constant while flowing the length of the weir, and if all the filaments in the entire cross-section had the same velocity, the foregoing equations would give the time required to reduce the level of the water from one stage to another, and this time multiplied by the velocity of flow in the sewer behind the weir would give the length of weir required. These ideal conditions, however, are not obtained in practice. The velocity in the sewer is gradually retarded as the head becomes less, and, consequently, the sill must be lengthened somewhat in order to perform the same amount of work.

At the point where the overflow chamber would be constructed, the center line of the main sewer is 11 ft. north of the center line of Walworth Street, in order to avoid cutting so heavily into the side hill a short distance up stream therefrom. In consequence of this fact, the upper end of the overflow chamber could not be made of as great width as would otherwise have been done. By taking advantage of the angle of intersection between Barber and Walworth Streets, a curved weir could be constructed which would facilitate the overflow of the storm water. By referring to the completed design, Plates LXV and LXVI, it will be seen that the dry-weather channel was built upon a curve gradually decreasing in width and size as it passes from the 14 ft. 9-in. sewer to the 5-ft. interceptor.

In order to avoid back-water for partial depths of flow, a number of calculations were made with varying depths of flow in the main sewer. As a result of a large number of these calculations for assumed relative elevations of inverts of main and intercepting sewers and of varying volumes of flow, the following conclusions for regulating the design of the overflow chamber were obtained:

Since it was not desirable to allow the velocity in the main sewer above the overflow chamber to be reduced below about 2.50 ft. per sec., it was necessary to make a drop of at least 1.50 ft. in passing from the invert of the 14 ft. 9-in. sewer to the invert of the 5-


PROFILE OF PLATFORM
FEB-10-1898
CROSS SECTION OF PLATFORM
FEB-10-1898
FLUME
Dead Water
PROFILLE OF PIPE LINE
FEB-10-1898
Initial Velocity 1.47 ft. per sec. Q=13.33 cu.ft. (h=0.994)
End of Pipe
No. of Branches
PROFILE OF PLATFORM
FEB-11-1898
Before Observations, Initial Velocity 1.06 Q=7.16 cu ft. (h=.657)
During " " " Q=7.37 " " (h=.664)
After " " " Q=7.44 " " (h=.674)
CROSS SECTION OF PLATFORM
FEB-11-1898
FLUME
Wave
Apex.
PROFILE OF PLATFORM
FEB-17-1898
During Observations Q=10.82 cu.ft.
CROSS SECTION OF PLATFORM
4 FT. WIDE
FEB-17-1898
FLUME
Apex.

ft. sewer. With this drop, the minimum velocity in the main sewer will be about 2.50 ft. per sec., when 60 cu. ft. per sec. are flowing. For a less quantity than 60 cu. ft. per sec. there will be an acceleration in the velocity above the junction for any small volumes of flow, and for no quantity less than 60 cu. ft. per sec. will the effect of back-water reduce the velocity to less than 2.50 ft. per sec. For volumes greater than 60 cu. ft. per sec. the sill of the overflow must be long enough to take out all but 60 cu. ft. per sec., which will remain in the sewer to be carried off by the interceptor. For the maximum discharge of 2 500 cu. ft. per sec. for a 14 ft. 9-in. sewer there will be no back-water effect. Hence the 14 ft. 9-in. sewer will flow unobstructed when nine-tenths full.

The elevation of the up-stream end of the weir, therefore, was placed 2.70 ft. above the invert of the 14 ft. 9-in. sewer, and is carried to an elevation of 4.50 ft. above the invert of the 5-ft. interceptor after the invert of the interceptor has been fixed at a proper elevation as above determined. The grade of the crest of the weir is 0.3 ft. per 100 ft. The form of cross-section of the dry-weather channel at the upper end begins as the segment of the 14 ft. 9-in. sewer, and, in passing down stream to the 5-ft. interceptor, gradually changes to the section of the 5-ft. sewer with the crest of the overflow sill nine-tenths of the diameter of the sewer above the invert. In order to avoid any back-water effect from the storm water overflowed, it is necessary that the weir should never act as a submerged weir. That is to say, the surface of the storm water in the overflow channel must always be lower than the crest of the weir. In order to effect this arrangement it was necessary to build about 2 000 ft. of 14 ft. 9-in. sewer above the overflow chamber upon a grade of 0.30 ft. per 100 ft. The storm-water branch below the overflow chamber was given a drop of about 12 ft. below the level of the sill, and was carried down the valley on a grade of 0.50 ft. per 100 ft. The overflow branch, therefore, was made 13 ft. 6 in. in diameter.

The overflow channel within the chamber, on account of the limiting width of the street, was made but 3 ft. wide at the upper end of the sill, and was carried down stream with the outer foundations of the sewer along the northerly line of Walworth Street, whereas the overflow sill and dry-weather channel gradually curved around to the southward, thus giving an increasing width to the storm overflow

channel. The grade of the invert for this portion of the channel was made to follow a vertical curve which was steepest at the upstream end and gradually flattened until it became tangent to a 0.5% grade at the lower end of the chamber. The water overflowed in this manner is discharged into the 13 ft. 6-in. storm outlet with a minimum amount of agitation.

With the large volume and velocity of the inflow it would not be permissible to construct columns rising from the overflow sill to support the arch. An unobstructed weir from end to end being necessary, the chamber is spanned with a single arch for about the upper half of its length. For the lower half, the arches for the storm overflow channel and the branch leading to the intercepting sewer meet in a large groin. The groin was designed as a strong rib of shale-brick masonry, bonded and interlocked with each of the arches which it supports, and keyed at its highest point with the highest point of the arch which spans the entire chamber.

On account of the foundations resting upon soft clay, it was deemed necessary to extend the masonry of the sewage channel down to a level with that of the storm overflow branch, and the foundation for the arch rib was brought to the same level and made continuous therewith. Plates LXVI and LXVII show the plan and several cross-sections of the chamber as it was designed. As will be mentioned later, in constructing the chamber, owing to unstable earth encountered, the excavations of the upper end were carried down to a level with those of the storm outlet and filled in solid from side to side with natural-cement concrete, thus making at the upper end about 10 ft. extra thickness of concrete foundation.

Abbey Street Viaduct Support.

Just west of Scranton Avenue, the sewer passes under the Abbey Street viaduct, a steel structure, the full width of the street, supported on steel trestlework. The roadway is about 98 ft. above the grade of the sewer, and the foundations are stone masonry pedestals. As it was not possible to avoid encountering one or more of these, it was necessary to provide some other means of support for the viaduct. The sewer was located so as to encounter only two pedestals. In one case, the center of the pedestal was only 1.15 ft. to one

FIG. 1.—FEB. 11TH, 1898. VOLUME, 11 CU. FT. PER SEC.

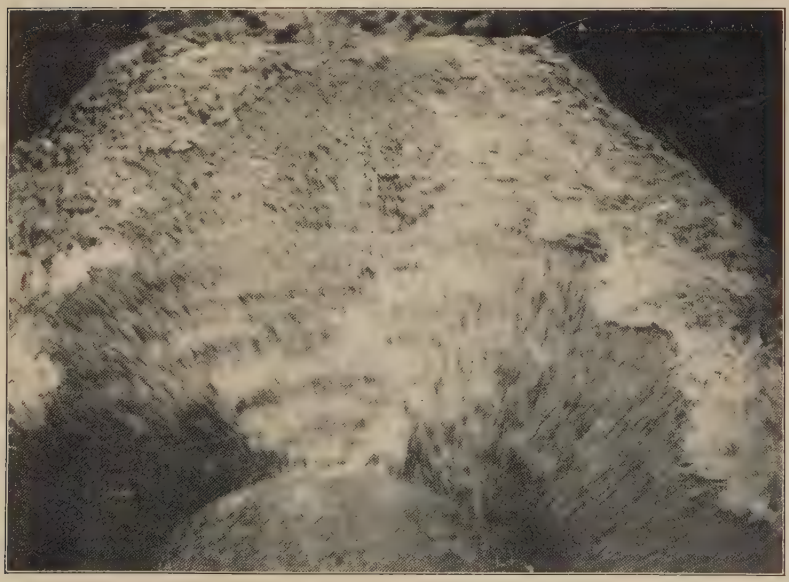


FIG. 2.—DOWN-STREAM VIEW OF DISCHARGE SHOWN IN FIG. 1, ABOVE.

side of the center of the sewer. In the other case the pedestal projected about 3 ft. inside the outer limits of the foundations, and it could be partially embedded in the masonry of the sewer foundation. For supporting the steel column resting upon the other pier, only two ways seemed feasible: either to strengthen the sewer walls sufficiently to carry the weight directly upon the sewer, or to build two piers, one on either side of the sewer, upon which would rest a suitable steel girder for carrying the column. The two methods were carefully compared, and, while it was found that two auxiliary piers with a connecting steel girder could be built for perhaps $1 000 less than an all-masonry structure, the latter method seemed to be preferable. Supporting the column upon a reinforced section of sewer secured a permanency not obtained by a metal structure. By resting the column directly upon the sewer, it was possible to replace the upper courses of the pedestal masonry, including the capstone, so that when the structure was completed and the earth brought to the required grade the visible portions of the masonry foundations of the viaduct were unaltered in appearance.

The problem was to support a dead load from the column of about 84 000 lb. and a calculated live load of about 126 000 lb. upon the arch, and yet retain the line of resistance safely within the outer face of the walls at a time when the sewer was assumed to be nine-tenths full of water. The pressure upon the clay foundations, moreover, could not be increased to more than the specified limit of 2 tons per sq. ft. It was found that these conditions could not be complied with by adding a simple girdle of masonry outside of the normal section of sewer masonry, and it became necessary to distribute the load over a width of about 40 ft. and a length, longitudinal with the sewer, of about 42 ft. The girdle of masonry at the top of the arch was made about 12 ft. long, and the thickness of the key was increased to 4 ft. for this length. About 6 ft. from either side of the center line of the pedestal this girdle of masonry terminates abruptly in a vertical face. From each of the four corners the extra thickness of masonry is gradually lengthened, until, upon reaching a level with the bottom of the foundation, it extends 21 ft. up stream and down stream from the column. By this means a saddle of masonry was built continuously with the sewer masonry which distributes the load over a width of 40 ft. and a length of 42 ft.

The other pedestal extends only to within about 2 ft. of the bottom of the sewer foundations, and on account of the horizontal offsets in the courses of the stone masonry, some provision had to be made to prevent a settlement of the sewer foundations from bearing upon the pedestal masonry and causing its settlement or tilting. For this purpose thin boards wrapped with building paper were laid upon the upper surfaces of the projecting ledges, and similar boards were placed against the vertical edges of the masonry courses. The concrete then was rammed in place, over and against these boards, and permitted to set, after which the boards were withdrawn, thus leaving a clearance between the two structures, and an opportunity for the sewer foundations to settle about $\frac{3}{4}$ in. before coming in contact with the masonry of the pier. The limiting planes separating the different kinds of masonry in the normal sewer are extended laterally and become the limiting planes of the same kind of masonry for that of the extra thickness required.

Widened Section at Outlet.

At the outlet into the Cuyahoga River, unusual difficulties were encountered. No less than twelve railway tracks are passed under, the seven nearest the river being on a level of about 10 ft. above the water. The available depth of water under the supporting structures for those tracks is only about 5 ft., which fact required a change in the cross-section of the sewer from a 16½-ft. circle to a flattened bottom channel 50 ft. wide. The railway tracks which limit the available head-room could easily be spanned by steel-girder construction, but the most serious problem was to design a channel to make the proper transition from the circular to the widened section. Such a change could not be made suddenly, as the shock and wave from 4 300 cu. ft. of water per sec. flowing with a velocity of 30 ft. per sec. would be sufficient to destroy almost any structure opposing its impact. That the change must be made gradually was plain, but in what length and through what intermediate forms of cross-section was not so apparent.

In order to aid in solving the difficulties, a number of experiments on a small scale were made in the winter of 1898. They were made in the flume which carried the flow of Walworth Run at

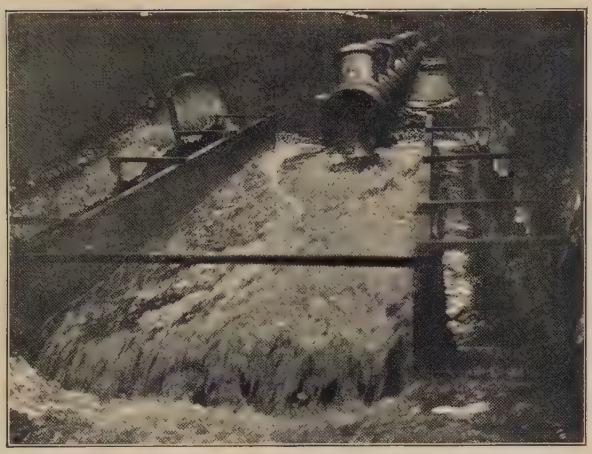


FIG. 1.—FEBRUARY 17TH, 1898. PLATFORM 4 FT. WIDE.
DISCHARGE, 2.45 CU. FT. PER SEC.

FIG. 2.—FEBRUARY 23D, 1898. PLATFORM 4 FT. WIDE.
DISCHARGE, 11.32 CU. FT. PER SEC.

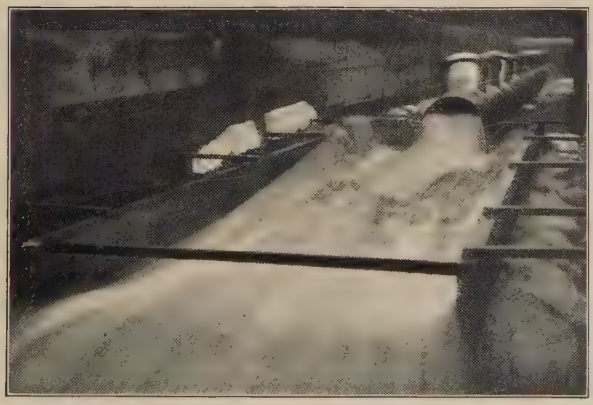


FIG. 3.—FEBRUARY 15TH, 1898. PLATFORM 4 FT. WIDE.
DISCHARGE, MORE THAN 15 CU. FT. PER SEC.

Scranton Avenue near the outlet. A weir was first constructed for determining the volume of water with which the experiments were to be made. A short distance down stream from this weir, a bulkhead was built in the flume, through which was inserted the upper end of an 18-in. sewer pipe, the joint being made tight by caulking.

Connecting with this, a line of 18-in. sewer pipe, each alternate length being either a Y-branch or a T-branch, extended down stream in the center of the flume for 73 ft. Each pipe was brought to the exact grade of 1.15 per cent. The pipe was sent from the yard of a local dealer, without special selection, but the line as laid was more uniform than could be expected in practice, though the irregularities in the size and shape of the several lengths were sufficient to give the flowing water quite a rough surface.

At the lower end of the pipe line, a false bottom of surfaced 2-in. plank was built in the bottom of the flume. This false bottom was about 6 ft. wide (the width of the flume) and 16 ft. long. The upper surface was continuous with the invert of the discharge end of the sewer pipe, level transversely, but with a longitudinal grade of 1.15%, the same as that of the sewer pipe. The upper surface of this false bottom was very carefully dressed off with a carpenter's plane, so that the water glided over it very smoothly. The object was to observe the shape, profile and behavior of water discharging from the 18-in. circle freely upon a smooth, flat plane.

In order to make it possible to observe the exact depth of water at any point on the platform, an overhead plane of reference was established, from which, with a steel tape and sharp-pointed plumb-bob, measurements could be taken to the water surface and to the platform. On either side of the flume, about 5 ft. above the bottom, a 2 by 12-in. surfaced plank was secured longitudinally with the flume, and with the 12-in. faces vertical. The upper edges were strained and spiked to the timbers of the flume so as to be very accurately level, the plank on the opposite side being brought exactly to the same elevation.

A line, vertically over the outlet end of the 18-in. pipe, was stretched at right angles across, from one plank to the other, and the point on either side marked zero. Foot and half-foot marks were then made on each board, so that any desired distance down stream from the discharge end of the pipe could be determined. A straight-

edge now was made to span the two guide-planks transversely with the flume, and was graduated so that the zero was exactly in line and above the center of the end of the pipe line.

By this arrangement of guide-planks, plumb-bob measurements could be made from the plane of reference to any desired point on the platform.

Upon discharging a measured volume over the platform, the distances from the plane of reference to the surface of the flowing water were measured at all points desired. The differences between these measurements and those given for the corresponding points to the surface of the platform gave the depth of water at each point.

Two principal sets of experiments were made: First, free discharge from the circular pipe upon the platform; and, second, a discharge modified by inserting a funnel-shaped tube in the end of the pipe, which served to make the transition from circle to plane more gradual. In each case two widths of platforms were used, one about 6 ft. wide, the full width of the flume, and the other reduced to 4 ft. by setting surfaced side-planks edgewise upon the platform. This gave about the same relative width between pipe and channel as would be required at the Walworth outlet.

The object of the long line of sewer pipe was to be sure that the velocity of the water had become constant before discharging at the free end. Incidentally, quite a number of observations were made on the value of entry head, friction head, relative surface and mean velocities, length required to establish uniform flow, etc., which, though of interest and value, are not pertinent here. The length of pipe in each case proved considerably longer than was necessary to give the water a steady flow. The volume admitted to the pipe was regulated by flash-boards in the flume above the measuring weir whereby an approximate regulation could be made, though there was at times quite an appreciable variation of volume during a particular experiment.

Fig. 1, Plate LXVIII, is a general view of the pipe line, taken January 28th, 1898, when the volume of water flowing was 2.74 cu. ft. per sec. The smooth spreading out of the water upon the platform and the recoil wave from the sides of the flume are clearly shown. On Plate LXIX is shown the profile of the water through the pipe, from the inlet, or dead water at the bulkhead, to a point

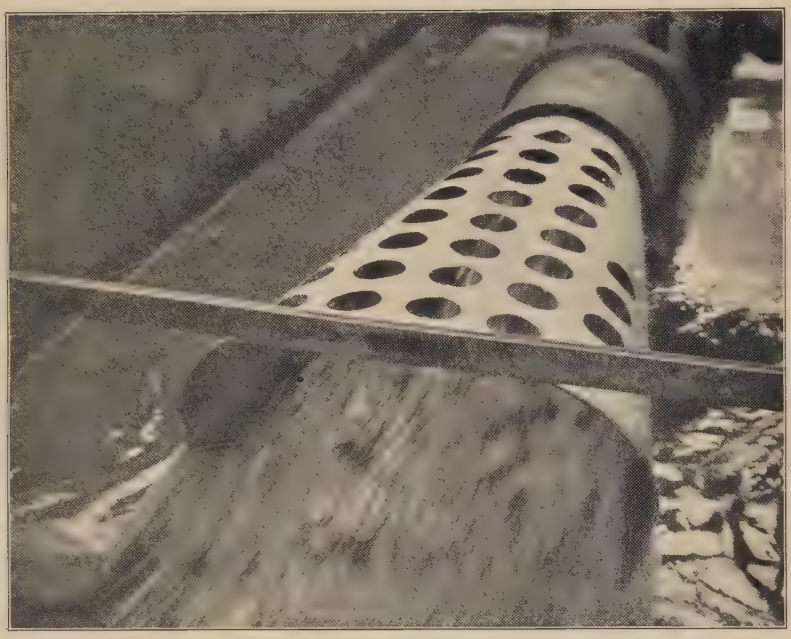


FIG. 1.—FEBRUARY 14TH, 1898. FUNNEL DISCHARGE. VOLUME, 12.13 CU. FT. PER SEC.

FIG. 2.—FEBRUARY 14TH, 1898. FUNNEL DISCHARGE ON 6-FT. PLATFORM.
VOLUME, 12.16 CU. FT. PER SEC.

on the platform 4 ft. from the discharge end of the pipe. Cross-sections of the spreading water on the platform, at distances 0.5, 1, 2 and 4 ft. from the end, are shown. The figures show the depth of flowing water at each point, those in the pipe being obtained by rod and engineer's level at the Y-branches, and those for the platform by measuring from the plane of reference with a steel tape. The initial surface velocity of the water approaching the weir, the head upon the weir measured from a fixed point to the surface of still water about 8 ft. above the weir, and the calculated quantity, in cubic feet per second, are also given.

The calculated area of the water section and the approximate mean velocity at each station, numbering down stream from the end of pipe line, are also shown.

On Plate LXIX are shown the results of the experiments of February 8th, 1898; on Plate LXX, those of February 10th and 11th. In each of these cases the discharge is upon the full width of the platform.

Fig. 1, Plate LXXI, is an instantaneous view of the discharge on February 11th, when the volume was 11 cu. ft. per sec.

Fig. 2, Plate LXXI, is the same discharge looking down stream over the end of the pipe. In both, the violent recoil from the sides of the flume is most conspicuous. The radiating lines of flow of the different filaments of water are also clearly shown.

Fig. 2, Plate LXVIII, is a time exposure (for a very short time) of the flow of February 14th, when the volume was 10 cu. ft. per sec. Although the fine detail is lost, a clear, general impression is given.

On Plate LXX are shown the results of the experiment of February 11th, when the volume flowing was somewhat more than 7 cu. ft. per sec.

The effect upon the spreading water, when the platform was narrowed to 4 ft. in width, is shown in Fig. 1, Plate LXXII, taken February 17th, 1898, with 2.45 cu. ft. per sec.; Fig. 2, Plate LXXII, taken February 23d, with 11.32 cu. ft., and Fig. 3, Plate LXXII, taken February 18th, with more than 15 cu. ft. per sec. flowing. In the latter case the pipe was surcharged to within 3 ft. of the outlet, and, through the upper portion of the line, the water was overflowing from the Y-branch.

The detail of the experiment of February 17th, with 10.82 cu. ft. per sec., is shown on Plate LXX.

The funnel for the second class of experiments was 4.5 ft. long, small enough at the upper end to fit tightly into the sewer pipe, and, at the larger end it was approximately an ellipse, 3 ft. by 1.65 ft. It is shown in position in Fig. 1, Plate LXXIII, while carrying 12.13 cu. ft. per sec. on February 14th. Through the galvanized sheet-iron, on the upper side, 4-in. holes, 6 in. apart each way, were cut so that the plumb-bob could be inserted for measuring to the water surface within the funnel. Fig. 2, Plate LXXIII, shows the effect of the funnel outlet on the 6-ft. platform on February 14th, with a flow of 12.16 cu. ft. per sec.

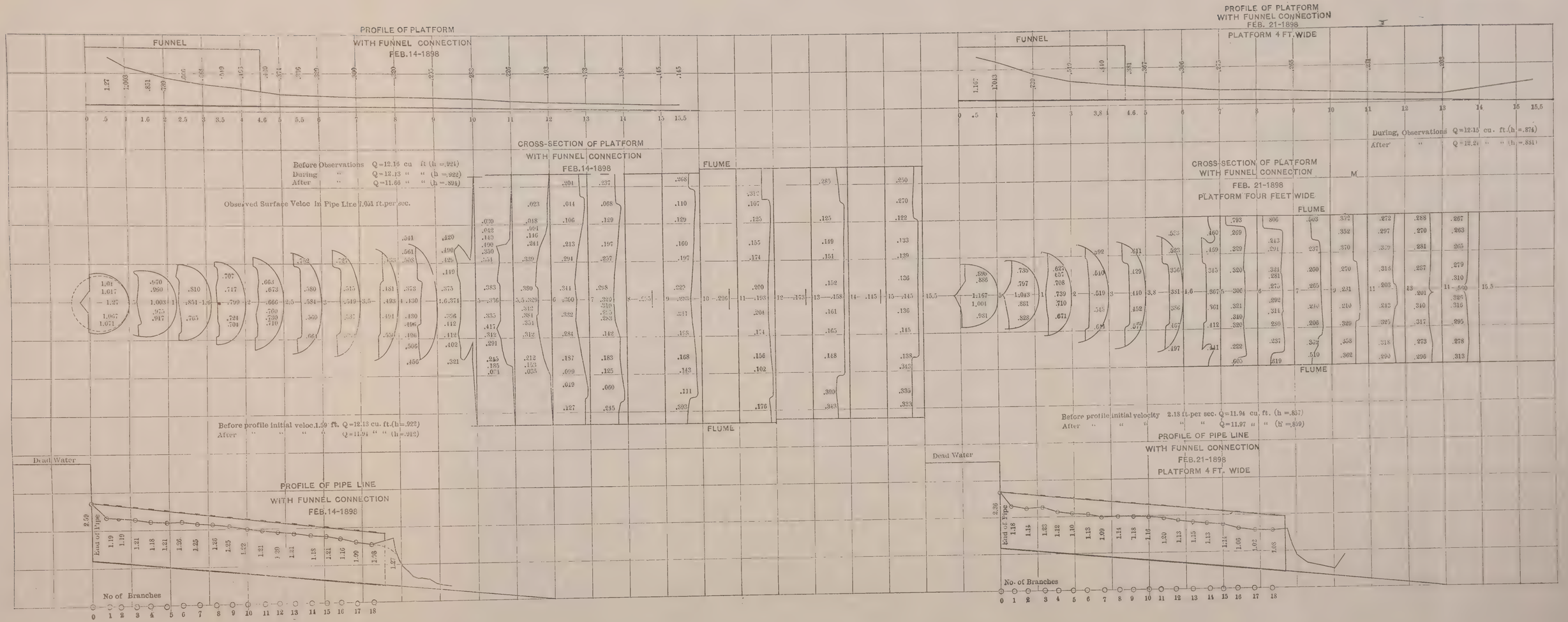
On Plate LXXIV are shown the details of the experiment of February 14th, when there was a flow of about 12 cu. ft. per sec. on the full width of the platform. On Plate LXXIV is also shown the experiment of February 21st, with about the same volume of flow, but with the platform channel only 4 ft. wide.

Fig. 1, Plate LXXV, is a view when the flow was 11.51 cu. ft. per sec., on February 23d, and Fig. 2, Plate LXXV, when the flow was about 15 cu. ft. per sec.

While these experiments do not pretend to any great refinement, they illustrate some valuable facts regarding the behavior of flowing water under the peculiar conditions considered. The most important is the imperative necessity of making the transition from the circular to the wide section very gradual. If made gradual enough, the back wave from the sides can be almost, if not entirely, avoided. Another fact shown is that the velocity of the mean cross-section during the transition probably tends to accelerate. In the experiments, the acceleration from the falling water surface exceeded the retardation from the friction of the enlarged perimeter of the channel.

In order to determine the velocity at each section of the sewer channel and thus the depth, the problem of the unsteady flow of the water must be solved. An approximate depth can be determined, on the assumption that the velocity remains unchanged through the changing sections, but this method will give depths too great, or too small, depending on whether the velocity is really accelerated or retarded.

PLATE LXXIV.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LV, No. 1011.
PARMLEY ON
WALWORTH SEWER.


PROFILE OF PLATFORM
WITH FUNNEL CONNECTION
FEB. 14-1898
FUNNEL
CROSS-SECTION OF PLATFORM
WITH FUNNEL CONNECTION
FEB. 14-1898
Before Observations Q=12.16 cu ft (h =.924)
During " Q=12.13 " " (h =.922)
After " Q=11.66 " " (h =.894)
Observed Surface Veloc in Pipe Line 1.651 ft. per sec.
FLUME
Before profile initial veloc. 1.59 ft. Q=12.13 cu. ft. (h=.922)
After " " " " Q=11.94 " " (h=.912)
Dead Water
PROFILE OF PIPE LINE
WITH FUNNEL CONNECTION
FEB. 14-1898
End of Pipe
No of Branches
PROFILE OF PLATFORM
WITH FUNNEL CONNECTION
FEB. 21-1898
PLATFORM 4 FT. WIDE
FUNNEL
During, Observations Q=12.15 cu. ft. (h =.874)
After " Q=12.21 " " (h =.881)
CROSS-SECTION OF PLATFORM
WITH FUNNEL CONNECTION
FEB. 21-1898
PLATFORM FOUR FEET WIDE
FLUME
Before profile initial velocity 2.18 ft. per sec. Q=11.94 cu. ft. (h =.857)
After " " " " Q=11.97 " " (h =.859)
PROFILE OF PIPE LINE
WITH FUNNEL CONNECTION
FEB. 21-1898
PLATFORM 4 FT. WIDE
Dead Water
End of Pipe
No. of Branches

Attempts to apply the discussions and formulas for unsteady flow, as given in works on hydraulics known to the writer, proved a failure; the results obtained were either negative or manifestly absurd.

The writer, therefore, made the following solution of the problem, which may be of value to other engineers.

No equation can be written which will give the length or velocity directly in terms of the area, slope of invert, hydraulic radius, etc., since the terms on each side of the equation are dependent on those of the other. Resort to a method of approximation, therefore, is necessary.

FIG. 6.

In Fig. 6, let $x\ m$ be a horizontal line; $x\ n$ be parallel to the invert, and thus equal to the slope of the channel; and $x\ y$ be the water surface. Let d_o be the depth at section A, and d_n the depth at section B, to be determined. Knowing the shape of the channel, if the velocity at B is determined, the depth, d_n, is easily found. Let v_0, A_0, R_0, C_0, etc., be the elements, velocity, area, hydraulic radius, coefficient of friction, etc., at section A, and let the same letters with the subscripts, n, represent the corresponding elements at section B. Now, if there were no friction in the channel between A and B the velocity at B would be represented by the equation, $V_n = \sqrt{2\,g\,h + v_o^2}$, in which h is the total fall of the water surface, or, $m\ y$ in Fig. 6. But, since there is friction, the actual velocity is less than that given by the formula, or, some of the head, $m\ y$, is used up as friction head. Now, suppose that the friction were uniform and the amount required to maintain the velocity, v_0, and assume that $m\ o$ represents this friction head, h_0, in the length, l, from A to B. Then an approximate value of the velocity at B, call it the first approximation, v_1, would be represented by the equation, $v_1 = \sqrt{2\,g\,(h - h_o) + v_o^2}$. But the friction increases with

the velocity, and so the last equation also gives too great a velocity. It is, however, much nearer the true velocity, v_n, than the value given by the first equation. The last equation will give a fair first approximation.

From Fig. 6 we have, $h = m\ n + n\ y = i\ l + (d_o - d_n)$, in which i is the inclination of the invert and l the length between sections A and B. If $h_o = m\ o$, the friction head between A and B, in order to maintain a uniform velocity, v_o, we would have

$$v_o = C_o \sqrt{R_o\ S},\ \text{or,}\ S = \frac{v_o^2}{C_o^2\ R_o},\ \text{and}\ h_o = S\ l = \frac{v_o^2\ l}{C_o^2\ R_o}.$$

In the equation for first approximation we may consider that $(d_o - d_n) = 0$, then, by making the substitutions indicated, we have a first approximation, v_1, for the velocity at B,

$$v_1 = \sqrt{2\ g \left\{ i\ l - \frac{v_o^2\ l}{C_o^2\ R_o} \right\} + v_o^2}.$$

A value, v_1, for the velocity at B is thus determined from which first approximations for the area, radius, coefficient of friction and depth, or, A_1, R_1, C_1, d_1, are easily found. With this value for d_1 a second approximation for the velocity, v_2, at B can be made, in which case we use the mean values,

$$v = \frac{v_o + v_1}{2},\ C = \frac{C_o + C_1}{2},\ \text{and}\ R = \frac{R_o + R_1}{2},$$

instead of v_o, C_o, and R_o, respectively. Making these substitutions for the values of h and h_o given above, we have for a second approximation,

$$v_2 = \sqrt{2\ g \left\{ \left[i\ l + (d_o - d_1) \right] - \frac{v^2\ l}{C^2\ R} \right\} + v_o^2}.$$

In like manner any number of approximations may be made and the general formula for the nth will be,

$$v_n = \sqrt{2\ g \left\{ \left[i\ l + (d_o - d_{n-1}) \right] - \frac{v^2\ l}{C^2\ R} \right\} + v_o^2},$$

in which $v = \frac{v_o + v_{n-1}}{2}$, $C = \frac{C_o + C_{n-1}}{2}$, and $R = \frac{R_o + R_{n-1}}{2}$.

If the inclination of the invert is considerable, only a few approximations will be necessary to obtain values such that $v_n = v_{n-1}$, or, to find the exact value, v_n, for the velocity and consequent depth at B. If the slope of the invert is very slight, the formula may

FIG. 1.—FEBRUARY 23D, 1898. FUNNEL DISCHARGE ON 4-FT. PLATFORM.
VOLUME, 11.51 CU. FT. PER SEC.

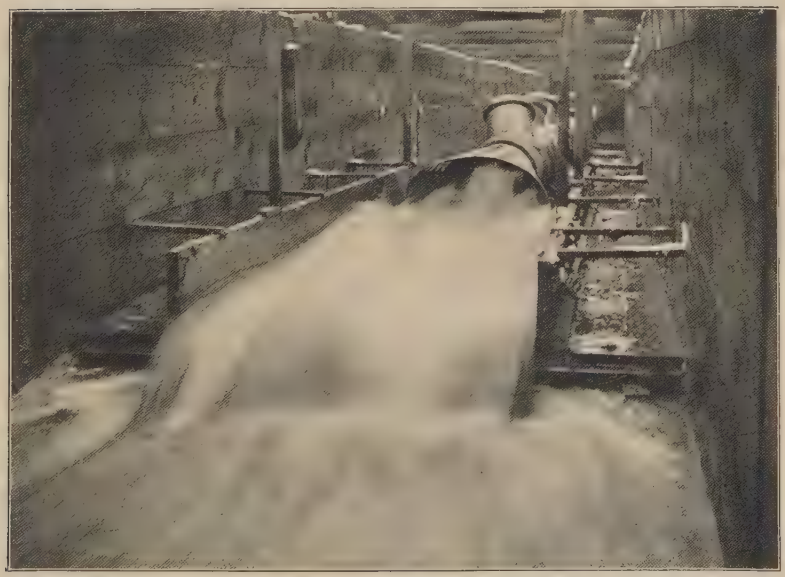


FIG. 2.—FEBRUARY 23D, 1898. FUNNEL DISCHARGE ON 4-FT. PLATFORM.
VOLUME, ABOUT 15 CU. FT. PER SEC.

give successive values alternately too large and too small, and such that each maximum value is less than the preceding one and each minimum value larger than the preceding one. It thus presents the case where the velocity head and friction head are alternately excessive. In this case, if a sufficient number of approximations were made, the true value for velocity and depth at B would be obtained, but the process is greatly shortened if, as soon as maximum and minimum values are determined, a mean between them be taken as the approximate value for use in the next approximation. By a few averages obtained in this manner the case can be quite readily solved. It may also be noticed that it is not necessary to make the first approximation, as a second approximation can be made directly by taking some reasonable value of d_1 by estimation and using that directly in the second approximation formula. The foregoing solution is for the case of accelerated velocity, but the formula is general, and will apply to a case of retarded velocity by observing the algebraic signs. The shape of the channel is immaterial; also, any change there may be between the two sections taken, as long as the change is gradual and the length is not taken so great that the average velocities and friction constants at the two sections are materially in error. It is necessary, of course, that there be no change of grade between the two sections.

The writer has applied this method of calculation to observed cases of unsteady flow and has found that it gave results agreeing as closely with the observed depths as could be expected with the necessary uncertainties in the areas, grades and coefficients of friction. As applied to the experiments above described and illustrated, it gives very satisfactory results if applied between two sections where the change in shape is not abrupt. Where a sudden change occurs, the depths obtained by the formula are too small, because a part of the total head has been consumed in producing waves, side currents, etc.

Two curves are shown on the profile of the outlet section, Plate LXXVI, the upper one being calculated for uniform velocity equal to that in the sewer before a change in grade or section occurs. The second and lower line shows depths as calculated for the case of acceleration. The true depths no doubt lie somewhere between the two lines, and probably very close to the lower one, because of the

very gradual changes in the sewer section and the small amount of energy wasted in lateral currents.

In designing the outlet, a great velocity was desirable, because the depth of water in the channel would be reduced. At Station 0, therefore, the grade of the invert was changed from 0.5 to 0.9 per cent. At Station — (0 + 50) the shape of the sewer begins to depart from that of a circle, and at Station — (1 + 00) it becomes a horse-shoe-shaped channel, 16.5 ft. wide and 16.5 ft. high. The dry-weather sewage channel in the bottom is obtained by retaining the lower segment, 0.6 ft. deep, of the 16.5-ft. circle. The floor of the channel on either side rises outward on a 4% slope until it meets the vertical side wall. The lines of junction between the planes forming the floor of the channel and the curved side walls are symmetrical reverse curves in plan and are on the grade required to give a 4% transverse slope to the floor.

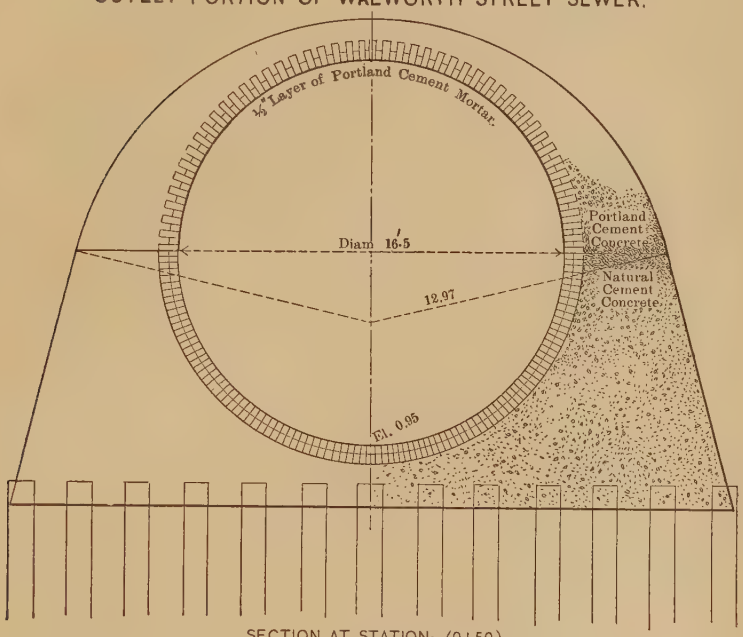
At Station — (1 + 00) the channel, while retaining the horse-shoe-shape of bottom, gradually widens, in symmetrical reverse curves, until at Station — (2 + 00) the width has been increased to 50 ft. The crown of the sewer continues on a 0.90% grade from Station — (1 + 00) until the height of the crown above the springing line of the arch is reduced to three-tenths of the span. From this point forward the crown is varied so as to keep the rise constantly three-tenths of the span. The grade of the springing line of the arch is 3.8%, which gradually reduces the height of the springing line above the floor of the sewer from 8.5 ft. at Station — (1 + 00) to 5 ft. at Station — (1 + 92).

The width of 50 ft., attained at Station — (2 + 00), continues to the river. At Station — (1 + 92) the arch terminates in a cut-stone parapet and wing walls, and the remainder of the distance to the river is bridged over with steel girders.

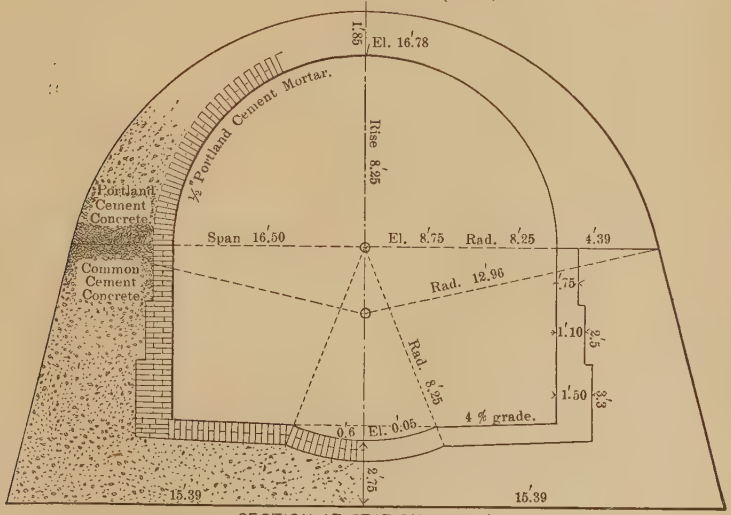
The main passenger track of the Erie Railroad passes over the arch with about 3 ft. of cover beneath the rails, and, on that account, the depth of the arch at the key was made 4 ft.

The steel girders for spanning the portion nearest the river, Plates LXXVI and LXXVIII, were designed by Mr. C. H. Haupt, Assistant Engineer, and are extra heavy for the span. This increased strength was required partly by the fact that they have less depth than ordinarily used for such a span, but principally because

OUTLET PORTION OF WALWORTH STREET SEWER.



SECTION AT STATION—(0+50)



SECTION AT STATION—(1+00)

FIGS. 7 AND 8.

the railway company required that they be strong enough to permit laying the tracks in any position. This makes it necessary for one girder to sustain practically the whole weight of an engine instead of the weight being supported by two girders.

At the point of discharge into the river, the channel is depressed, and discharges over an ogee portion, Plates LXXVI and LXXVIII. The ogee is designed so that the water will be discharged into the river at about mid-depth and in a horizontal direction. This tends to prevent any scouring effect on the bottom of the river channel, and also opposes the inertia of the river against the impact of the storm water of the sewer.

Coffer-Dam.—About 50 ft. of the outlet, including the ogee-shaped portion, was constructed inside of a coffer-dam. Guide-piles were driven so as to enclose the entire space. To them were bolted waling timbers which supported 6 by 12-in. tongued and grooved sheet-piling. The piling structure on the river side, and extending well into the banks, was made double and filled with puddle. The portion of the coffer-dam on the land side was enclosed with a single row of tongued sheeting. The structure is clearly shown by Fig. 2, Plate LXIII.

General Construction.

The work was divided into five sections and contracts. On account of several difficulties and special structures to be designed nearer the outlet, the uppermost section, extending from Gordon Avenue to about midway between Junction and Burton Streets, was let first. This section was about 4 400 ft. long, the diameter of the sewer ranging from 8 ft. to 11 ft. 6 in. Bids for this section were received on February 25th, 1897, and the work was let to Messrs. Wilson and Strack, of Cincinnati, Ohio. The principal quantities of work and the contract prices are shown in Table 3.

Ground was first broken, officially, with public ceremonies, by Mayor McKisson on the evening of March 27th, 1897. The work was carried on by the use of two cableway conveyors. A 5-ft. cubical concrete mixer, discharging into the cableway buckets, was used for part of the work, and part of the concrete was mixed by hand. A narrow-gauge railway was laid from the upper end, at a junction with the "Big Four Railway," down to Junction Street, within about 400 ft. of the lower end of the work.

PLATE LXXVI.
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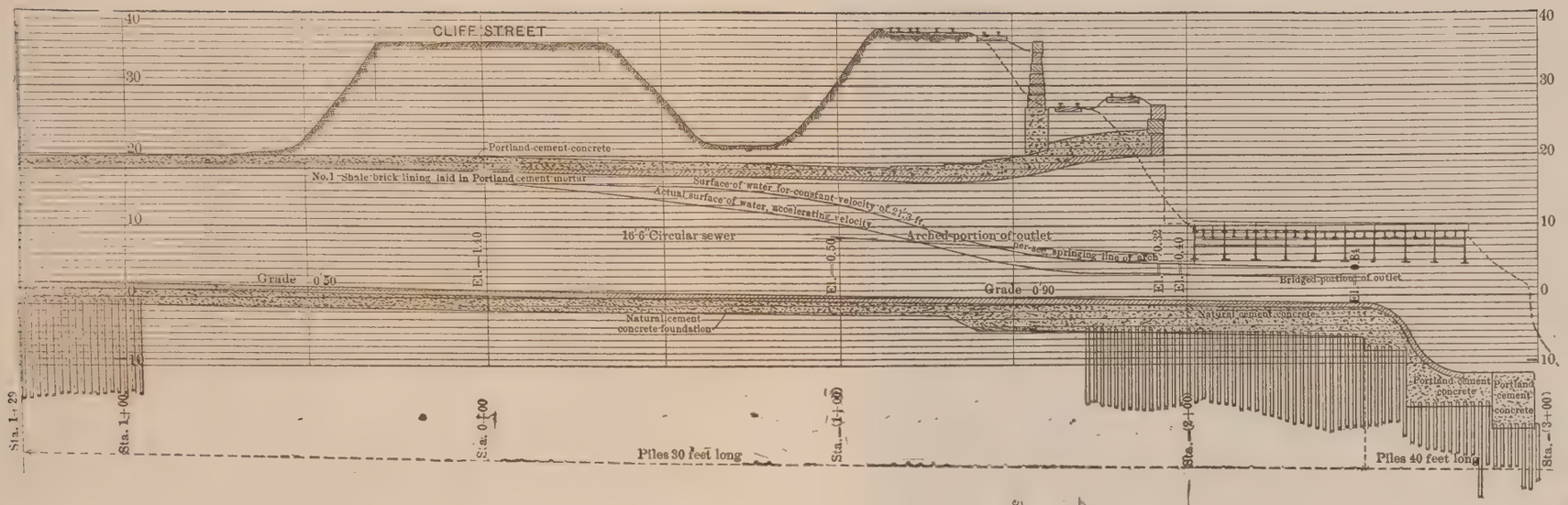
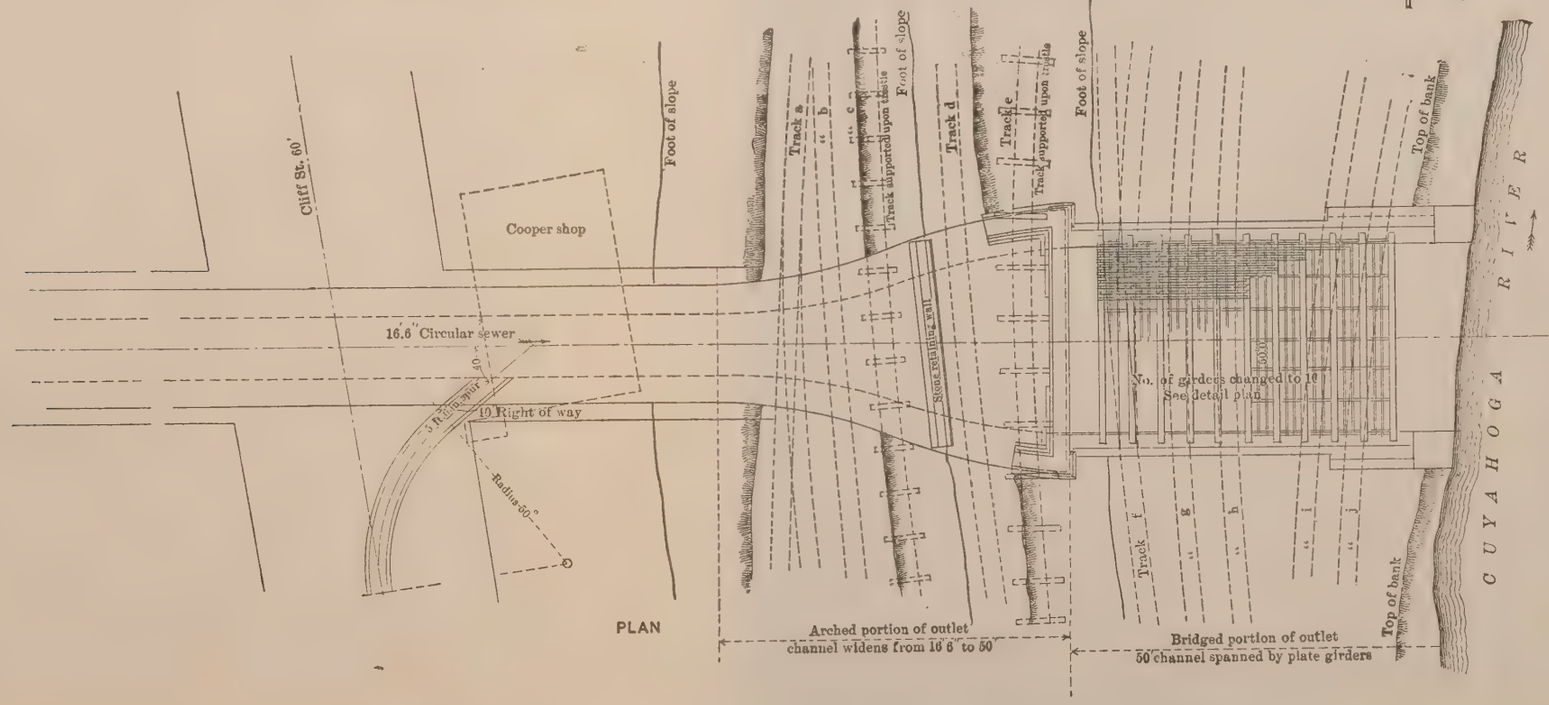
GENERAL PLAN OF THE WALWORTH STREET SEWER OUTLET SHOWING THE LOCATION OF THE INTERSECTING RAILROAD TRACKS





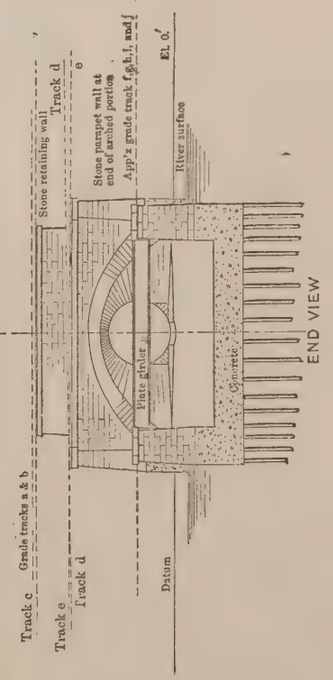


TABLE 3.—Quantities and Prices for Section 1.

Items.	Quantities.	Prices.
Excavation, including cost of sheeting and bracing..	57 600 cu. yd.	$0.30
Grillage lumber, 3-in. oak, in place.....	354 000 ft., B. M.	20.00
Natural-cement concrete, in foundations.....	9 255 cu. yd.	2.90
Shale brick, No. 1, laid in 1:2 Portland-cement mortar required for lining ring of sewer below arch and including second ring of brick in invert consisting of clay brick (not more than 15 per cent. absorption), also laid in 1:2 Portland-cement mortar.....	1 712 cu. yd.	7.50
Clay brick masonry, laid in 1:2 natural-cement mortar, required for side walls above concrete base and below arch.....	4 471 cu. yd.	4.85
Arch masonry, laid in 1:2½ Portland-cement mortar, the inner ring being No. 1 shale brick and the outer portions clay brick.....	4 582 cu. yd.	6.70
Cast-iron manhole caps and covers	8 250 lb.	0.02

The sewer was built upon a grillage consisting of transverse 3-in. oak planking laid upon 3 by 12-in. longitudinal stringers bedded in the clay bottom. To produce the cylindrical surface of the invert, different methods were tried. One method was to use a board form having the shape of a complete transverse section of the finished concrete. This form was set about 12 ft. in advance of the completed concrete, and then the concrete was rammed in place and tested frequently with a straight-edge.

Another method, which proved superior and was finally adopted, was to ram the concrete under a form not unlike an inverted segmental arch center. This form is shown in Fig. 1, Plate LXXX. After the form was moved forward to a new position the inner surface of the concrete was smoothed off with mortar.

Line and grade were given by stakes driven in the bottom of the excavation. For the concrete grade, 2 by 2-in. stakes were toe-nailed to the grillage, and the concrete was finished off flush with the tops of the stakes. The finished concrete bottom was afterward tested by the level so that the two rings of brickwork would come just to the invert grade of the sewer.

The side walls are of clay brick laid in natural-cement mortar, English bond, with three courses transverse to the sewer and every fourth course longitudinal. The courses pitch inward with a slope of 1 vertical to 4 horizontal, while the outer face is at right angles, or with a batter of 1 to 4. The bricks forming the curve, back of the single ring of No. 1 shale brick laid in 1:2 Portland-cement mortar,

are beveled roughly with a trowel, though that is a detail not required by the specifications. The backs of the side walls are plastered with natural-cement mortar.

Although clay brick were specified for the side walls, the outer ring of the invert and the outer portions of the arch, difficulty was found in obtaining brick which would not exceed the maximum limit of 15% absorption. As a result, the contractors were permitted to use shale brick having an absorption not exceeding 8%, in lieu of clay brick, for almost the entire contract. The cost per cubic yard for masonry was not greatly increased by the change, and the delays and trouble caused by the rejection of large quantities of clay brick were avoided, while the character of the work was really improved.

The arch was built upon ordinary centering with the ribs spaced at about 2.5-ft. centers. The ribs and lagging were separate, so that the forms could readily be moved forward piecemeal. V-shaped frames, upon which the centering rested, transferred the weight directly to the invert.

The lagging was lightly nailed to the ribs, so that it could be easily separated as soon as the supports for the ribs were removed.

The peculiar method of laying the brick in the arch caused the masons some trouble at first, but they soon became familiar with the bond, and the work proceeded without delay. Fig. 1, Plate LXXXI, shows the construction of a 17-in. ring of work outside of a 13-in. ring. As shown, the two rings have nearly reached a common radial plane, the outer ring, as already explained, containing one more course of brickwork than the inner one. The toothing on the end wall is also shown, and illustrates how the superimposed rings break joint.

As the arch nears the key, the courses become so steeply inclined that they would fall apart from the pressure of the fresh mortar if carried directly to the required radial thickness. The upper stretcher brick, therefore, are omitted temporarily until the section of the key has been completed along the line of the intrados. The upper surface of the ring, therefore, presents a toothed appearance, as shown in Fig. 2, Plate LXXXI, caused by the projecting header bricks. The stretcher bricks are now driven down to place in the mortar, and the upper surface of the ring is plastered over

PLATE LXXVII.
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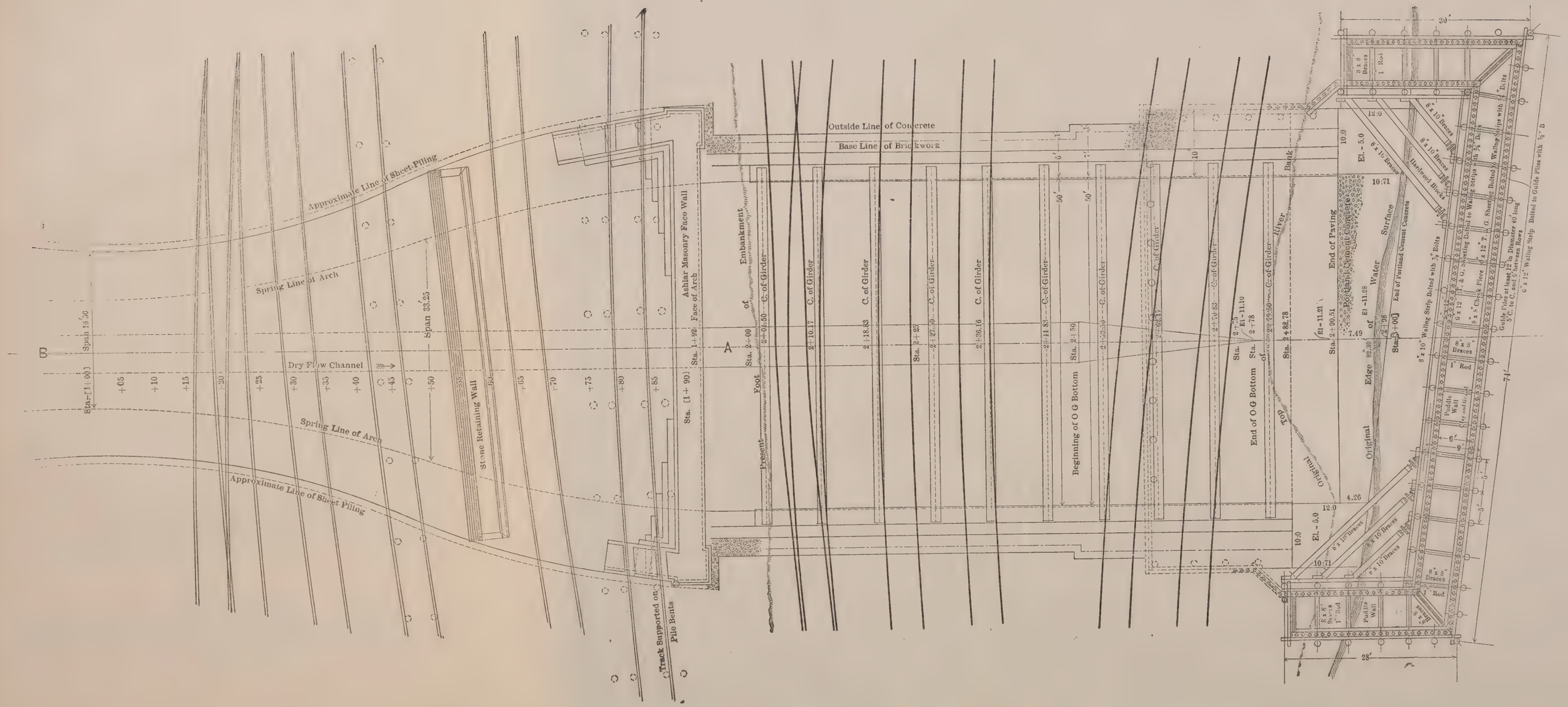
PLAN OF PORTION OF SEWER OUTLET BETWEEN STATIONS (1+00) AND —(3+00)
EXISTING PILING SHOWN BY DOTTED CIRCLES.



with mortar, thus completing the ring. The circular arc forming the figure of the extrados is obtained with a sweep, of the required radius, swinging on a nail at a point below the center of the arch. The sweep can be swung into position at will, and fixes the thickness of the arch at any point.

As explained before, the bricks are cut with a trowel so as to bring the cross-section of the extrados to a circular arc instead of allowing it to present a series of 4-in. steps where the thickness of the arch changes by one course of brick. Of course, this is only approximately accomplished. The brick cutting required is not as great as would at first appear, and the requirement, therefore, was not formidable. The entire back of the arch was plastered with a heavy coating of mortar, so as to present a smooth and continuous surface.

Walworth Run at times occupied the location required for the sewer. In these cases the stream was diverted into an artificial channel at one side. At a few other places it was necessary to cross the stream. This was usually accomplished by carrying the stream across the sewer trench in a flume, and omitting a short section of the sewer arch until the water could be diverted into the sewer through a storm inlet at the side.

At Junction and Burton Streets the location for the sewer conflicted with the old storm culverts of the stream. In these cases it was necessary to construct temporary tunnels, some distance to one side, through which the water was diverted from its regular channel. These tunnels were bricked up, making them essentially sewers 5 ft. in diameter. Although expensive, they relieved the work of the danger from flood. The old culverts were then broken down to permit of the building of the main sewer.

Work on Section 1 continued through the seasons of 1897 and 1898, and it was not until about January 1st, 1899, that it was completed, including all the necessary embankments, connections with intersected sewers, watercourses, etc.

The Second Section.

Section 2 extends from a point connecting with Section 1, about midway between Junction and Burton Streets, to within 100 ft. of Mill Street, and varies in diameter from 11 ft. 6 in. to 14 ft. 9 in.

Bids were received on August 6th, 1897, and the work was awarded to Messrs. Bramley and Gribben, of Cleveland.

The classification of the work was the same as on Section 1. The principal items are shown in Table 4.

TABLE 4.—Quantities and Prices for Section 2.

Items.	Quantities.	Prices.
Excavation	75 000 cu. yd.	$0 45
Grillage lumber	485 000 ft., B. M.	20.00
Natural-cement concrete	16 000 cu. yd.	2.80
Invert masonry	2 600 cu. yd.	6.50
Side walls	8 000 cu. yd.	5.80
Arch masonry	8 200 cu. yd.	6.25

The general methods followed were so similar to those used by the contractors on Section 1 that no special description need be given, except in a few particulars.

Instead of using a narrow-gauge railway, a standard-gauge track was built alongside the entire length of the sewer and connected with the "Big Four Railway."

A 4-ft. cubical concrete mixer, with an engine on a flat car, was equipped with overhead platform, hopper, etc. Permanently coupled to this car was another carrying the boiler, a hoisting engine, and a boom derrick. This equipment was pushed along, as the work progressed, and delivered the concrete from the mixer directly into the bottom of the ditch through a chute. The stone, sand and cement were hoisted by the derrick, to the platform, and to the hopper above the mixer, directly from the cars alongside. The whole equipment worked well, and appeared to be as efficient as any that could have been adopted.

Construction was begun about September 1st, 1897, at the east end of Section 2. A cable machine, similar to that used by Messrs. Wilson and Strack on Section 1, was adopted, and, during the autumn of 1897, a length of about 492 ft. of sewer was built. The pit for the overflow chamber was also excavated and a small portion of the concrete bottom deposited, but, owing to the severity of the weather, work was stopped until the spring of 1898, the pit being allowed to fill with water.

PLATE LXXVIII.
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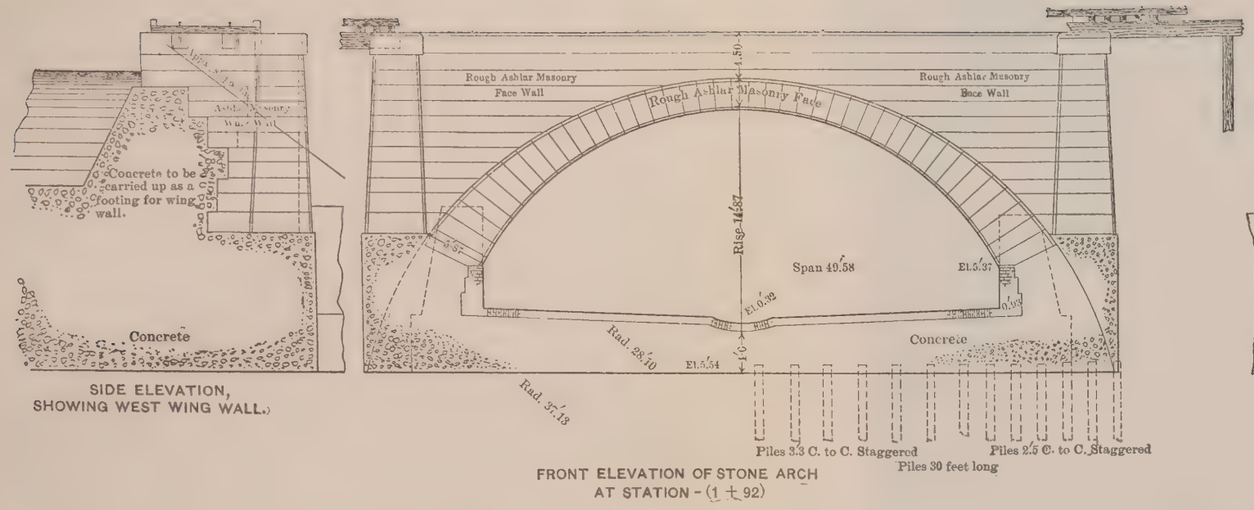
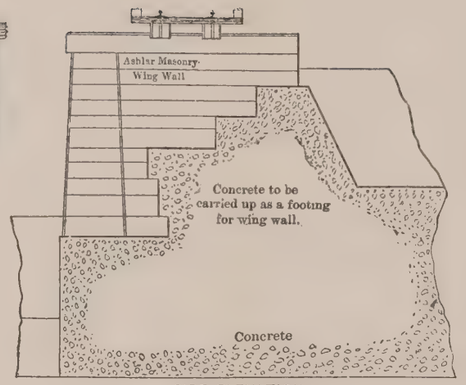
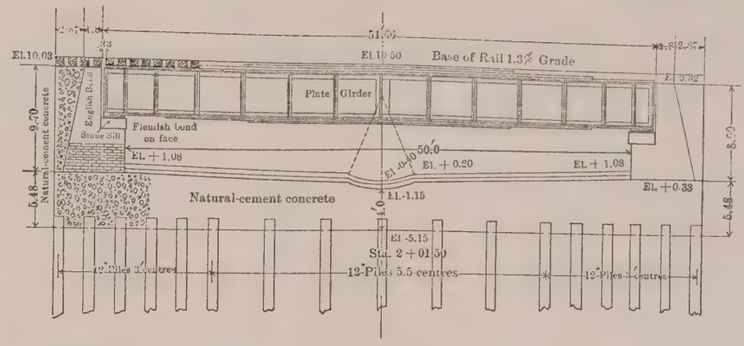


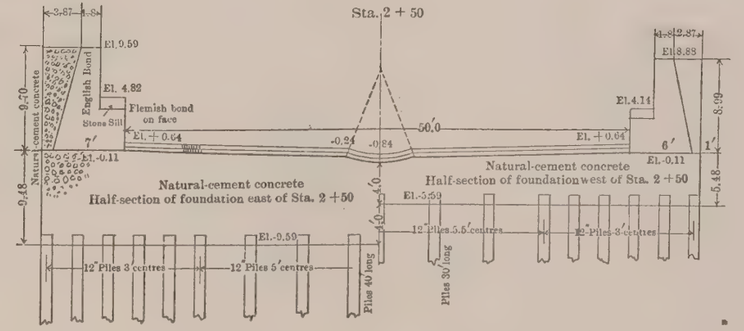
SIDE ELEVATION,
SHOWING WEST WING WALL.

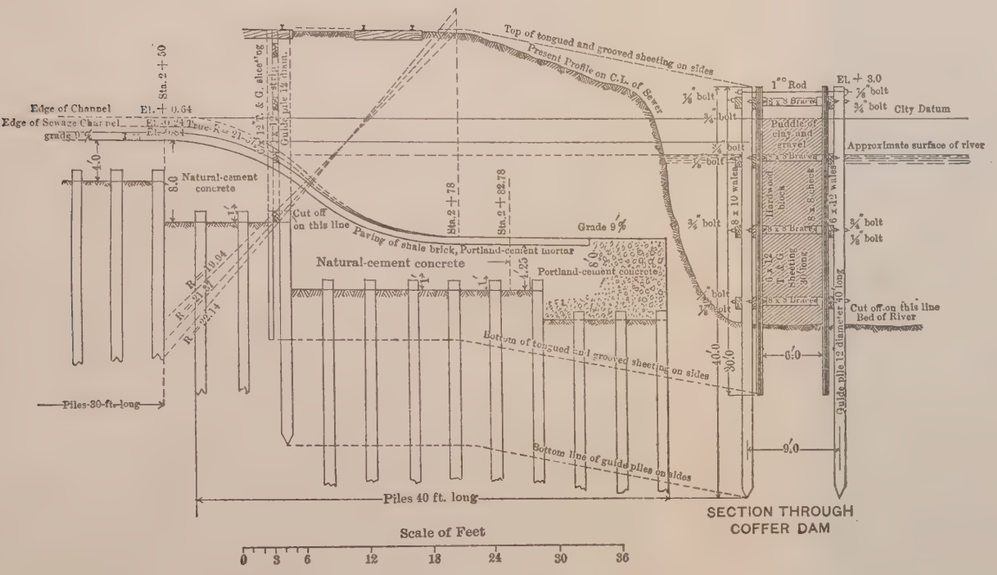
FRONT ELEVATION OF STONE ARCH
AT STATION - (1 + 92)



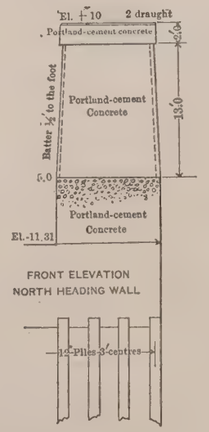
SIDE ELEVATION.
SHOWING EAST WING WALL.



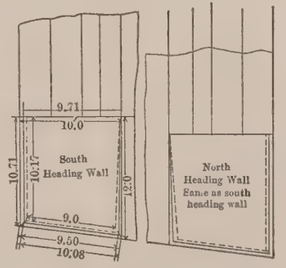



Edge of Channel
Edge of Sewage Channel
El. + 0.64
Natural-cement concrete
Top of tongued and grooved sheeting on sides
Present Profile on C.L. of Sewer
Cut off on this line
Paving of shale brick, Portland-cement mortar
Natural-cement concrete
Portland-cement concrete
Grade 9%
Sta. 2 + 78
Sta. 2 + 82.78
Bottom of tongued and grooved sheeting on sides
Bottom line of guide piles on sides
Piles 30-ft. long
Piles 40 ft. long
1" Rod
El. + 3.0
City Datum
Approximate surface of river
Puddle of clay and gravel
Cut off on this line
Bed of River
Guide pile 12" diameter 30.0 long
Scale of Feet
0 3 6 12 18 24 30 36

SECTION THROUGH
COFFER DAM

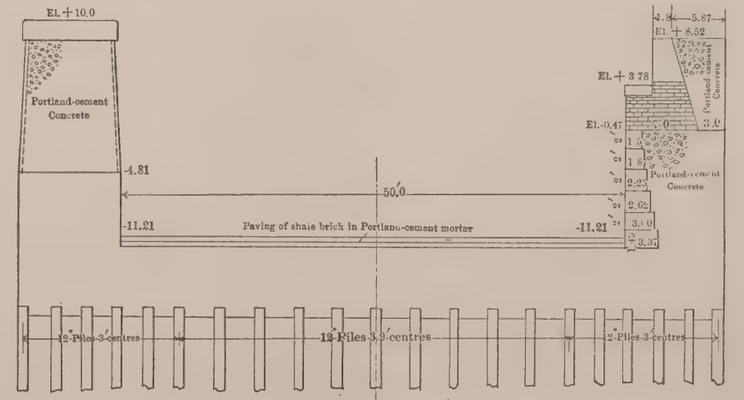


FRONT ELEVATION
NORTH HEADING WALL



PLAN OF HEADING WALLS

WALWORTH STREET
SEWER OUTFALL



SECTION AT STA. 2 + 90.51

On account of soft ground, and the fact that the grade of the 14 ft. 9-in. sewer entering the upper end of the overflow chamber was nearly 10 ft. above that of the storm overflow outlet, it was necessary to excavate 7.3 ft. deeper than the sub-grade shown upon the plans. The foundations for the overflow chamber, therefore, were carried down to practically the same level at both ends. This required the use of a large extra amount of natural-cement concrete, but it was deemed unsafe to omit the precaution. The excavation was filled in solid against the sheeting, all of which was left in place.

The Flemish bond arch rings were carried up in the same manner as for the ordinary sewer arch, the key being formed by the intersection of the various courses at their junction with each other. Where the arches terminated in the rib forming the groin the bricks were all carefully interlocked so as to give the most rigid connection possible. Owing to the fact that the 5-ft. intercepting sewer is not yet completed to this point, a 3-ft. temporary channel was built across from the lower end of the dry-weather channel and discharged back into the storm outlet, in order that the overflow sill might be completed at once. This sill is composed of a fine, white sandstone from the quarries of Peninsula, Ohio, each block being 5 ft. in length and cut carefully to the proper curves. Fig. 1, Plate LXV, shows an interior view of the chamber looking up stream, and Fig. 2, Plate LXV, a corresponding view from the upper end looking down stream.

Westward from the overflow chamber for the entire distance to a point about 425 ft. west of Burton Street, a greater or less quantity of extra concrete foundation was found to be necessary, in order to carry the foundations down to the bed of hard, blue clay. The extra depth of concrete thus required varied from 7 to 1.25 ft., with an average of from 2 to 3 ft.

A large brewery stands on the southerly bluff line just east of Rhodes Avenue, and the trench was cut into the slope about one-third of the way up from the valley. Extra heavy sheeting and bracing became necessary, therefore, in order to protect the side hill, with the great weight of the brewery, and the railway tracks which passed along the edge of the bluff immediately above. For this portion of the work, the concrete and brickwork were built up

solidly against the sheeting at the side next to the hill, in order to prevent any possible movement of the hillside. Although there was a slight cracking near the top of the bank, there was no serious settlement of the building or the railway.

It was expected that the switch track from the Cleveland, Cincinnati, Chicago and St. Louis Railway would be maintained by temporary bridging while the sewer was being built. The contractor, however, made arrangements with the railway company and the brewery interests whereby he was permitted to cut out the switch track temporarily.

During the season of 1898, the sewer was constructed westward from Mill Street to the east edge of the embankment of the C., C., C. and St. L. Railway at Burton Street. A length of about 340 ft. west from Burton Street was also constructed. During the winter of 1898 and 1899, the work west of Burton Street was carried on continuously, with the exception of short intervals of severe weather. Steam was carried from the concrete mixer or from the locomotive boiler through pipes to hot-water barrels and into pipes buried in the sand piles, so that an abundance of hot water and hot sand was available at all times. The trench was kept tightly planked over, and salamanders were kept burning inside.

Connection between Sections 1 and 2 was made on May 12th, 1899, thus completing the work of Section 2, except the portion through the Burton Street and "Big Four Railway" embankments. The part at Burton Street was built in July, 1899. The portion across the right of way of the C., C., C. and St. L. Railway is described under the heading: "Tunnel under Big Four Railway."

Third and Fourth Sections.

Bids were received for constructing Section 3, extending from 100 ft. west of Mill Street down stream to a point 822.7 ft. east from Willey Street, on October 1st, 1897; and for Section 4, continuing down stream from the last-named point to within 100 ft. of Cliff Street, near the outlet, on March 15th, 1899. As Messrs. Bramley and Gribben were the lowest bidders for both these sections, the construction of both will be considered together. The principal quantities and the contract prices for each are shown in Table 5.

FIG. 1.—WALWORTH OUTLET, LOOKING DOWN STREAM. CHANNEL 50 FT. WIDE. RAILWAY GIRDERS IN THE DISTANCE.

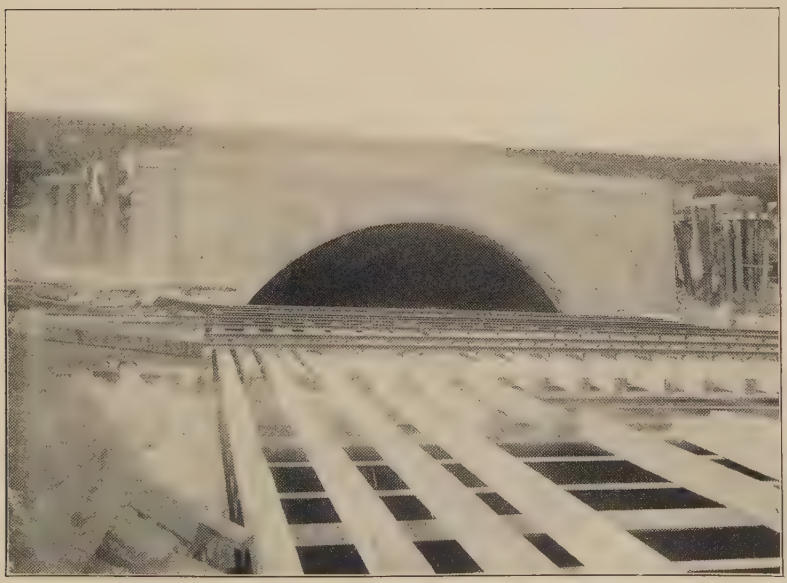


FIG. 2.—WALWORTH OUTLET. VIEW OF TERMINAL ARCH, SPAN 50 FT. ALSO SHOWING STEEL GIRDERS AND BRIDGING PARTIALLY COMPLETED.

TABLE 5.—Quantities and Prices for Sections 3 and 4.

Items.	Third Section.		Fourth Section.	
	Approximate quantities.	Prices bid.	Approximate quantities.	Prices bid.
Excavation	67 500 cu. yd.	$0.40	39 000 cu. yd.	$0.70
Concrete	14 400 " "	2.80	10 000 " "	4.00
Shale-brick masonry	1 900 " "	6.50	950 " "	8.25
Clay-brick masonry	6 500 " "	4.75	3 600 " "	6.50
Brick masonry	6 600 " "	6.25	3 400 " "	7.50
Grillage	345 000 ft., B. M.	20.00	178 000 ft., B. M.	20.00

The cable was set up just west from Brevier Street, and work began about April 1st, 1899. The work was carried on vigorously and continuously during the season, and was completed about December 1st, 1899. There were no features of construction on this portion of the work which need special mention, except at the point where the sewer passes under the viaduct of the New York, Chicago and St. Louis (Nickel Plate) Railway. At this point one of the bridge abutments conflicted with the location for the sewer. Under agreement between the city and the railway company, at the time of the opening of Walworth Street, the railway changed their bridging before the construction of the sewer had been commenced at this point.

Work was begun at Cliff Street on September 1st, 1899. At the level of the foundations of the sewer as planned, soft clay, with a mixture of loam, sand, filled material and oil from the former works of the Standard Oil Company, were encountered. An extra depth of 9.9 ft. of concrete was found necessary in order to get down to the hard clay. The trench was filled solidly with natural-cement concrete to the full width between the sheeting, thus giving a thickness of 11.9 ft. of concrete under the brickwork of the invert. It might have been cheaper to use a pile foundation, but as no price for piling was included in the contract, it was finally decided to fill the trench with concrete. This extra depth, however, gradually decreased until the normal depth of foundation was reached, about 700 ft. westward.

Where the sewer passed under the Abbey Street viaduct, it was somewhat of a problem to carry the weight of one of the trestle columns during the construction of the sewer. The contractor

solved the problem in his own way, and very satisfactorily. Traffic on the viaduct was not interrupted except by erecting barriers so as to deflect the traffic to the other side of the roadway where the weight would be carried directly to the undisturbed foundations. The post which rested upon the pier to be removed was suspended by passing a cable from the foot of the post diagonally upward to the top of the opposite post, and other cables to the tops of both posts composing the next pair of the trestle bents. By means of turn-buckles, the weight of the columns and the capstone of the pedestal attached was lifted free from the pedestal. Timber struts were also put in along with the cables in order to take up vibration. The arrangement of cable and struts is shown in Figs. 1 and 2, Plate LXXXII. The construction of the sewer was carried forward as near as practicable to the pier to be removed, and the excavation and the foundations at the sides were carried forward a short distance on either side and beyond the pedestal. Fig. 2, Plate LXXXII, shows clearly this stage of the construction. The pedestal was then removed, leaving the capstone hanging to the foot of the viaduct column, and the sewer structure was carried through to completion without interference underneath. Fig. 2, Plate LXXXII, shows the construction of the masonry of the arch, including the saddle-shaped reinforcing masonry at the sides. After the arch had been completed, the upper portion of the pedestal was rebuilt upon the back of the arch, and, after the arch had sufficiently hardened, the turn-buckles were gradually loosened and the weight brought to bear upon the new structure.

Immediately up stream from the viaduct, for a distance of about 250 ft., the clay was supposed to be sufficiently hard to carry the weight of the sewer. After the arch had been constructed, however, and the back-filling from the cableway piled in a high ridge along and over the sewer, it was discovered that there were cracks in both the crown and the invert of the sewer. The arch cracked for a distance of about 300 ft., the invert for a distance of about 100 ft. There were also a few other cracks extending several feet from the crown on either side. As soon as this was discovered, the depth of the foundation was increased until an extra depth of about 3 ft. of concrete, or a total of 5 ft. was reached, after which there was no cracking.

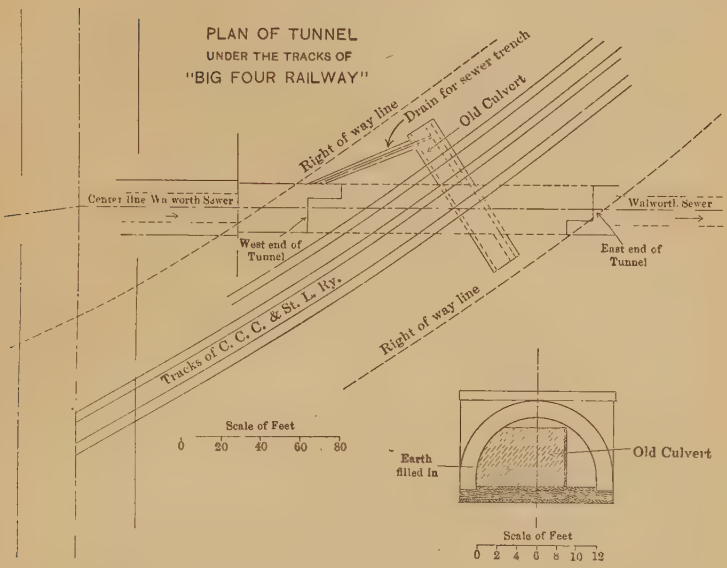


FIG. 9.

TIMBERING FOR TUNNEL UNDER THE "BIG FOUR RAILWAY" TRACKS

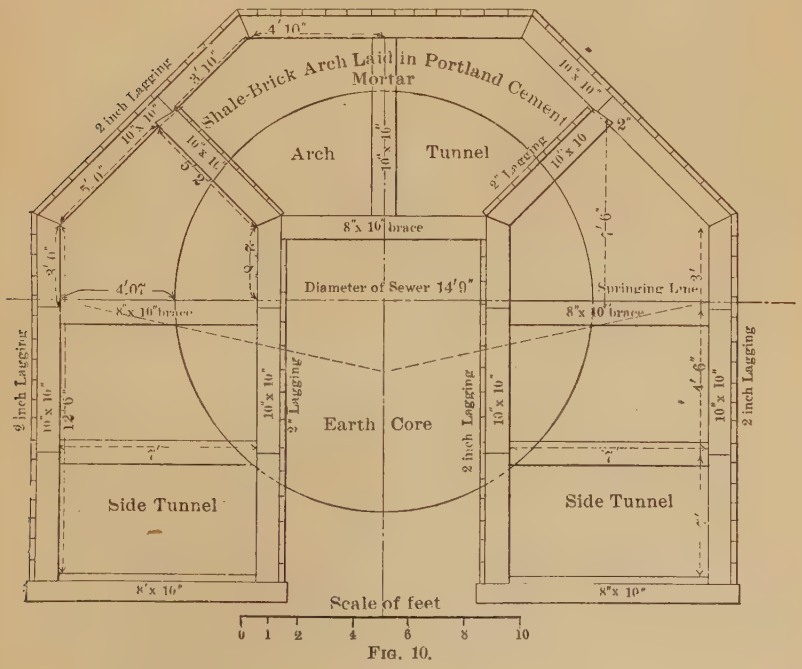


FIG. 10.

Work was carried on continuously through the winter of 1900 and 1901, and connection at Brevier Street was made on March 7th, 1901, thus completing the work of Messrs. Bramley and Gribben, with the exception of the portion in tunnel under the tracks of the "Big Four Railway."

Tunnel under "Big Four Railway."

Where the sewer intersects the C., C., C. and St. L. Railway, it was planned and contracted for on the expectation that the railway would support its tracks over the trench upon temporary trestle-work. On account of serious differences between the railway and the city, it became necessary to build about 190 ft. of the sewer in tunnel. This tunnel proved to be one of the most difficult parts of the entire work.

Under special resolution of the City Council, a supplemental agreement was made, under which the contractor was to do the work for cost, plus 15% for profit and use of equipment. Accordingly, work was begun about August 9th, 1900; it was completed on July 1st, 1901.

The location of the tunnel and of the short sections of open cutting at each end are shown in Fig. 9.

The length of the tunnel proper is 144 ft., with 16 ft. of open cut at the east end and 32 ft. at the west end, making a total length of 192 ft. The tunnel, as designed and built, is shown by Fig. 10, which, with only slight modification, was adhered to throughout.

As soon as the pit at the east end had been sunk and a grillage in the bottom laid, the work of tunneling was begun and carried on by 10-hour night and day shifts.

The side tunnels were commenced about September 5th and completed about December 19th, 1900. The ground penetrated was treacherous in the extreme, the surface of the ground-water being near the springing line of the sewer. Above this level the material was a very fine, dry sand, which required the closest lagging and bulkheading as the work progressed. In fact, the sand sifted through the cracks in the planking so badly that in many places they had to be caulked with hay. Below the ground-water level the soil changed more to a soft, silty substance which resembled

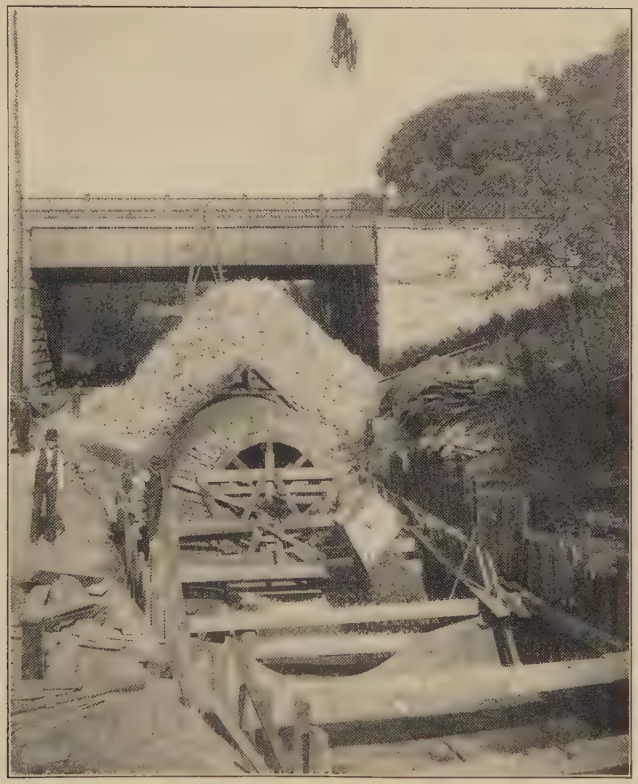


FIG. 1.—SEWER WEST OF SWISS ST., 9 FT. 9 IN. IN DIAMETER, SHOWING FIRST INVERT FORM USED.

FIG. 2.—GENERAL VIEW LOOKING DOWN WALWORTH VALLEY, NEAR OVERFLOW CHAMBER.

wet quicksand and which, to prevent caving, required even greater precautions than the dry material above.

The timbering used consisted of 10 by 10-in. sets with 2-in. lagging. The sets were spaced 3 ft. apart from center to center, and were framed so that the side struts for the roof heading rested directly on the apex joints of the top drifts of the two side tunnels. Of course, care was taken to place the timbers of the side tunnels opposite, so that the timbers of the roof tunnel would come transverse to the center line.

After the top heading of each side tunnel was driven in some distance, the middle drift was carried forward, and, still later, the bottom drift. As a rule, only one face in each side tunnel was worked at a time.

The method of supporting the timbering of the upper headings while the next lower heading was being driven is of some interest. For example, the side struts for the top heading were carried on a 3 by 10-in. mud-sill, the timber set appearing as in Fig. 11.

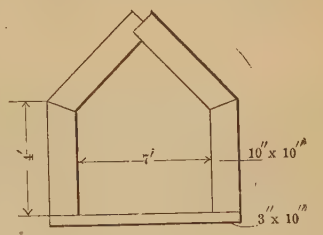


FIG. 11.

Two methods were used for holding up the upper timbers, the first being as follows: A 10 by 10-in. cross-brace was fitted between the lower ends of the side posts and immediately above the mud-sill, and held down by heavy cleats at the ends. Longitudinal timbers were then slipped under the mud-sill, the advance ends of the timbers being carried on the ground floor of the top drift and the rear ends upon jacks which rested on the floor of the lower drift. By turning up on the jacks, the weight of the upper drift was carried entirely upon the two longitudinal timbers. The ends of the mud-sill were then sawed off and the next lower section of the side struts inserted and carried upon a new mud-sill. The jacks were then removed and the 10 by 10-in. cross-timber was driven down to cover the joint, as shown by Fig. 12. The finished appearance of the timber sets of the two upper drifts is shown in Fig. 13. The second method, and one which was adopted for most of the work, was superior to the one described, and is as follows: The longitudinal

timbers were laid on top instead of under the mud-sill. The advance end of the timbers, as before, rested upon the floor of the upper drift, and the rear ends were carried on the top of the cross-brace of a completed bent of tunnel timbering. Chains were wrapped around the cross-brace and timbers, and the weight was carried by wedging up under the chains. In this way two bents of upper timbers were supported. The method is illustrated in Fig. 14.

About midway in the tunnel an old stone culvert was encoun-

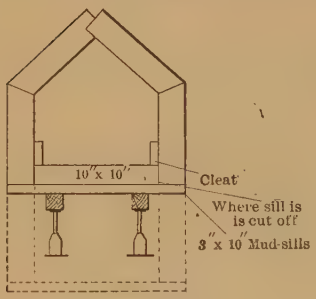


FIG. 12.

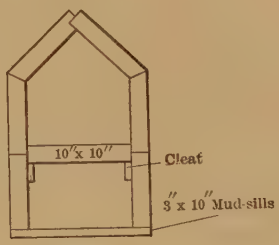
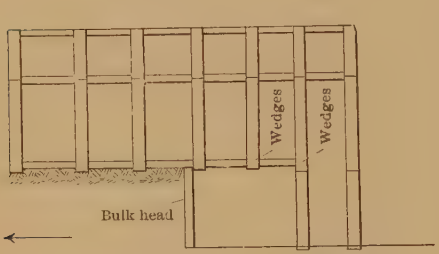


FIG. 13.



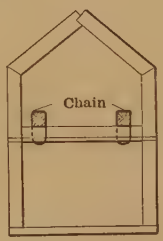


FIG. 14.

tered, which extended diagonally across the work. The culvert was supported upon a solid layer of transverse 12 by 12-in. oak timbers, with a flooring of 3 by 12-in. oak longitudinal planking. Old stumps were also frequently encountered. These were invariably very hard and sound. The old culvert and the stumps were a great annoyance, and impeded progress seriously.

The roof tunnel was driven after the side tunnels had been completed. Work on this drift was begun on December 20th, 1900, and finished on February 23d, 1901. On account of the caving nature of the ground, the face was bulkheaded in three parts—a middle

FIG. 1.– BUILDING 17-IN. ARCH RING OUTSIDE OF COMPLETED 13-IN. RING, FLEMISH BOND.

FIG. 2.—HEADER AND STRETCHER BRICKWORK AT TOP OF ARCH.

and two side sections. The bulkheads for each of these sections were pushed forward about 3 ft. and braced in position. Only enough of the earth at the roof was mined out to allow the roof timber and the side inclined posts to be set. In this way the roof was secured and the bottom part of the heading mined out in safety. As soon as the floor level was reached, the middle prop was placed.

The west open cut, lying at the foot of the railway embankment, was begun in January, 1901, and roofed over with planking. The broken stone and sand from the siding were shoveled through chutes from the cars directly upon the plank bottom of the excavation, where they were mixed and shoveled or wheeled into place. The large amount of timber bracing in the open cut and in the tunnels interfered greatly with the movements of the men, and some changes had to be made in the timbering from time to time to make passageways. The concrete for the side tunnels was wheeled in from the west end and packed tightly about all the timbering. That for the side walls was dumped from a plankway laid along the floor of the upper drift of the side tunnels.

The concrete sides were carried up to a radial plane 14° above a horizontal through the middle of the sewer. These planes inclined from the extrados of the arch downward and inward on a slope of 1 vertical to 4 horizontal.

The remainder of the arch was built of shale-brick masonry laid in 1:2½ Portland-cement mortar. The bricks were ranged in Flemish bond, as already described for the arch of the sewer in open cut. In the tunnel, however, the brickwork was carried out to a full bearing against all roof timbers. The spaces between the timber sets were packed with earth. The spaces between the vertical side posts were built in solid with brick masonry.

The centering consisted of three layers of 2 by 12-in. planks bolted together, forming ribs 6 in. thick, without any radial or cross-bracing. The outer layers of planking were one-fifth of a semi-circle in length and the middle thickness one-third of a semi-circle. The ribs were bolted together in three sections, and were thus easily taken down and re-erected. From 20 to 40 ft. of centering were set up at a time, with the ribs spaced about 3 ft. apart, so as to clear the tunnel timbering.

After the brickwork of the arch was built up to the inner roof

timbers of the side tunnels, screw jacks were set upon the arch ribs just above the inner rafters of the side tunnels. They were then turned up until the inner rafter was loosened and removed, when the brickwork was continued, giving full support to the inclined roof timber of the central tunnel as it progressed. As soon as the brickwork passed the joint between the two inclined roof timbers, the jack was removed and the entire weight carried upon the centering. The timber ribs were so stiff that no appreciable settlement could be detected when the full weight of the tunnel roof was transferred to them.

The arch was keyed by the use of jack-lagging in the ordinary manner. As the vertical props in the roof tunnel were encountered, they were sawed in two and removed.

The excavation of the core was begun on April 20th and finished on May 9th, 1901. An elevator for mine cars was erected in the shaft at the east end and the earth was removed by these cars and a tram track on the floor of the excavation. The cross-braces of the side tunnels were left built solidly into the concrete of the sides, and their ends protrude through the side walls. On account of soft foundations, the excavation for the entire tunnel work was carried from 1 to 3 ft. deeper than would otherwise have been necessary.

The concrete for the foundation under the invert was put in place by building a tram track upon timbers laid across the sewer at about the level of the lower tier of protruding timbers shown in Fig. 2, Plate LXXXIII. The concrete was brought down in cars and dumped into the bottom from this track. The work was carried on from the farther end of the tunnel, working backward toward the shaft. The protruding timbers were then cut off far enough back to permit the brick lining of the tunnel to be laid. After all the masonry had been completed in the tunnel, the portion of the sewer in open cut at the east end was built up, thus completing the tunnel.

Progress reports and book accounts of the expenses of the work, kept in the engineer's office, have made it possible to compile Table 6, showing approximately the cost of the principal parts of the work. The costs shown in Table 6 are for labor only, but, in the case of the tunnel excavation, they include the cost of framing and putting all timberwork in place, and, in the case of the core

FIG. 1.—CONSTRUCTION OF SEWER AT ABBEY VIADUCT, REQUIRING REMOVAL OF PEDESTAL. VIEW LOOKING DOWN STREAM.

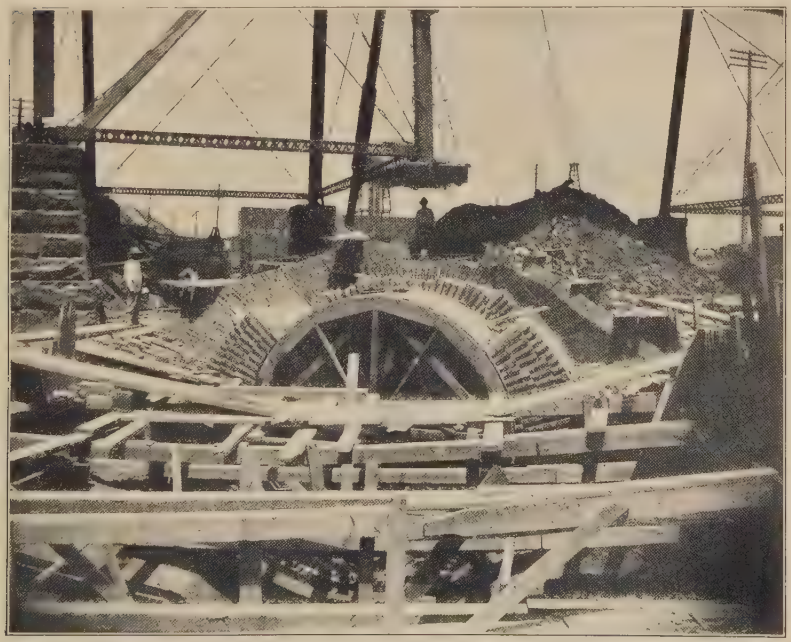


FIG. 2.—CONSTRUCTION UNDER LEG OF TRESTLE SUPPORT OF ABBEY VIADUCT, AND SHOWING EXTRA SADDLE OF MASONRY TO SUPPORT WEIGHT OF VIADUCT. VIEW LOOKING DOWN STREAM.

excavation, the removal of the central part of the tunnel timbering. The costs for the masonry and concrete include the cost of engineers and watchmen on the railway tracks, and other expenses in and about the work during the time it was in progress. It was not possible to be very accurate in any of the figures, for the reason that there was at times doubt as to what part of the work certain expenses should be charged. The shifting of workmen from one part to another at irregular intervals also led to some confusion, but the figures given are as nearly correct as can ordinarily be obtained for work of such character.

TABLE 6.

Items.	Quantities.	Cost per Yard.
Excavation in side drifts	1 706.67 cu. yd.	$5.72
Excavation in center drift	517.87 " "	6.01
Core excavation	499.84 " "	1.25
Excavation in west open cut	572.97 " "	1.84
Concrete, labor	1 096.10 " "	1.36
Arch masonry	611.15 " "	5.46
Total cost		$19 400.28

In explanation of the high cost of the west open cut, it may be stated that the material was very sticky, wet clay, and, owing to the width and depth of the excavation, required from six to eight men in line to scaffold it out. The cost of labor for the arch masonry included the cost of bringing the materials into the tunnel, storing them during the cold weather and rehandling them when the masonry was built.

The entire tunneling operations were carried on, under the direction of the engineer in charge, by a force of tunnelmen who had had much experience in other tunnels in Cleveland and elsewhere.

Construction of Section 5, or the Outlet Section.

As a length of 430 ft. at the outlet was not let until after most of the upper portions of the work had been constructed, some changes were made in the masonry for this portion. These changes were all made for the sake of economy, and not because of faults in the portions already built.

During the preceding three years, the cost of brick masonry for sewers had increased, while the cost of both natural and Portland cement had decreased. The outlet section, therefore, was constructed almost entirely of concrete. Natural-cement concrete was used for the foundations and up to the center of the sewer at the sides. The archwork above the center was built of Portland-cement concrete with an inside layer of header and stretcher brickwork. The lining of the invert is of the hardest shale brick, the same as for other portions of the work. The arch was built with a paper and thin Portland-cement mortar lining, as for the other portions of the sewer.

On account of the soft foundations and large extra quantity of concrete that became necessary in Section 4, a pile foundation was used for all the outlet sections.

Except for the central portion of the sewer, the piles are 2.5 ft. from center to center, and staggered. This arrangement makes the rows about 2.2 ft. apart, and the calculated weight upon each pile ranges from 15 to 18 tons.

For the central portion, where the weight sustained is less, the distances between the rows remain the same, but the piles in each row are spaced 3 ft. apart, with the piles staggered.

This section was awarded to Messrs. W. C. Gayer and Company, of Cleveland, and was begun about October, 1900. The specifications required the contractor to include all cost of supporting the railroad tracks in the items of sewer construction. As a consequence, there was a great diversity of opinion among the various bidders as to the allowance therefor. Some placed it on one item and some on another.

Table 7 shows the quantities and contract prices.

The contractors did not install cable machinery, as had been done for the preceding sections. The westerly portion, or up stream from the railway tracks, was taken out with wheel and slip-scrapers, assisted near the bottom of the excavation by an engine and long cable.

On account of water, the excavation was carried to within about 4 ft. of the finished subgrade when the work of pile-driving was begun. The sides were supported by cross-braces and by round piling, driven along the margins of the trench, about 5 ft. from

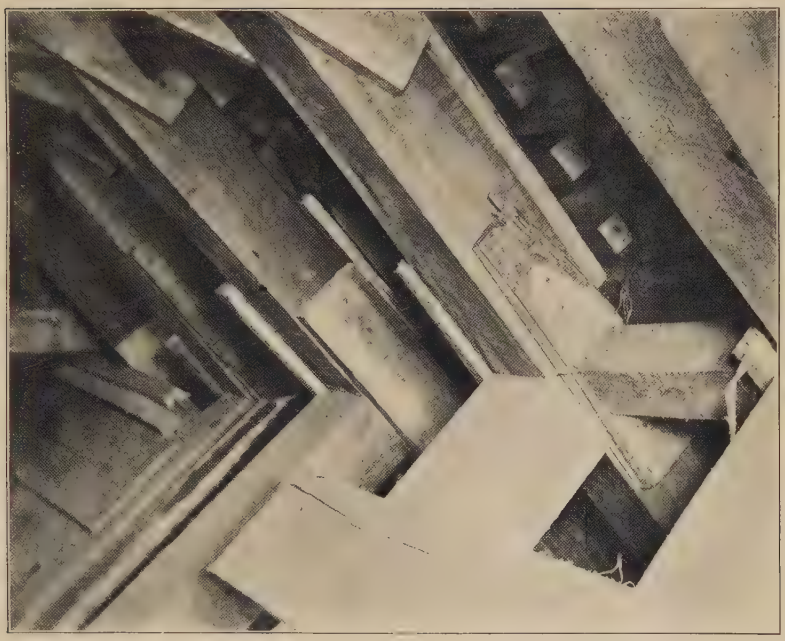


FIG. 1.—TUNNEL UNDER "BIG FOUR RAILWAY." TIMBERING AT PEAK OF NORTH SIDE TUNNEL, AND SHOWING, ALSO, SIDE ROOF TIMBERS OF TOP TUNNEL HEADING.

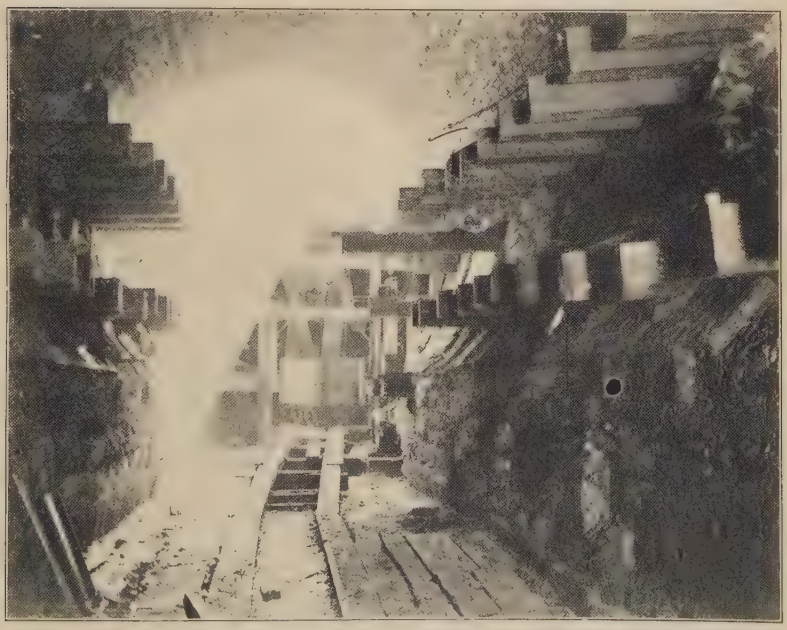


FIG. 2.—TUNNEL UNDER "BIG FOUR RAILWAY." INTERIOR VIEW SHOWING CORE REMOVED BEFORE THE INVERT WAS BUILT. INSIDE DIAMETER, 14 FT. 9 IN.

center to center. As the earth was removed, horizontal planking was inserted behind the piling. A portion of the foundation piling was driven by the use of a long iron follower, with the pile-driver in an elevated position, but it was found that nearly double the work could be done by placing the driver in the bottom of the ditch.

TABLE 7.—Quantities and Contract Prices.

	Estimated Quantities.	Unit Prices.	Amounts.
Excavation	22 000 cu. yd.	$1.25	$27 500.00
No. 1 shale-brick masonry	1 100 " "	9.00	9 900.00
No. 2 " " "	20 " "	8.00	160.00
Portland-cement concrete	1 900 " "	7.00	13 300.00
Natural " "	4 300 " "	4.50	19 350.00
Ashlar masonry	320 " "	10.00	3 200.00
Sill blocks	100 cu. ft.	2.00	200.00
Rubble masonry	95 cu. yd.	8.00	760.00
Steel girders	280 000 lb.	0.05	14 000.00
Oak flooring	50 000 ft., B. M.	30.00	1 500.00
Puddle	500 cu. yd.	2.00	1 000.00
Wrought iron	11 000 lb.	0.15	1 650.00
Oak, tongued and grooved, sheet-piling	72 000 ft., B. M.	100.00	7 200.00
Waling and bracing for coffer-dam	25 000 " "	100.00	2 500.00
Foundation piling	85 000 ft.	0.25	21 250.00
Coffer-dam piling	3 000 ft.	0.50	1 500.00
Sheeting and bracing	75 000 ft., B. M.	20.00	1 500.00
Catch-basins	2	30.00	60.00
Catch-basin pipe, 12-in.	40	1.00	40.00
Cast-iron inlet covers	900 lb.	0.04	36.00
Cast-iron manhole covers	800 lb.	0.04	32.00
Total of contract			$126 638.00

In order to test the stability of the foundation and the supporting power of the piles, a test pile was loaded with pig iron. No appreciable settlement occurred under a load of 25 tons, and the driving of other piles 5 ft. distant caused the loaded pile to rise.

Where the sewer passes under the upper set of railway tracks, there was considerable difficulty in making the excavation and driving the piling for the foundation. The railway company supported the most westerly track by using one of the 55-ft. steel plate girders for the 50-ft. channel at the outlet. The second track was supported on an 80-ft. plate girder, and the next two tracks were carried on 100-ft. steel through trusses; these trusses and the 80-ft. plate girder were old bridges which the railway company had on hand. As the contract required that all cost of bridging the railway tracks was to be included in the various items bid upon the work, the con-

tractor made his own arrangements for placing and removing these structures. After the sewer was built, the earth was filled in for the most part by dumping through the bridging from cars on the railway track. A portion of the filling, however, was scraped in from the west side with teams.

The pile-driving under the bridges supporting the railway tracks was particularly tedious and difficult. The piles were in 25-ft. lengths, as longer ones could not be handled. The excavation was made with sloping banks, in preference to using vertical sheeting and bracing, and marginal piles were driven to support the bridges and prevent the banks from caving. The vibrations caused some settlements, and the work required careful watching to prevent accidents to trains passing overhead.

The piling was cut off to a subgrade and embedded 1 ft. in the foundation concrete. After the side walls were built and the arch centering placed for the portions under the bridges, a cubical concrete mixer was erected on an elevated platform. The materials were supplied to the mixer by derricks, and, when mixed, the concrete was delivered to tram cars operated on top of the arch centering.

The site of the coffer-dam was roughly excavated by a river dredge. Most of the sheet-piling for the coffer-dam was driven during the fall and early winter of 1900. Wakefield sheet-piling of three thicknesses of 2-in. elm, maple or beech planking was substituted for 6-in. tongued oak timber sheeting, the contractor accepting $15 per thousand less than the contract price.

The structure was braced and tied as called for in the plan for the part above the water line, but the cross-bracing and bolting below the water line, on account of the difficulty in placing it, was omitted. During the spring of 1901, the space between the inner and outer rows of sheet-piling was filled with blue clay dredged from the river bottom. Later, when the coffer-dam was pumped out, it was found that many of the round piles in the inside row had been seriously bent, and several of them broken, by the pressure of the puddle. It was evident from the indications that the failure occurred before the water was pumped out of the dam, which showed that the pressure of the puddle wall was much greater than that of the water. If the plan had provided for strong metal ties below the water line to

FIG. 1.—TUNNEL UNDER "BIG FOUR RAILWAY." BULKHEADED TOP DRIFT, SHOWING TIMBERING.

FIG. 2.—TUNNEL UNDER "BIG FOUR RAILWAY," SHOWING ARCH MASONRY AND SECTIONAL RIBS.

prevent the inner and outer walls of sheet-piling from spreading, the failure could not have occurred. On the other hand, if the bolts and waling timbers below the water surface, called for by the plan, had been provided in the structure as built, this defect would not have been discovered at all, or would have been much less serious. Aside from this defect, the plan fulfilled all the requirements. No serious leaks occurred, and, by interior bracing, the bent and broken piles were sustained in position.

The foundation piles were driven by carrying the driver upon interior cross-braces and timbers, all of which were supported upon temporary piles standing at an elevation about 5 ft. above the water level. A follower was used to force the piles down to the proper grade, but was a source of endless trouble, as timbers were soon splintered and several iron followers were broken. The spacing of the piles, too, was very irregular, and this method of driving is not recommended, either from the standpoint of the city or the contractor. Much more rapid and satisfactory progress was made where the pile-driver could be set down close to the work.

TABLE 8.—Total Cost of Walworth Sewer.

Section 1, Wilson and Strack, Contractors..	$113 966.25
Section 2, Bramley and Gribben, " ..	277 769.83
Section 3, " " " " ..	179 170.19
Section 4, " " " " ..	139 939.55
Section 5, W. C. Gayer and Company, " ..	128 068.41
Total to contractors.....	$838 914.23
Add cost of inspection, and miscellaneous expenses	31 567.42
Total cost.....	$870 481.65

During the latter part of the work on the outlet section (Section 5) considerable difficulty arose, as the progress was not satisfactory to the city. The contract time for completion was July 1st, 1901, and as the work was still unfinished, in July, 1902, the city took partial possession and completed the sewer. This was done by

employing a special foreman who worked directly under the orders of the engineer in charge. This course on the part of the city led to prolonged litigation with the contractors. This litigation was settled by arbitration in December, 1905, at the figures given in Table 8. Table 8, therefore, gives the final cost of the sewer.

The concrete for all the foundations within the coffer-dam was made with Portland cement. The channel and walls were built up as shown on the plans without any difficulty. The terminal buttresses on each side of the channel are of solid Portland-cement concrete, and make very strong and suitable protection against snubbing from passing vessels.

The steel bridging was sublet to the King Bridge Company, of Cleveland, and was erected without any incident of interest. The bridge seats were of Portland-cement mortar, instead of cut stone, as provided in the plan, and produced a result superior to stone.

DISCUSSION.

Mr. Force. C. G. Force, M. Am. Soc. C. E. (by letter).—Notwithstanding the fact that the writer's official connection with the Department of Public Works of Cleveland was severed prior to the completion of the Walworth Street sewer, he has not allowed his professional interest in that work to abate, and he now desires to express his gratification on its successful completion, and his appreciation of the scholarly manner in which the author has prepared this paper.

As adding to that portion of the paper devoted to the historical side of the work, the writer refers to his annual report, as Chief Engineer of the Department of Public Works of Cleveland, for the year 1892, pages 205, 206, and 207, on the subject of Walworth Street sewer.*

Inasmuch as the opinion has been expressed by sewerage engineers that the capacity of this sewer is unnecessarily large, that the velocity allowed is too great, and that the arch and walls of the sewer are excessively heavy, the writer regrets that the time at his disposal, at present, will not admit of a review of his observations and conclusions upon which the controlling features of the design of the sewer are based.

Mr. Gregory. C. E. Gregory, Assoc. M. Am. Soc. C. E.—The unusually great rate of run-off provided for in the Walworth sewer does not seem to be entirely justified by the reasons set forth by the author. While it is true that it is desirable under the circumstances to provide for the worst conditions ever likely to occur, and to allow for the tendency of storms to follow along the direction of the trunk sewer, it has not been shown clearly just how severe are the storms which have been provided for, nor how much the motion of the storms traveling down the ravine would affect the time in which the water would concentrate, and consequently the rate of run-off.

The formula selected for determining the run-off from the laterals is more safe than either the Burkli-Ziegler or McMath formulas, inasmuch as it makes the value of f slightly increase as a function of the size of the water-shed, as it does also in the New York formula; while in the two other formulas mentioned it decreases as a function of the acreage. The rainfall provided for, however, as shown by the equations of the intensity curves, is the same as for the Burkli-Ziegler formula, as is shown by the following demonstration.

As was shown in the speaker's discussion of the paper by C. W. Sherman, M. Am. Soc. C. E., on rainfall in Boston,† the average velocity for any water-shed (using the author's notation) is expressed by $84\sqrt[5]{AS^2}$. The value for f for completely built-up areas having

* This report is on file in the Library of the Society.

† *Transactions*, Am. Soc. C. E., Vol. LIV, p. 183.

Mr. Gregory. 90% impervious surface was shown by the Hering gaugings of the Sixth Avenue sewer to be 0.50 of the average intensity of rainfall for the time required for the water to concentrate at any required point. This value of f, however, has been observed to increase to as high as 80 or 90% of the average rate of rainfall, but this has occurred on short-time water-sheds, after a rain lasting, say, ten times the time of the water-shed, and consequently having a resultant run-off much less than for the maximum short-time rain of four or five times greater intensity with f at 0.50. It has been quite generally accepted that the relation of run-off to rainfall varies directly with the percentage of impervious area, but the speaker thinks it may be reasonably expected that a long-time rain on a long-time water-shed should cause a slightly greater percentage of run-off than a short rain, for the reason that the rainfall on the areas near the outlet will have gone far beyond the time required for its concentration, and consequently they will have a greater value for f than the most distant areas, the first run-off of which has just reached the outlet. This possibility is mentioned to account for the increase of f as a function of the acreage in the author's formula. The speaker knows of no reliable experiments to prove or disprove this relation.

Write the formula, $Q = A\,f\,R\sqrt[6]{\frac{S^{\frac{3}{2}}}{A}}$, and if $f = 0.62\frac{1}{2}$ and $R = 4$, then $f\,R = 2.50$, but f should be 0.5, therefore $R = 5$. The right hand member may be divided into a formula giving intensity of rainfall, $R\sqrt[6]{\frac{S^{\frac{3}{2}}}{A}}$, the percentage running off, f, and the tributary area, A. For a small area, such as would render the value of the radical unity, the intensity of rainfall would be equal to the value of R, or at the rate of 5 in. per hour, a very high rate for a 10-minute rain.

It now remains to be seen how nearly $5.0\sqrt[6]{\frac{S^{\frac{3}{2}}}{A}}$ corresponds with the probable intensity of precipitation. It has often been shown that i is a function of t, or that $i = f\,t$, that $t = \frac{l}{v}$, in which i is average intensity of rainfall; l, length of water-shed; v, average velocity, in feet per minute, and that, by substitution in standard formulas for flow, $v = 84\sqrt[5]{A\,S^2}$; therefore $t = \frac{l}{84\sqrt[5]{A\,S^2}}$. Express l in terms of A, as follows: In a rectangle the length of which in feet is x, the width being $\frac{x}{2}$, the area in square feet is expressed by $\frac{x^2}{2}$. If the

two sides slope toward a central conduit, the greatest length of sewer will be $x + \frac{1}{2} \times \frac{x}{2}$, or $\frac{5}{4}\, x = l$, and the area, in terms of l, is $\frac{8}{25}\, l^2$. The area in acres, therefore, is $\frac{\frac{8}{25}\, l^2}{43\,560}$, or $A = \frac{l^2}{136\,100}$ and $l = 349\,\sqrt{A}$. Put this value in place of l, and add 10 minutes for the water to reach the sewers, and we have $t = \frac{349\,\sqrt{A}}{84\sqrt[5]{A\ S^2}}$ $= 4.2\sqrt[5]{\frac{A^2}{S^2}} + 10$, or $t - 10 = 4.2\sqrt[5]{\frac{A^2}{S^2}}$. Mr. Gregory.

Therefore $S = \left(\frac{4.2}{t - 10}\right)^{\frac{10}{4}} A^{\frac{3}{4}}$. Substitute for S, in $R\sqrt[6]{\frac{S^2}{A}}$ and $i = R\sqrt[6]{\left(\frac{4.2}{t - 10}\right)^{\frac{15}{4}} A^{\frac{1}{8}}}$, or $i = \frac{2.45\ R}{(t - 10)^{\frac{5}{8}}} \times A^{\frac{1}{48}}$. If f is considered a function of A, then this formula provides for rains of average intensity, as expressed by $i = \frac{12.25}{(t - 10)^{\frac{5}{8}}}$, and the percentage running off by $f A^{\frac{1}{48}}$ or $0.5\ A^{\frac{1}{48}}$.

In like manner the Burkli-Ziegler formula gives a rainfall curve of $\frac{12.25}{(t - 10)^{\frac{5}{8}}}$, but the percentage of run-off varies as $\frac{f}{A^{\frac{1}{16}}}$, decreasing as A increases. If the numerator is kept constant, but the length of the water-shed is changed to, say, nine times its breadth, the value of R becomes $\frac{12.25}{8^{\frac{5}{8}}} = 3.36$, or two-thirds of the value for the rectangle twice as long as wide, and about half the value for a square, and shows the unreliability of a formula of this form. It also varies with the value of the coefficient of roughness in the formula for flow of water, and is only correctly applicable to average-shaped water-sheds with ordinary conduits to carry off rain water.

The effect of a storm traveling down the line of the main sewer was correctly stated by the author when he said that the time required for the concentration of the water at the outlet would be shortened, but it is neither shown nor estimated how much it would be shortened or what effect the shortening would have on the run-off.

To show just what the effect would be requires a careful analysis

Mr. Gregory.

of the probable rainfall, and its effect on each branch. If the storm center or zone of great intensity traveled down the valley at a rate equal to the velocity in the sewer it would doubtless concentrate the maximum flow, from the areas directly adjoining, in the main sewer. The storm, however, could not at the same time move along the main sewer and normal to it along the laterals. The maximum from each of these branches would occur only after an average maximum rain for the time of the branch, which would be after the maximum flow from the adjoining areas had passed on. As the far greater part of the areas is tributary to the branches, the maximum in the main could occur only when the branches were contributing their maximum quantities. The total time of the water-shed in this case depends upon the large branch in Mill Street, south, and would be unaffected by the motion of the storm over all points above this branch. Just what the effect of the other branches and the main line areas above this point would be can only be shown by tracing each along its course to the outlet. It seems to be obvious that it is wrong and misleading to consider that the maximum concentrations from all the branches, the times of which vary through a wide range, would or could be brought together at the same time in the trunk sewer.

The author argues that the smaller branches at the Clark-Gordon junction should have a maximum at the same time as the longer main branch because the greatest intensity might occur at the end of a 20-minute period. If this reasoning (although it sounds plausible) should be carried to its conclusion, then the entire system would be designed for the maximum of the smallest branch. It is true that the maximum might occur at the end of the period, but it is also true that, if it did, there are equal areas at the farther end of any water-shed of average shape which will have correspondingly less intensity, and the whole will average for the time of the largest branch.

Experiments show that the maximum is reached, even in storms where the greatest intensity occurs at the latter end of a storm, only after such time, after the great intensity, as is required for the water to concentrate from the most distant point. Fig. 15 shows such a storm, gauged in the Sixth Avenue sewer, New York City, where the area is 221 acres, 90% of which is composed of impervious roofs or pavements, and the time of concentration is 25 minutes. The diagram is plotted by taking the times as abscissas and the rates, in cubic feet per acre, of both rainfall and run-off, as ordinates, and shows clearly just what actually happened in a similar case.

Starting with the Clark and Gordon Streets junction, there is a maximum concentration in 21 minutes, assuming that the storm travels along the trunk sewer at a rate equal to the velocity in the sewer, the zone of great intensity will reach the Alum Street branch,

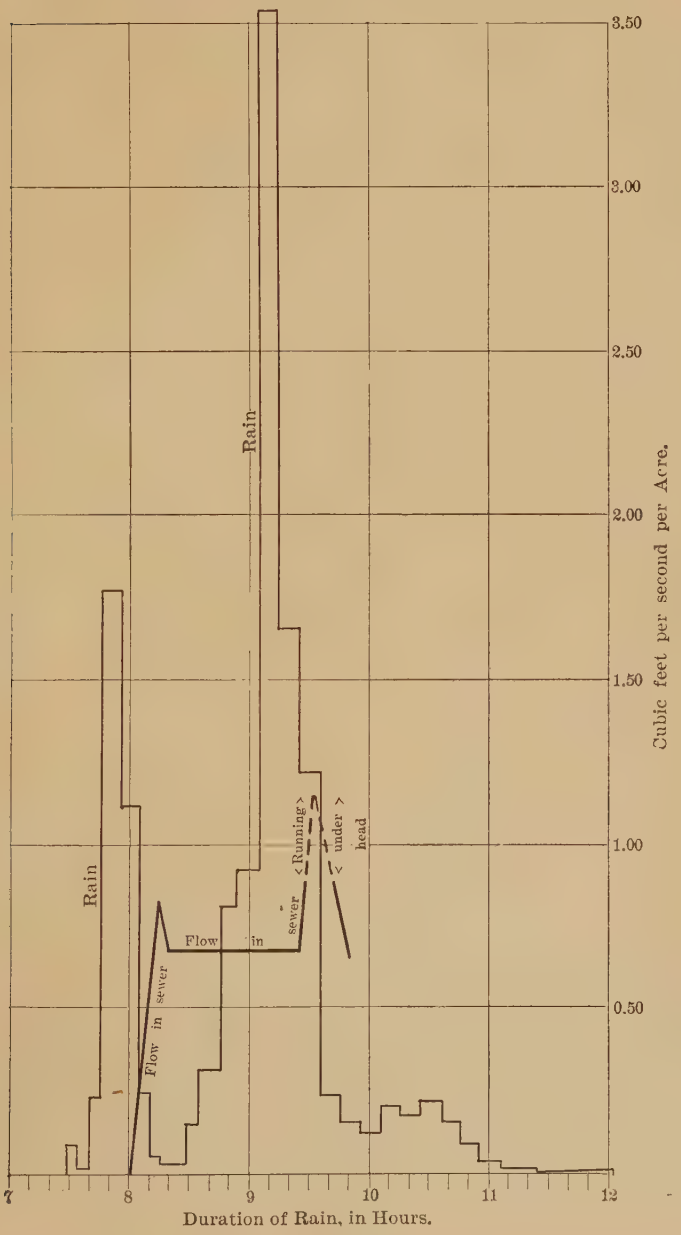

STORM OF AUGUST 4th, 1888, IN SIXTH AVENUE SEWER.
FROM 221 ACRES, 90% IMPERVIOUS ROOFS AND PAVEMENTS.
TIME OF CONCENTRATION, 25 MINUTES.
3.50
3.00
2.50
2.00
1.50
1.00
0.50
Cubic feet per second per Acre.
Rain
Rain
Flow in sewer
<Running under > head
Flow in sewer
7
8
9
10
11
12
Duration of Rain, in Hours.

FIG. 15.

Mr. Gregory.

say, 1 minute later, then the maximum run-off in the Alum Street branch will occur 21 + 1 = 22 minutes after the maximum rain at the Clark and Gordon Streets junction and 1 minute later than the maximum run-off. Allowing 1 minute for the wave to reach Alum Street from Clark and Gordon Streets, it will arrive in 22 minutes, or just as the maximum from Alum Street arrives at the same point, and will be equivalent to a 21-minute rain over the entire area down to and including Alum Street. The small branches between here and Junction Street, south, will all have their maxima before the wave from Alum Street reaches there, and consequently must be estimated for the time of Alum Street, 21 minutes. The branch at Junction Street, south, requires 25 minutes to concentrate, and the zone of intensity will arrive 3 minutes later than at Alum Street, and the maximum concentration will occur 3 + 25 = 28 minutes later than the maximum rain on the Alum Street branch. The maximum from Alum Street will arrive 21 + 3 = 24 minutes after the maximum rain on Alum Street, or 4 minutes before the maximum occurs in Junction Street, and will be equivalent to a rain lasting 21 + 4, or 25 minutes; or, in other words, the maximum run-off just below Junction Street will occur for a rain of 25 minutes' duration covering Junction Street, north and south, and all areas above this point. Following this method to the end, the results show that the time for concentration at any point is the time of the longest time branch which has been passed; and the time for the total area is the time for the Mill Street, south, branch, or 45 minutes, and 3 minutes less than if the intensity had occurred

TABLE 9.

(1) Names of Branches.	(2) Time required for maximum in each sewer, in minutes.	(3) Time required for maximum in trunk sewer, in minutes.	(4) Total Acres.	(5) Total cubic feet provided for.	(6) Rate of rainfall provided for.	(7) Rate of rainfall by Talbot's formula: $i = \frac{105}{t + 15}$	(8) Rate of rainfall by Sherman's formula: $i = \frac{38.64}{t^{0.657}}$
Clark-Gordon	21	21	485	815	3.4	2.91	4.87
Alum	21	21	546	958	3.5	2.91	4.87
Five small branches	11	21	653	1 193	3.66	2.91	4.87
Junction	25	25	911	1 637	3.6	2.63	4.40
Burton	32	32	1 434	2 390	3.33	2.23	3.70
Pollock	12	32	1 453	2 434	3.34	2.23	3.70
Mill	45	45	2 292	3 388	2.86	1.75	2.80
Pearl and Brevier	19	45	2 508	3 801	3.02	1.75	2.80
Kenilworth	39	45	2 924	4 301	2.94	1.75	2.80
Scranton and Cliff	19	45	2 982	4 475	3.00	1.75	2.80
		48				1.7	2.70
		12				3.9	7.5

simultaneously over the entire area. Table 9 shows the times of the several branches, the acres tributary, the run-off provided for, and the rate of rainfall required to produce this run-off. Mr. Gregory.

Column 7 shows the corresponding rate of rainfall taken from the Talbot formula for intensities; and, for comparison, Column 8 shows the intensities for C. W. Sherman's maximum curve for Boston, which crosses the curve of the storm provided for, in order to agree with a time of less than 45 minutes. The intensity for 48 minutes for either curve gives but slightly less value than for 45 minutes, and shows the final effect of the movement of the storm down the valley.

L. J. Le Conte, M. Am. Soc. C. E.* (by letter).—The proper dimensions to be given to storm-water sewers is a subject of vast public interest, and is particularly interesting in this case. Mr. Le Conte.

As to the apportionment of cost, the writer has little to say, except perhaps in relation to the frontage tax of 25 cents per ft., which seems to him to be undesirable. Private connections with a district sewer should be absolutely prohibited by law. Each and every property holder should be compelled to connect with his own street sewer which he paid for by frontage tax alone. The method adopted of putting half the balance on the districts and the remaining half upon the general fund, seems to be about as fair a distribution of the burden as it is possible to make.

The writer was much impressed by the data given in Table 2. The natural surface slopes seem to be in no case less than 16 ft. per mile, and they average fully 39 ft. per mile. These generous slopes suggest the propriety of carefully controlling surface drainage, as far as possible, and of checking the flow of surface waters and distributing them in the street gutters, so that they may be kept there as long as possible, or until a point is finally reached where the street flooding would become a public nuisance, and then, and not till then, these surface waters should be turned gradually into the nearest main sewers. By this means the coefficient of retardation is enormously affected, and the sizes of storm-water sewers can be correspondingly reduced.

Every city engineer is confronted with the fact that street improvements, sidewalks, new buildings, etc., have a powerful tendency toward hastening the concentration of surface drainage, which is exactly what is not wanted. Hence he should do everything legitimately and physically possible to retard concentration, by scattering the surface drainage throughout the street gutters. If this is done systematically, it is really surprising how much can be accomplished in many cases that come up, especially where there are good surface slopes to assist. Of course, the whole subject is now in process of

* This discussion was presented before the San Francisco Association of Members, Am. Soc. C. E., at its meeting of October 21st, 1905.

Mr. Le Conte. development, and existing facts are just beginning to be put into good shape for digestion. There is really great need for a good set of reliable coefficients which will enable one to apply the formulas intelligently to very many cases. When an engineer, after much research, has obtained a reasonably good formula and a full set of reliable coefficients, he is naturally averse to making any radical change, unless there are good and substantial reasons for so doing. He can easily explain special cases by changes in the coefficients.

The main sewer velocities, 17 to 21 ft. per sec., certainly call for first-class construction in every particular. The writer has some doubts as to the ability of the brick inverts to withstand these high velocities when the storm waters are charged with sandy debris from the catch-basins. He has seen so many cases of failure, in this respect, that he has uniformly advised lining the inverts with granite blocks closely laid in 1:1 Portland-cement mortar.

The author's remarks about concrete are very timely and to the point. Reinforced concrete is making rapid strides in new construction, and is generally thoroughly reliable and economical. In a great many cases brickwork cannot compete with it. There is a great future open to this class of construction, the true merits of the work being, principally, efficiency and economy, both of which are bound to tell in the long run.

Mr. Parmley. WALTER CAMP PARMLEY, M. AM. SOC. C. E. (by letter).—The writer believes that Mr. Gregory's criticisms are not well taken. Little reliance should be placed on refined mathematical deductions unless they are based on definite and ascertainable facts, which are sufficiently numerous or general in character to warrant building algebraic equations upon them. In the problem of run-off, there are so many local conditions in any given case that no mathematical expression can be more than a rough approximation to the desired truth. Any formula resting on such variable and indeterminate data, therefore, should be interpreted with a broad reasonableness which takes account of elements that cannot be stated in exact algebraic form.

To take $f = 0.50$, as obtained from one experiment only, and attempt to measure with it the correctness of the data on which the Walworth sewer was designed is unwarranted and unscientific. The observations of several important sewer systems in Cleveland, recorded by the writer,* show conclusively that there were instances where the value of f must have equaled, if not actually exceeded, unity. The writer believes, therefore, that these observations are sufficient to justify the values of f used in designing the sewer. As it is difficult to see how the rate of run-off can exceed the rate of precipitation, the writer believes that the most rational value

* *Journal* of the Association of Engineering Societies, March, 1898, pp. 220-223.

Mr. Parmley.

to assume is to take $f = 1.00$, for that fraction of the total area which is impervious, and to add to this fraction a certain percentage of the remainder, or pervious portion, of the area. Thus, suppose that one-half of a given area is impervious and that the remainder will give one-half the rate of run-off given by the impervious area, then the value of f will be $1.00 \times 0.50 + 0.50 \times 0.50 = 0.75$.

The only rational run-off formula is $Q = f\,i\,A$, first discussed by A. N. Talbot, M. Am. Soc. C. E., the value of f being a constant, as determined above; i, or y in the paper referred to, expressing the intensity of precipitation for a nearly uniform rain of sufficient length to cause all parts of the area to contribute to the flood wave at a given point; and A the number of acres drained. By assuming some fixed ratio of length to width, to represent the average sewerage area, all the exponential formulas may be reduced to the rational form. Inasmuch as the intensity involves the element of time, it is manifest that any other value of this ratio will lead to somewhat different equations. For an assumed area three times as long as wide, the following equations were obtained for the rational form of the several formulas in the paper referred to:

(Parmley),

$$Q = f\left(\frac{1.47\,R}{\sqrt[3]{t-8}}\right)A, \text{ in which } t = \frac{3.2\sqrt{A}}{S^{\frac{3}{4}}} + 8, \text{ approximately.}$$

(Burkli),

$$Q = f\left(\frac{1.79\,R}{\sqrt{t-8}}\right)A, \text{ in which } t = \frac{3.2\sqrt{A}}{\sqrt{S}} + 8, \text{ approximately.}$$

(McMath),

$$Q = f\left(\frac{1.59\,R}{(t-8)^{\frac{2}{5}}}\right)A, \text{ in which } t = \frac{3.2\sqrt{A}}{\sqrt{S}} + 8, \text{ approximately.}$$

(N. Y. Diagrams),

$$Q = f\left(\frac{1.42\,R}{(t-8)^{0.3}}\right)A, \text{ in which } t = \frac{3.2\sqrt{A}}{S^{0.9}} + 8, \text{ approximately.}$$

In each of these formulas the part within parentheses represents the intensity, i, of a uniform rain of duration sufficient to concentrate the run-off from the entire area. It is, therefore, the rain curve of the formula.

As illustrative of the foregoing statements, the following examples may be taken: Suppose $R = 4$, and $S = 4$, then,

(Parmley) $$Q = f\,R\sqrt[4]{S}\,A^{\frac{5}{6}} = 5.65\,f\,A^{\frac{5}{6}}$$

and, if $A = 500$, $Q = 1\,000\,f$.

Mr. Parmley. Using the rational form above,

$$t = \frac{3.2\sqrt{500}}{4^{\frac{3}{4}}} + 8 = 33.5 \text{ min.},$$

and,

$$Q = \frac{5.88\, f}{\sqrt[3]{33.5 - 8}} \times 500 = 1\,000\, f,$$

thus agreeing with the result given by the exponential formula.

Again, applying the same constants to the Burkli formula,

$$Q = 4\, f^4\sqrt{4}\, A^{\frac{3}{4}} = 598\, f,$$

and from the rational form,

$$t = \frac{3.2\sqrt{500}}{\sqrt{4}} + 8 = 44 \text{ min.},$$

and

$$Q = \frac{1.79 \times 4 \times f}{\sqrt{44 - 8}} \times 500 = 599\, f,$$

as before, thus showing that the reduction from the exponential to the rational form is correct.

Now it will be seen that the parts within parentheses, which are the rain curves, are not the same in the Burkli as in the formula used in designing the Walworth sewer, as stated by Mr. Gregory. These curves, when $R = 4$, are Nos. 6 and 8, respectively, in Fig. 16, which gives a comparison of several curves considered in designing the sewer.

For the case where the length is twice the breadth, Mr. Gregory reduces the Walworth formula to the rational form,

$$Q = f\left(\frac{2.45\, R}{(t - 10)^{\frac{5}{8}}} \times A^{\frac{1}{48}}\right) A,$$

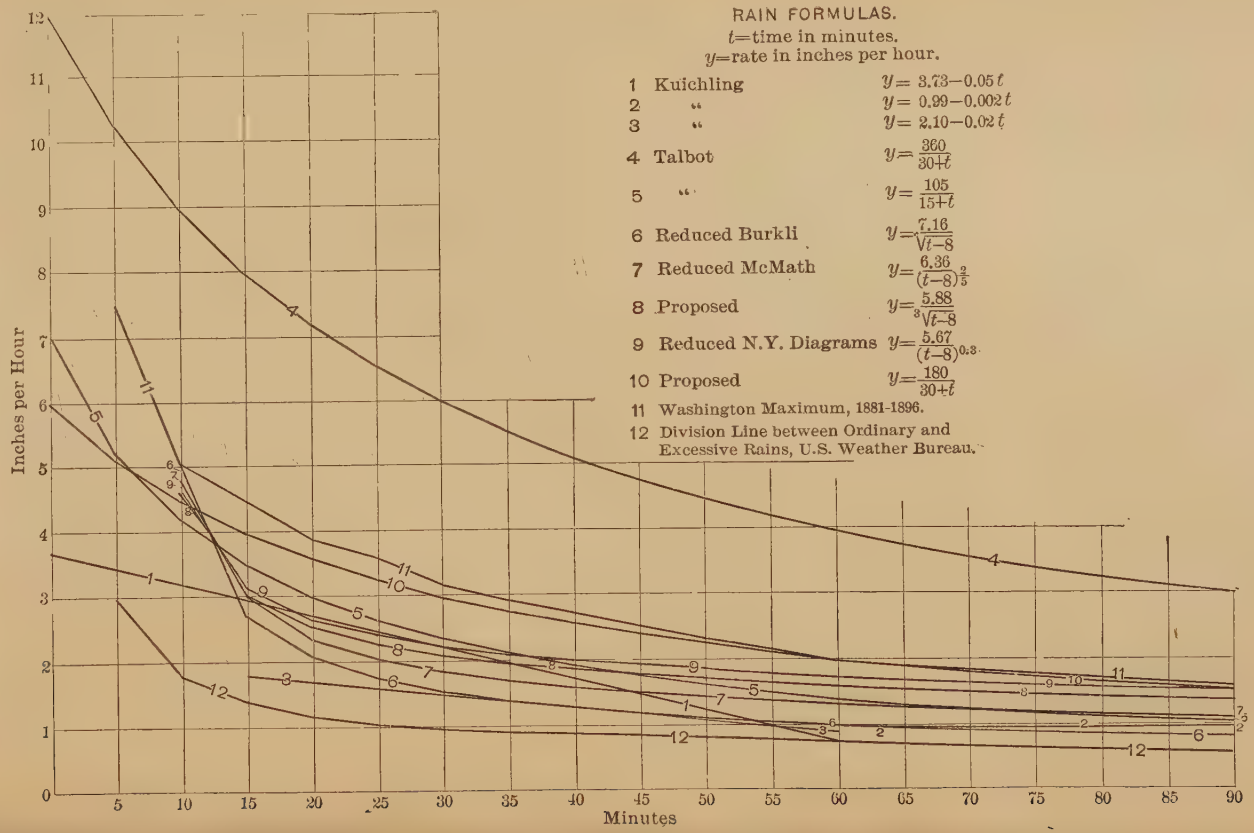
and the Burkli to the form,

$$Q = f\left(\frac{2.45\, R}{(t - 10)^{\frac{5}{8}}} \times \frac{1}{A^{\frac{1}{16}}}\right) A.$$

His error consists in stating that the rain curve is a part instead of the entire expression within parentheses. Considering the entire expression within parentheses as the rain curve, as it should be in order to be of the form $Q = f\, i\, A$, gives an increase in the curve of intensity, as is above cited in the reductions actually used in the Walworth designs. The difficulty with Mr. Gregory's expressions is that, since t varies with the value of A, no equation is given for this relation.

Mr. Gregory's criticism of the additive method used in proportioning the Walworth sewer might have force if the times required for concentration, sizes of lateral sewers, exact grades, etc., were all definitely known, and if Jupiter Pluvius would agree to follow a

Mr. Parmley.


RAIN FORMULAS.
t=time in minutes.
y=rate in inches per hour.
1 Kuichling y= 3.73−0.05 t
2 " y= 0.99−0.002 t
3 " y= 2.10−0.02 t
4 Talbot y= 360/(30+t)
5 " y= 105/(15+t)
6 Reduced Burkli y= 7.16/(√t−8)
7 Reduced McMath y= 6.36/(t−8)^2/3
8 Proposed y= 5.88/(∛t−8)
9 Reduced N.Y. Diagrams y= 5.67/(t−8)^0.3
10 Proposed y= 180/(30+t)
11 Washington Maximum, 1881-1896.
12 Division Line between Ordinary and Excessive Rains, U.S. Weather Bureau.
Inches per Hour
Minutes
0 1 2 3 4 5 6 7 8 9 10 11 12
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90

Mr. Parmley.

prearranged plan of precipitation. As a matter of fact, exact data cannot be predicated, and Jupiter refuses to be controlled by the Engineer. To take less than reasonable maxima for the bad behavior of the elements leads to no profitable conclusions.

If anything further than the records of the United States Weather Bureau for a complete decade, and the application of those records to the design of the sewer, as was made, were necessary to convince the writer that the Walworth sewer was not built too large, this was furnished by a summer afternoon shower. It is to be remembered that at the present time only a few of the lateral sewers, or pavements, are built, that the locations for some of the streets are not determined, and that there are hundreds of acres of undeveloped land tributary to the sewer. Yet at Burton Street a sudden rain dash, not so severe as to cause more than passing remark, and not lasting more than 20 minutes, caused the 14 ft. 9-in. sewer to flow one-third full. The effect of the phenomenon upon the writer was to make him almost tremble lest a serious error had been made in not building the sewer even larger.

In reference to Mr. Le Conte's observations, the State law provides for a frontage tax for local sewerage benefits, and a general district tax for main sewerage, the local tax being fixed at a maximum of $3 per front foot. In the present case there is frontage directly tributary to the sewer which will receive the usual local sewerage benefits. For these benefits it was only right that the property should pay, but, for the reason stated, the tax was finally reduced to a nominal amount.

The shale bricks used for the sewer were specially hard, vitrified bricks, which the writer believes afford nearly or quite as enduring a surface as granite. They were carefully laid in 1:2 Portland-cement mortar, and, unless, by some defect, the water gets under them, so as to exert a lifting force, it is hard to believe that they will not prove satisfactory.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

Paper No. 1012.

THE THEORY OF FRAMEWORKS WITH RECTANGULAR PANELS, AND ITS APPLICATION TO BUILDINGS WHICH HAVE TO RESIST WIND.*

By Ernst F. Jonson, Assoc. M. Am. Soc. C. E.

With Discussion by H. T. Forchhammer and Ernst F. Jonson.

Frameworks with rectangular panels and stiff joints, such as are generally used in modern high buildings, are, of course, statically undetermined structures. The stresses in each member, therefore, are functions of those in all the other members, so that no member can be figured except in relation to all the others, and only on the basis of a preliminary and more or less incorrect design, from which, by successive modifications, a practically correct one may be evolved.

It is the writer's purpose, first, to develop the exact theory of frameworks with rectangular panels, and then to suggest such short-cuts as may be of use in actual designing.

In the following discussion all distortions due to direct stresses will be neglected, as they are insignificant when compared with those due to bending.

* Presented at the meeting of November 1st, 1905.

Let Fig. 1 represent a portion of a straight beam supported at three or more points, and acted upon by the two concentrated loads, W_1 and W_2, the two uniformly distributed loads, $a\ w_1$ and $b\ w_2$, and three external moments at A, B, and C. Let M_1, M_2, M_3, and M_4 be the bending moments produced in the beam at the points of support. These latter moments will be considered positive when producing compression in the top of the beam. Let δ_1 and δ_2 be the elevation of the supports, D and E, above the base line, $A\ B\ C$.

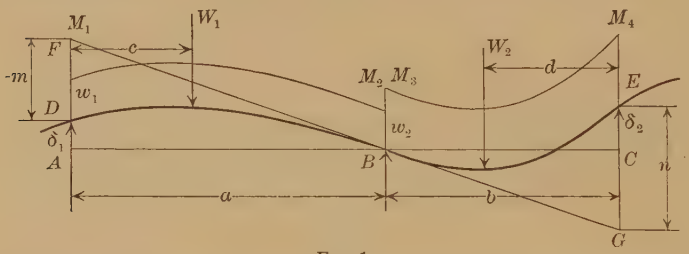


Fig. 1.

No matter what the condition of bending in this beam may be, the deflections, $-m$ and n, at D and E, measured from the tangent, $F\ G$, must be such as to bring the beam into contact with the supports, D and E. Hence we have

$$\frac{-m+\delta_1}{a} = \frac{n-\delta_2}{b},$$

$$\frac{\delta_1}{a} + \frac{\delta_2}{b} = \frac{m}{a} + \frac{n}{b} \quad \text{.....I}$$

In a beam with uniform cross-section, the deflection of one point measured from the tangent at another point is the sum of the moments of the bending moment on each unit of length between the two points, around the former point, divided by the modulus of elasticity and the moment of inertia.

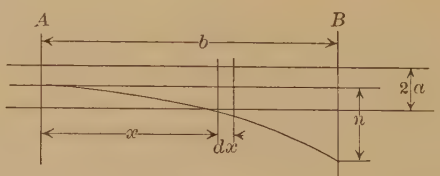


Fig. 2.

To demonstrate: Let Fig. 2 represent a portion, b, of a beam of the depth, $2a$, subject to a variable bending moment, M, which

produces a deflection, n, from the tangent at A. Then the unit stress at the extreme fiber is

$$S_1 = \frac{M\,a}{I},$$

and the unit stress at a distance, 1, from the neutral axis is

$$S = \frac{M}{I}.$$

The corresponding elongation in a distance, $d\,x$, is

$$d\,e = \frac{M}{E\,I}\,d\,x.$$

The corresponding deflection at B is

$$d\,n = \frac{M\,(b - x)}{E\,I}\,d\,x.$$

Hence the total deflection is

$$n = \frac{1}{E\,I}\int_o^b M\,(b - x)\,d\,x \ldots\ldots\ldots\ldots\ldots\ldots\text{II}$$

E being the modulus of elasticity, and I the moment of inertia.

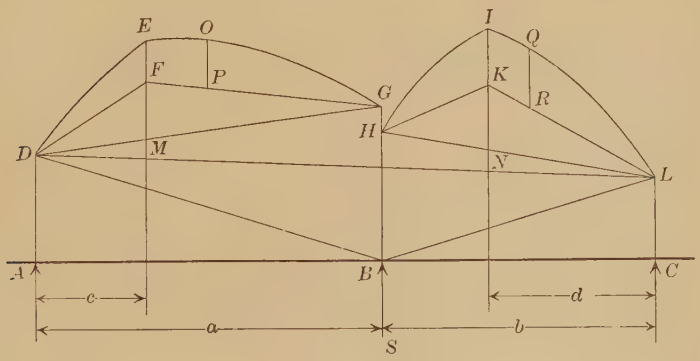


FIG. 3.

Let Fig. 3 be a moment diagram of the beam in Fig. 1. Then we have

$$A\,D = M_1 \qquad B\,G = M_2 \qquad B\,H = M_3$$

$$C\,L = M_4 \qquad F\,M = \frac{W_1\,c\,(a - c)}{a} \qquad K\,N = \frac{W_2\,d\,(b - d)}{b}$$

$$O\,P = \frac{w_1\,a^2}{8} \qquad Q\,R\ \frac{w_2\,b^2}{8}.$$

The moment of the area, $A\ D\ E\ O\ G\ B$, around A will then be

$$\frac{M_1\ a^2}{6} + \frac{2\ M_2\ a^2}{6} + \frac{w_1\ a^4}{24} + \frac{W_1\ c\ (a^2 - c^2)}{6}.$$

And the moment of the area, $B\ H\ I\ Q\ L\ C$, around C will be

$$\frac{2\ M_3\ b^2}{6} + \frac{M_4\ b^2}{6} + \frac{w_2\ b^4}{24} + \frac{W_2\ d\ (b^2 - d^2)}{6}.$$

Let I_1 and I_2 be the moments of inertia of the beam from A to B and B to C, respectively, and let there be any number of loads, W_1 and W_2; then will the deflections at A and C be

$$m = \frac{1}{6\ E\ I_1}\left[M_1\ a^2 + 2\ M_2\ a^2 + \frac{w_1\ a^4}{4} + \sum W_1\ c\ (a^2 - c^2)\right]$$

$$n = \frac{1}{6\ E\ I_2}\left[2\ M_3\ b^2 + M_4\ b^2 + \frac{w_2\ b^4}{4} + \sum W_2\ d\ (b^2 - d^2)\right].$$

Inserting these values of m and n in Equation I, we have

$$6\ E\left(\frac{\delta_1}{a} + \frac{\delta_2}{b}\right) = \frac{1}{I_1}\left[M_1\ a + 2\ M_2\ a + \frac{w_1\ a^3}{4} + \sum \frac{W_1\ c\ (a^2 - c^2)}{a}\right] + \frac{1}{I_2}\left[2\ M_3\ b + M_4\ b + \frac{w_2\ b^3}{4} + \sum \frac{W_2\ d\ (b^2 - d^2)}{b}\right] \quad \text{III}$$

This equation we will call the "theorem of four moments," as it is nothing else than a more general form of the theorem of three moments.

By making $M_2 = M_3$ and $I_1 = I_2$ we get the ordinary form of that theorem.

From the conditions upon which this question was based, it is evident that it is true, not only for a straight beam, but also for a beam forming any angle at B, Fig. 1, as long as the base line, $A\ B\ C$, is drawn with the same angle at B and the moments and distances, δ, are taken positive on the same side of the two parts of the beam.

In order to make the theorem of four moments easily applicable to frameworks, we will write it in two more forms, one for each of the two pairs of opposite corners of a rectangular panel. This will enable us always to keep the left-hand side of the vertical members the positive side, just as the top is in the case of horizontal members. The distance, δ, at the horizontal end, not being needed, will be equal to zero.

Form 2, for the upper left-hand and the lower right-hand corners, **Fig. 4:**

$$6\,E\,\frac{\delta}{a} = \frac{1}{I_1}\left[M_1\,a + 2\,M_2\,a + \frac{w_1\,a^3}{4} + \sum\frac{W_1\,c\,(a^2 - c^2)}{a}\right] + \frac{1}{I_2}\left[2\,M_3\,b + M_4\,b + \frac{w_2\,b^3}{4} + \sum\frac{W_2\,d\,(b^2 - d^2)}{b}\right] \ldots\ldots\ldots \text{IV}$$

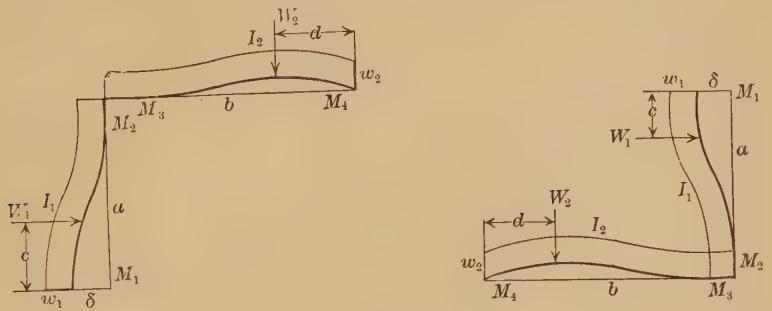


FIG. 4.

Form 3, for the upper right-hand and the lower left-hand corners, **Fig. 5:**

$$6\,E\,\frac{\delta}{a} = \frac{1}{I_1}\left[M_1\,a + 2\,M_2\,a + \frac{w_1\,a^3}{4} + \sum\frac{W_1\,c\,(a^2 - c^2)}{a}\right] - \frac{1}{I_2}\left[2\,M_3\,b + M_4\,b + \frac{w_2\,b^3}{4} + \sum\frac{W_2\,d\,(b^2 - d^2)}{b}\right] \ldots\ldots\ldots\ldots \text{V}$$

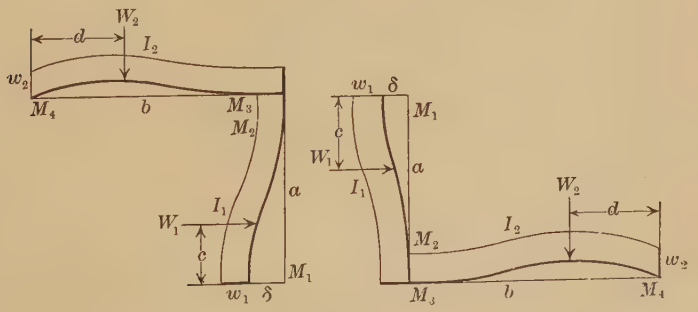


FIG. 5.

By means of Equations III, IV, and V, the moments may be found in any framework composed of quadrangular panels.

Let Fig. 6 represent a framework acted upon by three horizontal wind forces, W_1, W_2, and W_3.

Let $M_{26} = x$, $M_{28} = y$, and $M_{30} = z$.

Since the sum of the moments around each intersection point must be equal to zero, we have

$$x + M_{31} = 0 \ldots\ldots\ldots\ldots\ldots\ldots \text{VI}$$

$$M_{32} - y - M_{33} = 0 \ldots\ldots\ldots \text{VII}$$

$$M_{34} - z = 0 \ldots\ldots\ldots\ldots\ldots \text{VIII}$$

According to the first form of the theorem of four moments, we have

$$\frac{a}{I_{16}}(M_{31} + 2\,M_{32}) + \frac{b}{I_{17}}(2\,M_{33} + M_{34}) = 0 \ldots\ldots\ldots \text{IX}$$

From these four equations we find the values of M_{31}, M_{32}, M_{33}, and M_{34}, expressed in x, y, and z.

Since the lengths of the girders remain constant, the deflection, δ, must be constant for all the columns. Hence we have, according to the second and third forms of the theorem of four moments,

$$\frac{h_3}{I_{13}}(M_{25} + 2\,M_{26}) - \frac{a}{I_{16}}(2\,M_{31} + M_{32}) = \frac{h_3}{I_{14}}(M_{27} + 2\,M_{28}) + \frac{a}{I_{16}}(2\,M_{32} + M_{31}) \ldots\ldots\ldots\ldots\ldots \text{X}$$

and a similar equation for the second panel.

We also know that the wind-shear moment must be equal to the difference between the sum of the lower and that of the upper column moments.

$$h_3(W_1 + W_2 + W_3) = M_{26} + M_{28} + M_{30} - M_{25} - M_{27} - M_{29} \ldots \text{XI}$$

From these three equations we find the values of M_{25}, M_{27}, and M_{29}, expressed in x, y, and z.

By means of the following four applications of the theorem of four moments we find the values of M_{21}, M_{22}, M_{23}, and M_{24}, expressed in x, y, and z.

$$\frac{a}{I_{11}}(M_{21} + 2\,M_{22}) + \frac{b}{I_{12}}(2\,M_{23} + M_{24}) = 0 \ldots\ldots\ldots \text{XII}$$

$$-\frac{h_3}{I_{13}}(M_{26} + 2\,M_{25}) - \frac{a}{I_{11}}(2\,M_{21} + M_{22}) = \frac{a}{I_{11}}(M_{21} + 2\,M_{22}) - \frac{h_3}{I_{14}}(2\,M_{27} + M_{28}) \ldots\ldots\ldots\ldots\ldots \text{XIII}$$

$$-\frac{h_3}{I_{14}}(M_{28} + 2\,M_{27}) - \frac{b}{I_{12}}(2\,M_{23} + M_{24}) = \frac{b}{I_{12}}(M_{23} + 2\,M_{24}) - \frac{h_3}{I_{15}}(2\,M_{29} + M_{30}) \ldots\ldots\ldots\ldots\ldots \text{XIV}$$

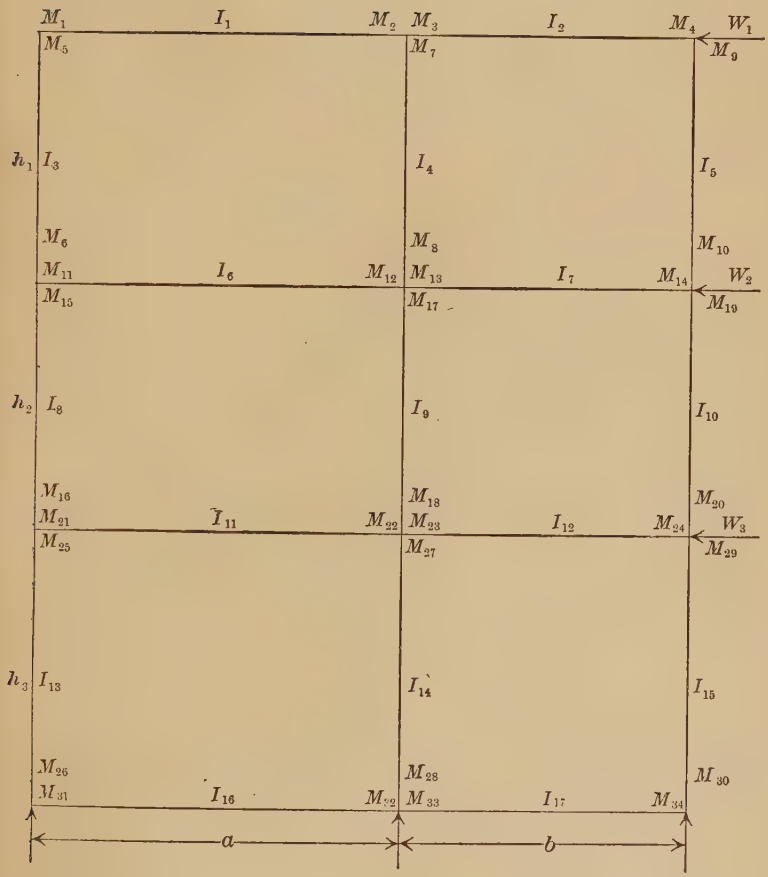

M_1 I_1 M_2 M_3 I_2 M_4 W_1
M_5 M_7 M_9
h_1 I_3 I_4 I_5
M_6 M_8 M_{10}
M_{11} I_6 M_{12} M_{13} I_7 M_{14} W_2
M_{15} M_{17} M_{19}
h_2 I_8 I_9 I_{10}
M_{16} M_{18} M_{20}
M_{21} I_{11} M_{22} M_{23} I_{12} M_{24} W_3
M_{25} M_{27} M_{29}
h_3 I_{13} I_{14} I_{15}
M_{26} M_{28} M_{30}
M_{31} I_{16} M_{32} M_{33} I_{17} M_{34}
a b

FIG. 6.

$$-\frac{a}{I_{16}}(M_{32}+2\,M_{31})+\frac{h_3}{I_{13}}(2\,M_{26}+M_{25})=-\frac{h_3}{I_{13}}(M_{26}+2\,M_{25})-\frac{a}{I_{11}}(2\,M_{21}+M_{22})\ldots\ldots\ldots\ldots\ldots\ldots\text{XV}$$

From M_{21}, and M_{25}, we find M_{16}.

From M_{22}, M_{23}, and M_{27}, we find M_{18}.

From M_{24}, and M_{29}, we find M_{20}.

By repeating the foregoing operation for each tier of panels, we get the values of all the moments, expressed in x, y, and z.

Then we set

$$M_1-M_5=0\ldots\ldots\ldots\ldots\ldots\text{XVI}$$

$$M_2-M_3+M_7=0\ldots\ldots\ldots\ldots\text{XVII}$$

$$M_4+M_9=0\ldots\ldots\ldots\ldots\ldots\text{XVIII}$$

By solving these three equations for x, y, and z, and introducing the values thus found into the expressions for the various moments, we get the actual values of these moments.

When all the moments are known, the direct stresses and reactions may be calculated directly from them.

Let Fig. 1 represent two members, $A\ B$ and $B\ C$, of a framework. Then we have (taking tension positive),

$$S_5-S_6=x+y\ldots\ldots\ldots\ldots\ldots\text{XIX}$$

where S_5 and S_6 are the direct stresses in two perpendicular members, above and below B, respectively, and x and y the shear at B, in the members, $A\ B$ and $B\ C$, respectively.

$$x=\frac{M_1-M_2}{a},$$

$$y=\frac{M_4-M_3}{b}.$$

Substituting these values of x and y in Equation XIX, we get

$$S_5-S_6=\frac{M_1-M_2}{a}+\frac{M_4-M_3}{b}\ldots\ldots\ldots\ldots\text{XX}$$

By starting from the top, the direct stresses in all the vertical members, as well as the vertical reactions, will be found through Equation XX; likewise the horizontal ones, by starting from either side.

The moments due to the vertical loads are found in a similar way, but with the difference that the elements due to the loads must

be retained in the expressions for the elastic condition of the horizontal members in Equations IX to XV, inclusive.

The direct stresses due to the vertical loads may be found in the same way as those due to wind, with the exception that the loads, W_1, W_2, $w_1 a$, and $w_2 b$, must be taken into account.

So that we have (see Fig. 1),

$$x = \frac{M_1 - M_2}{a} + \frac{w_1 a}{2} + \sum \frac{W_1 c}{a}$$

$$y = \frac{M_4 - M_3}{b} + \frac{w_2 b}{2} + \sum \frac{W_2 d.}{b}$$

Introducing these values of x and y into Equation XIX, we get

$$S_5 - S_6 = \frac{M_1 - M_2}{a} + \frac{M_4 - M_3}{b} + \frac{w_1 a}{2} + \frac{w_2 b}{2} + \sum \frac{W_1 c}{a} + \sum \frac{W_2 d}{b} \quad \text{.....XXI}$$

If the bottom member be left out and the lower end of the columns fixed, as is generally the case, we can set M_{31}, M_{32}, M_{33}, and M_{34} equal to zero, and then proceed as usual.

The foregoing method of calculating frameworks with rectangular panels may, in the case of most actual problems, be greatly simplified and yet retain a sufficient degree of accuracy.

In most buildings the moment of inertia of the various columns and girders may be assumed to bear the same relation to each other on the different floors. For instance (see Fig. 6),

$$\frac{I_3}{I_4} = \frac{I_8}{I_9}; \quad \frac{I_6}{I_7} = \frac{I_{11}}{I_{12}}, \text{ and so on.}$$

If this is the case it is evident that the moments due to wind will be related to each other in the same way.

In buildings where there is not too much difference in story heights and floor loads we may also assume that the point of no bending lies at the middle of the column.

Hence we have

$$M_{15} = M_6 \frac{(W_1 + W_2)\, h_2}{W_1\, h_1} \quad \text{.....XXII}$$

$$M_{17} = M_8 \frac{(W_1 + W_2)\, h_2}{W_1\, h_1} \quad \text{.....XXIII}$$

$$M_{19} = M_{10} \frac{(W_1 + W_2)\, h^2}{W_1\, h_1} \quad \text{.....XXIV}$$

$$M_{11} = M_{15} - M_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{XXV}$$
$$M_{12} = M_8 + M_{13} - M_{17} \ldots\ldots\ldots\ldots\ldots\ldots \text{XXVI}$$
$$M_{14} = M_{10} - M_{19} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{XXVII}$$

According to the theorem of four moments,

$$\frac{a}{I_6}\left(M_{11} + 2\,M_{12}\right) + \frac{b}{I_7}\left(2\,M_{13} + M_{14}\right) = 0 \ldots\ldots \text{XXVIII}$$

Since the deflection of all columns must be the same, we have, from the second and third forms of the theorem of four moments,

$$\frac{h_1}{I_3} M_6 - \frac{a}{I_6}\left(2\,M_{11} + M_{12}\right) = \frac{h_1}{I_4} M_8 + \frac{a}{I_6}\left(2\,M_{12} + M_{11}\right) \ldots\ldots \text{XXIX}$$
$$\frac{h_1}{I_3} M_6 - \frac{a}{I_6}\left(2\,M_{11} + M_{12}\right) = \frac{h_1}{I_5} M_{10} + \frac{b}{I_7}\left(2\,M_{14} + M_{13}\right) \ldots\ldots \text{XXX}$$

From these nine equations the values of all the moments are found in terms of M_6, and this relation may be used on each floor.

Finally, we have,

$$M_6 + M_8 + M_{10} = \frac{W_1 h_1}{2} \ldots\ldots\ldots\ldots\ldots \text{XXXI}$$

and so on, for each story.

To find the direct stresses in the vertical members, it is necessary only to figure the differences between the lower and upper column stresses at one floor, whence those for the other floors may be found by proportion, as they are proportional to the wind moment at the middle of the story.

The direct stresses in the horizontal members will be

$$S_6 = -\frac{M_6}{M_6 + M_8 + M_{10}}\left(W_1 + W_2\right) \ldots\ldots\ldots \text{XXXII}$$
$$S_7 = -\frac{M_6 + M_8}{M_6 + M_8 + M_{10}}\left(W_1 + W_2\right) \ldots\ldots\ldots \text{XXXIII}$$

and so on, for each floor.

When the vertical load on the girders is such that it may be considered uniformly distributed, the moments in the girders and columns due to this load may be found in the following way:

Taking a typical floor (third floor, Fig. 6), we may assume

$$M_6 = -M_{15'},$$
$$M_8 = -M_{17'},$$
$$M_{10} = -M_{19'},$$

and that the point of no bending in the columns, above and below, lies at a distance, $n = \frac{1}{4}(h_1 + h_2)$, from the girder.

Then we have

$$M_6 = -\frac{1}{2} M_{11} \quad \text{XXXIV}$$

$$M_8 = \frac{1}{2}\left(M_{12} - M_{13}\right) \quad \text{XXXV}$$

$$M_{10} = \frac{1}{2} M_{14} \quad \text{XXXVI}$$

According to the theorem of four moments, we have

$$\frac{a}{I_6}\left(M_{11} + 2\,M_{12} + \frac{w\,a^2}{4}\right) + \frac{b}{I_7}\left(2\,M_{13} + M_{14} + \frac{w\,b^2}{4}\right) = 0 \quad \text{XXXVII}$$

And, since the deflection at the point of no bending in the columns may be assumed to be zero, we have

$$\frac{n}{I_3} 2\,M_6 - \frac{a}{I_6}\left(2\,M_{11} + M_{12} + \frac{w\,a^2}{4}\right) = 0 \quad \text{XXXVIII}$$

$$\frac{n}{I_4} 2\,M_8 + \frac{a}{I_6}\left(2\,M_{12} + M_{11} + \frac{w\,a^2}{4}\right) = 0 \quad \text{XXXIX}$$

$$\frac{n}{I_5} 2\,M_{10} + \frac{b}{I_7}\left(2\,M_{14} + M_{13} + \frac{w\,b^2}{4}\right) = 0 \quad \text{XL}$$

From these seven equations the values of all the moments may be found for the floor in question. Those for the other floors may be taken as proportional to w.

The differences in the direct stresses in the vertical members must be calculated for one floor, and may then be taken by proportion for the other ones. The direct stresses in the horizontal members may be neglected.

If the building happens to be symmetrical about its center line, the foregoing method will, of course, be further simplified.

In making the preliminary design, the wind moment on the outside columns may be taken as 75% of that on the inside ones. This is, of course, only a very rough approximation, and must not be used as a final figure.

DISCUSSION.

Mr. Forchhammer.

H. T. Forchhammer, Assoc. M. Am. Soc. C. E.—The whole theory, as presented in this paper, is based on the supposition that all connections between girders and columns are stiff. This is not the case, generally. Most frequently the connecting members are angles, as in Fig. 7. This connection is not rigid, but flexible; if it carries a moment it will open on the tension side and close on the compression side, thus changing the angle between the girder and the column.

For all frameworks with such flexible connections, the theory—as developed by Mr. Jonson—will give absolutely wrong results. As such connections are almost always used at the present time, and, as Mr. Jonson's theory can easily be extended to them, the speaker will develop the theory for such connections in the general way.

In Fig. 8 is shown a horizontal section through the connection (on the line *a*—*a* of Fig. 7).

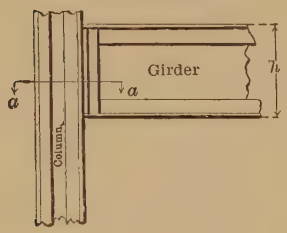


Fig. 7.

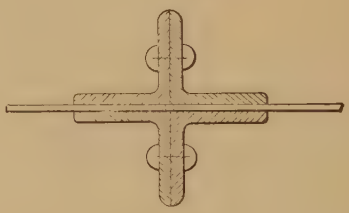


Fig. 8.

In Fig. 9 is shown the deformation due to tension.

First, an expression will be found for the opening, *o*, due to a tension, 2 *x*, per running inch of the angles. It will be assumed that the bent flanges of the angles have fixed supports at the center of the rivet and at the back of the angle. (This is not absolutely correct, but only a fairly good approximation.)

Midway between these two points will be a point of contra-flexure, where the bending moment is zero and the shear is *x*.

It is well known that the deflection of a beam, of length, *l*, with a fixed support and a load, *P*, on the unsupported end is $\frac{P\,l^3}{3\,E\,I}$, where *E* is the modulus of elasticity and *I* the moment of inertia of the beam.

Each angle flange, from the point of contra-flexure to the fixed support, is such a beam, where $P = x$, $l = \frac{a}{2}$, $I = \frac{1}{12} \times 1 \times t^3$; and

Mr. Forchhammer.

the deflection $= \dfrac{x \times \left(\frac{a}{2}\right)^3}{3\,E \times \frac{1}{12}\,t^3} = \dfrac{a^3}{2\,E\,t^3} \times x$. As the opening, o, is due to four such deflections, $o = \dfrac{2\,a^3}{E\,t^3}\,x$.

Now, consider the connection shown in Fig. 7 as being under the influence of a moment, M, which tends to close the connection at the top of the girder and open it at the bottom. It will be assumed that the deformation of the connection is a rotation around the very top point, and that the tension is increasing as shown by a straight line.

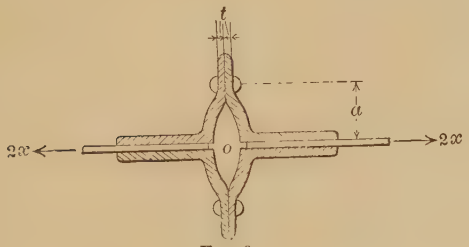


FIG. 9.

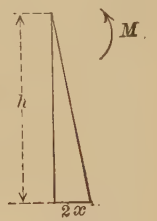


FIG. 10.

A diagram of the tension stresses is shown in Fig. 10, and, when the tension at the bottom of the girder is $2\,x$ per inch, it is easily seen that

$$M = 2\,x \times \frac{h}{2} \times \frac{2}{3}\,h = \frac{2}{3}\,h^2\,x, \text{ or } x = \frac{3\,M}{2\,h^2}.$$

Substituting this value for x in the expression, previously derived for o, gives $o = \dfrac{2\,a^3}{E\,t^3} \times \dfrac{3\,M}{2\,h^2} = \dfrac{3\,a^3}{E\,h^2\,t^3} \times M$.

The change in the angle between the girder and column, due to the flexibility of the connection, will therefore be:

$$\alpha = \frac{o}{h} = \frac{3\,a^3}{E\,h^3\,t^3} \times M.$$

The coefficient of M can easily be figured in each separate case, a being the distance from the back of the angle to the nearest pitch line, t the thickness of the angles, and h the depth of the connection.

Consider x as being positive when M is positive; or α is positive when due to a yielding from a positive moment.

To apply this to Mr. Jonson's theory, refer to his Fig. 1.*

The only difference between Fig. 1 and Fig. 11 is that in Fig. 11

* *Proceedings*, Am. Soc. C. E., for September, 1905, p. 499.

Mr. Forchhammer.

there is an angle, α, between the two tangents at B. As $y = b\,\alpha$, the equation will be:

$$\frac{-m+\delta_1}{a} = \frac{n-\delta_2+\alpha\,b}{b},$$

$$\frac{\delta_1}{a}+\frac{\delta_2}{b}-\alpha = \frac{m}{a}+\frac{n}{b}.$$

Mr. Jonson's Equation III, therefore, will only be changed by substituting $\frac{\delta_1}{a}+\frac{\delta_2}{b}-\alpha$ for $\frac{\delta_1}{a}+\frac{\delta_2}{b}$.

This change can now easily be carried through all Mr. Jonson's formulas, and, as α is dependent only on the moment which is carried through the connection, the solving of the equations will not be made much more difficult.

This theory, as developed by Mr. Jonson, and supplemented by the speaker, is only applicable where there are no partitions in the bents, or where the partitions are so weak that they cannot at all be trusted as cross-framing.

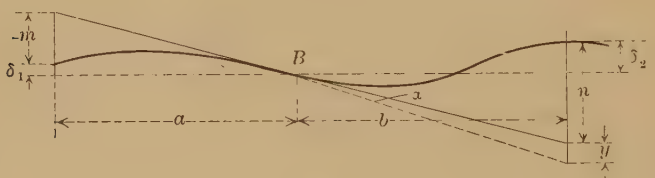


FIG. 11.

As to whether rigid or flexible connections are preferable in such frameworks, the speaker's opinion is that the rigidity obtained by rigid connections necessarily is followed by the uncertainty of statically undetermined structures. The stresses due to the uneven settlement of columns, for instance, may run very high.

The speaker's opinion is that the best framework is obtained by using rigid connections at the side columns and flexible connections at the inside columns. The rigidity of the connection to the side columns can never do much harm, as the columns are generally very flexible, compared with the rigid girders; and would better resist the horizontal forces.

One point in this paper needs further explanation: Mr. Jonson says: "The distance, δ, at the horizontal end, not being needed, will be equal to zero."

This, illustrated by Figs. 4 and 5, makes one believe that the author assumes that the corner-tangents are horizontal and vertical, respectively, after, as they were before, the deformation of the framework.

Mr. Forchhammer.

This assumption is not right, and is also unnecessary, in order to get the results obtained by the author.

In Equation III, the left-hand side of the equation reads $6E\left(\frac{\delta_1}{a}+\frac{\delta_2}{b}\right)$, but, from Fig. 12, it will easily be seen that $\frac{y}{a}=\frac{\delta_2}{b}$, and $\frac{\delta_1}{a}+\frac{\delta_2}{b}=\frac{\delta_1+y}{a}=\frac{\delta}{a}$, which, substituted in Equation III, gives $6E\left(\frac{\delta_1}{a}+\frac{\delta_2}{b}\right)=6E\times\frac{\delta}{a}$, and this is identical with the left-hand side of Equation IV. Considering flexible connections, this left-hand side will read $6E\times\left(\frac{\delta}{a}-\alpha\right)$, as stated above.

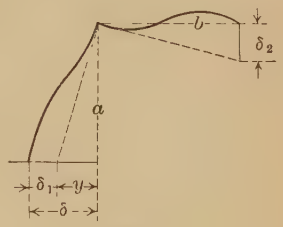


FIG. 12.

Mr. Jonson.

ERNST F. JONSON, ASSOC. M. AM. SOC. C. E. (by letter).—It is true, as Mr. Forchhammer says, that stiff connected frameworks are not generally used, but neither are buildings generally figured for wind. If, however, it is considered necessary to figure a building for wind, and it should be so considered in the case of all high or exposed buildings, it is the writer's opinion, for reasons which follow, that a stiff connected frame is the only proper and practicable solution of the problem. Hence the theory proposed by the writer was intended only for such frameworks.

As to Mr. Forchhammer's supplementary theory, it is evident that his application of the angle, α, to the writer's Equation I is not correct. If it be assumed that there is no yielding in the column itself, then one-half of α should be added on each side of the point, B, because there is one pair of connection angles on each side of the column. Equation I then becomes:

$$\frac{-m+\delta_1-\frac{a\,\alpha_2}{2}}{a}=\frac{n-\delta_2+\frac{b\,\alpha_2}{2}}{b}$$

$$\frac{\delta_1}{a}+\frac{\delta_2}{b}=\frac{m}{a}+\frac{n}{b}+\frac{\alpha_2+\alpha_3}{2}\quad\ldots\ldots\ldots\ldots\text{I}\alpha$$

If the yielding in the column itself is considered, a different α should be worked out for each kind of column. Mr. Forchhammer's formula cannot be applied because it is based on two pairs of short angles.

If the α element is examined, it is seen that it is practically proportional to M because the other elements would most likely be constant in any one tier of girders. Hence, in wind calculations, where the angles, α, all act in the same direction, they could affect the result only to the extent of the difference in the end moments of the

Mr. Jonson.

girders. Further, these end moments would necessarily be small, where simple angle connections are sufficient, especially as these connections would already be highly strained, most likely beyond the elastic limit, by the load deflection of the girder. For instance, in the case of a 24-in., 80-lb., I-beam, 16 ft. long, connected at each end to a vertical support by two 4 by 4 by $\frac{7}{16}$-in. angles, 18 in. long, these angles would be stressed to 16 000 lb. before one-tenth of the safe load is put on the girder.

From these considerations it is evident that the element, α, is not of much practical importance, and also that the method of construction from which it is derived is altogether unsuitable where any considerable wind is to be resisted.

As to the question of partitions, raised by Mr. Forchhammer, the writer thinks that they should not be considered at all, for they are likely to be removed at any time, and if they are not, and resist the wind, they will simply reduce the wind stresses in the steel frame.

It is true that very high stresses, due to uneven settlement, might arise in a frame with stiff connections, unless the foundation is extended to rock, or the foundation grillage is made continuous and sufficiently stiff to prevent uneven settlement, but that is simply an argument for taking either of these precautions. To concentrate all the wind resistance in the outside columns would not overcome this difficulty. The effect of uneven settlement might be even more serious in this case, for the outside columns, in which the whole stiffness of the structure lies, might be seriously overstrained. If there is to be a distortion beyond the elastic limit, it seems to the writer that a frame where all connections are stiff would be in less danger. However, uneven settlement in high buildings is a very serious thing in any case, and should be guarded against by all possible means.

If, however, the method of construction proposed by Mr. Forchhammer is thought desirable, it may be calculated by means of the writer's theory. We simply have to apply Equations IV and V to the outside corners of the outside panels. Of course, if the frame is absolutely symmetrical this is not necessary, for it is known then that each end column takes one-half of the wind. By using only two columns we get higher unit stresses from wind, and hence less stiffness.

As to Mr. Forchhammer's criticism of the illustrations, Figs. 4 and 5, it is not intended to imply that the straight lines representing the members of the frame before bending should necessarily be tangents to the curves, and it would have been better to show them at a perceptible angle.

However, this does not in any way affect the argument. The derivation of the theorem of four moments presupposes that the

angles, $F\ B\ G$ and $D\ B\ E$ (Fig. 1), are equal, but it is otherwise independent of their magnitude, and δ may be anything, hence zero. Imagine the line, $A\ B\ C$ (Fig. 1), as revolving around B, and it will be seen that δ_1 and δ_2 may be anything, as long as $\frac{\delta_1}{a} + \frac{\delta_2}{b}$ is constant. Hence the derivation given by Mr. Forchhammer is quite unnecessary. Mr. Jonson.

It may seem strange that there should be a break, $G\ H$, in the moment diagram, Fig. 3, but this break is due to the stiffness of the column. If there were no difference between M_{12} and M_{13}, in Fig. 6, there could be no difference between M_8 and M_{17}, and hence no resistance to wind.

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TRANSACTIONS.

Paper No. 1013.

AN EXAMPLE OF THE LEGITIMATE USE OF WATER FOR DOMESTIC PURPOSES.*

By K. F. Cooper, Jun. Am. Soc. C. E.

With Discussion by Messrs. F. W. Dalrymple, Alexander Potter, V. H. Hewes, M. R. Sherrerd, G. L. Christian, Edwin Duryea, Jr., Theodore Belzner, and James Owen.

The question as to what constitutes a legitimate supply of water for domestic purposes is one in which every water-supply engineer and every water-works official is interested. The following facts and figures relate to the quantity of water used at the Lick Observatory, and show the effect of proper regulation, and of the willing co-operation by consumers, on the quantity used, and how a somewhat limited supply is made to fulfill all requirements.

The Lick Observatory is situated on Mt. Hamilton, a peak in the Coast Range, in Santa Clara County, California. Its elevation is 4 209 ft.; mean annual temperature, 52°; the coldest month is January, with a mean temperature of 40°; the warmest month is July, with a mean temperature of 69°; and the mean annual rainfall is about 32 in., more than two-thirds of which falls in December, January, February and March. July and August are usually dry and cloudless.

The observatory is maintained by the State of California, in connection with the State University. The population at the ob-

* Presented at the meeting of October 18th, 1905.

servatory averages 52 people, and consists of the astronomers, assistants, machinists and laborers, with their families. The dwellings, for the most part, are cottages of from four to ten rooms. There are nine families and mess associations.

Two separate water systems are in use. One, a rain-water system, is used only for fire protection and for developing hydraulic power used in the operation of the large telescope and the platform on which it is mounted. This system will not be considered herein. The other, the ordinary domestic supply system, consists of springs down on the side of the mountain, from which the water is pumped to two brick and cement tanks, each having a capacity of 80 000 gal. These tanks are placed on a peak to the east of the observatory, and are 400 ft. above the source of supply and 65 ft. above the level of the houses. A full and careful record of the quantity of water pumped to these tanks has been kept for a number of years. This quantity is computed from gauge readings in the tanks. These tanks are tight and well covered, so that the evaporation and leakage amount to little. The average yearly pumpage is 650 000 gal., or an average of 1 780 gal. per day.

As the supply is not great, the consumption and waste have to be looked after very carefully. The main carrying the water from the tanks to the point of distribution is an ordinary 2½-in. iron pipe, and is § mile long. There is practically no leakage from this pipe and its connections. The Director of the observatory has complete charge of the system, and the rules governing the use of water have to be strictly observed. Any defects in plumbing are reported and repaired at once. As the consumers are interested in preventing waste, regulation is comparatively easy. The water is furnished free to consumers, without any meters or other regulating devices. Seventeen bathrooms are in use, and in each of them are stationary wash-stands and closets; three-quarters of all the bed rooms have running hot and cold water; and the laundries are also provided with running hot and cold water. Wash-rooms and closets are provided for the six thousand or more tourists visiting the observatory annually. A large proportion of the total water supply from the springs is used for photographic purposes. Some water is used in the operation of a gasolene engine, in the machine shop, in the occasional operation of a steam wood-saw, and for the six horses

used in the work about the place. Practically all the laundering for the inhabitants is done on the mountain. During the summer there is some sprinkling and irrigation. It will thus be seen that, with the exception of the unusually large proportion used for photography, the conditions approximate very closely to those in small towns and in the residence portions of some cities.

Of the 650 000 gal., average yearly pumpage, from one-fourth to one-third is used for photographic purposes. Deducting, say, 30% for the quantity used in photography, a use entirely distinct from any domestic purpose, there remain 455 000 gal. as the average yearly consumption of 52 people, or a total of 1 248 gal., or 24 gal. per capita per day. This, as before stated, includes the quantity used for sprinkling, which amounts to about 5% of the total, or, say, 32 000 gal. per year; also that used for engines, horses, and by visitors. The actual consumption for domestic purposes is probably not more than 20 gal. per capita per day. The rate of consumption is about the same during the whole year; the users are more careful of the water in the summer, but more is then used for the laundry and for bathing. There is little if any restriction in the legitimate use of the water. All that is needed can be used, but actual waste is almost entirely eliminated.

These facts are worthy of special notice only because the conditions of use approximate those in portions of most towns and some cities; because they give authoritative figures on what constitutes legitimate use under the conditions represented; and because they show what can be done toward preventing waste by intelligent regulation and the willing co-operation of the consumers. The information was obtained through the courtesy of Dr. W. W. Campbell, Director of the Observatory.

The foregoing statements are summarized in Table 1.

In considering the per capita quantities in Table 1, it should be borne in mind that the service includes a much larger number of taps, proportionally, than is usual. The proportion of bath-tubs, one to every three persons, is also unusually large.

The 6 000 tourists per year, not taken into account in Table 1, must also use a very appreciable quantity of the water. Their average number is 16.5 persons daily, with a use of water equivalent to perhaps 5 persons. All tourists, also all provisions, freight, and a

daily mail, are brought to the observatory by teams, and these, also, must increase the water consumption appreciably.

TABLE 1.—Water Supply of Lick Observatory, Exclusive of Fire Protection and Power Purposes.

(Resident Population Averages 52 Persons.)

Consumption for:	Quantity Consumed, in Gallons.		
	Per year.	Per day.	Per capita of resident population daily.
All purposes	650 000	1 780	34.3
All purposes, except photography	455 000	1 248	24.0
Ordinary domestic uses (estimated only).	379 600	1 040	20.0

It is probable, therefore, that after eliminating all excess uses of water at Lick Observatory, for purposes not usual in such large proportions in most towns and cities, the quantity remaining for the usual domestic and town purposes, exclusive of fire protection, will not exceed 20 gal. daily per capita of resident population.

DISCUSSION.

Mr. Dalrymple

F. W. DALRYMPLE, ASSOC. M. AM. SOC. C. E.—The speaker presents an example of the actual use of water, both for manufacturing and domestic purposes, on a considerably larger scale than that given by the author.

The City of Bayonne, N. J., has a population of 42 260, by the recent State census. The water for public uses is delivered to the city pipes by the New York and New Jersey Water Company, and is distributed to the consumers through pipe lines owned by the city.

Every tap, of whatever class, is provided with a meter, and it is thus possible to ascertain the quantity of water used by the different classes of consumers.

There are about twenty large manufacturing concerns within the city, the principal one being the Standard Oil Company, whose consumption of water ranges from 1 100 000 to 1 500 000 gal. per day.

The population is made up largely of the workers in these factories, but there are also many residents who do business in New York. The water consumption per capita, in gallons per day, for the quarter ending October 1st, 1905, was as follows:

For domestic uses	35.0
For manufacturing purposes	56.9
Total	91.9

The total number of gallons used per day was 3 884 000.

The figures for domestic use include the water consumption of a number of small manufacturing concerns which could not be separated from the others.

Mr. Potter.

ALEXANDER POTTER, ASSOC. M. AM. SOC. C. E.—An important factor affecting the consumption of water is the pressure or head on the system under consideration. The greater the head the greater the waste in drawing water from the faucet, for it is an almost universal practice to let the water run a while before using. The greater the head the greater the leakage, also, through the pipe joints in the streets. In the plant described, the head is stated to be 65 ft., a condition favorable to low losses from the causes mentioned. The observations made by the speaker on small waterworks systems warrant this conclusion.

The speaker has questioned the wisdom of the practice followed by a number of well-known engineers in estimating even higher than existing per capita consumptions in designing additional water supplies and trunk sewers. The conditions in American cities must in time approach more closely those of European cities, and projects

planned for a maximum service in 25 or 30 years should be based upon this inevitable reduction in per capita consumption. American people are too wasteful of water. At present they do not study the small economies, as they will sooner or later. Preventable waste of water is a study which is still in embryo. Notwithstanding that many meters are already installed, their use has scarcely begun, for they seem to be the only means at hand to insure against preventable waste. We are more careful of our own and of what we must pay for than we are of our neighbors' or of common property. Whether water is cheap or expensive to deliver to consumers, it should be paid for strictly on the basis of quantity used. When there is an intelligent appreciation of the correctness of this principle, by the people generally, there will be a greater incentive to economize. Mr. Potter.

The question is not simply one of finance, but the more serious one of the possible exhaustion of the available sources of potable water. There are many towns, approaching the limits of their existing supplies, having no place to which they can turn for additional water, except at vastly increased cost.

The lavish use of water not only affects the water supply, but also the cost of construction of large outlet sewers built upon the separate system. The lavish use of water is a two-edged sword, which makes itself felt both coming and going. It is the duty of engineers to mould public opinion and educate the people toward greater economies in these matters.

V. H. Hewes, M. Am. Soc. C. E.—Mr. Owen having called attention to the difficulty of having the caulking well done, and also the large losses due to leaks caused by faulty caulking, it has occurred to the speaker that, when there are so many small portable compressor plants on the market, by using a pneumatic tool, the caulking could be done more quickly and more perfectly than when the work is done by hand-driving, thus avoiding the difficulty of making tight joints on the under side of the pipe. Mr. Hewes.

Another waste of water, pointed out by Mr. Potter in a communication to a daily newspaper, is due to the street cleaning department sluicing or washing the dirt from the streets into the sewers with hose attached to fire-hydrants.

M. R. Sherrerd, M. Am. Soc. C. E.—In designing a water plant, any plan proposing to give, even for a small town, only 30 gal. per capita per day will be insufficient to meet the requirements of the distribution system, particularly for fire protection. It may be possible, in small, exclusively residential sections, to restrict the use of water by the general introduction of meters to, say, 50 gal. per capita. Small plants for 2 000 people might be able to supply sufficient water for domestic purposes, and with, say, 1 000 000 gal. storage, or reserve pumping capacity, could also furnish water for Mr. Sherrerd.

Mr. Sherrerd. fire protection on the basis of a supply of less than 200 000 gal. per day. Yet the size of the street mains should be calculated for a rate of flow sufficient to give at least three fire streams in all parts of the system. This will necessitate sufficient pipe capacity to give a flow at the rate of 1 000 000 gal. in 24 hours above the maximum consumption for ordinary purposes.

In comparing the relative cost of the construction of small and large plants, the additional proportion of expense, in providing mains sufficient for fire protection in the smaller plants, should not be overlooked.

Generally speaking, for designs of plants for cities, it seems to be now reasonably safe to estimate on the basis of 100 gal. per capita for the population which is expected to be supplied, allowing for the growth of twenty years. Some cities, where meters have been installed on practically all the taps, have been able to reduce the per capita consumption to between 50 and 60 gal., while others, notably those along the Great Lakes, where the supply of water is practically unlimited, run as high as 280 gal. per capita, but without the right to consider themselves any cleaner therefor. In fact, the unnecessary consumption of soft coal incident to the pumping of this wasted quantity of water, which causes an additional amount of dirt and soot in the city, is the only justification for the high consumption of water—which reasoning might be carried on *ad infinitum*.

Naturally, in discussing public water supplies, the question of the use of meters, and their effect on the reduction of waste, presents itself. In Newark, N. J., with a population of 250 000 and a total of about 30 000 taps, the compulsory introduction of meters during 1898 and 1899 on about 15% of the wasteful consumers selected by inspection, resulted in holding the consumption of the city, even including the natural growth, practically constant for five years. The increase, prior to that time, had been 2 000 000 gal. per day per year, so that at the end of five years the actual saving was about 10 000 000 gal. per day. This would certainly indicate that the best inspector that can be installed is the meter, for when a man's pocketbook is touched a vulnerable point is found, and he will see that the waste of water on his premises is reduced to a minimum.

Incidentally, the introduction of meters naturally leads up to the subject of what may be considered a fair minimum rate or a fair minimum quantity of water to be allowed to each tap, for which payment must be made. Some cities have established a frontage tax, which is assessed on all lots in front of which water pipes are laid. Other cities depend entirely on their water rates for revenue. The frontage tax is perhaps the fairest way to assess

Mr. Sherrerd.

the cost of the distribution system, as the construction of water mains is a benefit to real estate. However, where the law may not permit such an assessment, and it is essential to establish a minimum charge for water, a fair method of arriving at the basis of this charge seems to the speaker to be the following: Apportion the annual cost of the interest and sinking fund charges of the distribution system equally among the number of taps. An analysis of these charges in several cities has shown this proportional amount to be about $4 per tap per year, and, therefore, it is suggested that a fair basis for a minimum charge for metered water is $1 per quarter on each meter, in addition to the meter rate, and that an allowance of 500 cu. ft. per quarter, or 3 750 gal., for which payment must be made at meter rates, establishes a fair minimum, which in most cities is also equivalent to the basic assessed rate, thus putting the meter consumer on the same plane with the assessed.

Referring to the discussion concerning the waste of water in New York City, the speaker has always been of the opinion that the large water front surrounding the city is more or less responsible for the large waste in the distribution system. Certainly, in cities not so favorably located with a large water front, serious leaks in the street mains sooner or later show themselves on the surface, whereas in New York City leaks in the system along the river fronts easily find their way to the rivers or harbor without giving any surface indication of their existence.

Mr. Christian.

G. L. Christian, M. Am. Soc. C. E.—The speaker's conclusions as to the legitimate use of water for domestic purposes agree substantially with the experience obtained in Yonkers, N. Y., from 1891 to 1904, inclusive.

Yonkers is a progressive, prosperous, manufacturing city, with many miles of beautiful streets, fine residences and well-kept lawns. Its population is composed of people in all walks of life, from the poorest to the wealthiest. Those who predominate, and to such an extent as to stamp their individuality upon the community, belong to what may be termed the middle class of society, many of whom own their own homes, although the majority do not. Probably 80% of the houses are connected with the sewers.

Since June, 1892, all water used in the city has been metered, except that used in public buildings, for public baths, street sprinkling, building purposes and fires; and the records of the Water Board, from 1891 to 1904, inclusive, show that the number of persons using the water has increased from 30 000 in 1891 to 59 000 in 1904, while the domestic consumption per day per capita of actual users has remained stationary, never varying more than 2 gal. from year to year, and averaging 20 gal. for the whole period.

Mr. Christian. During the same time the per capita consumption for manufacturing purposes has varied from 11 to 35 gal., being 24 gal. in 1904, and averaging 26 gal. The water used must all be pumped, and the total consumption, as measured by plunger displacement, without any allowance for slip, has varied between 76 gal. in 1891 and 102 gal. in 1897, returning to 88 gal. in 1904, and averaging 87 gal. for all the years under discussion.

The late Joseph A. Lockwood, Superintendent of the Yonkers Water-Works, in his annual report for 1891, says:

"Two hundred private residences, with modern plumbing and with sewer connections, without stables, but including use of hose for sprinkling, average 22 ft. (165 gal.) daily, a use of about 30 gal. per head per day during the year."

And, again, from his annual report for 1893:

"Having had some fifteen years' experience in supplying water to the consumer by frontage and fixtures rates and by meter, at the same time we realized the absurdity of the first and the justice of the latter; therefore the adoption of the resolution, taking effect June 1, 1892, that thereafter water would be supplied by meter only, for domestic and manufacturing or mechanical purposes. The system continues to be satisfactory to the department and, with few exceptions, to the consumer.

"All the meters (3 248) are read quarterly, and some seventy of the larger ones monthly, at a cost for the year of $521.00 to take the statements. The value of the meter system is plainly shown by our records, where we have numerous cases, where from leaks and waste, bills ordinarily $1.00 to $4.00 for three months were from $25 to $30 for the same time, and one where the average bill for the quarter for four years had been $10 and $12 became more than $100, or the daily average use increased from 60 to 80 ft. per day, to 850 daily for three months.

"In most cases the owners or tenants of the premises are not aware of leaks, or, if they are, think them unimportant until the quarterly bill is rendered, and then there is no time lost in stopping the waste, because the meter is registering it, and the water wasted must be paid for. We have had three Jenks' drinking fountains put up during the year and supplied with water through a ¾-in. tap. The valve on the supply pipe is closed, so as to, as far as possible, supply only sufficient water during the time that they are most used, to keep water in the bowl. We have had meters on two of them for five months, and find that in one, under a head of 275 ft., an average of 765 ft. of water daily was run, and in the other, under a head of 200 ft., an average of 406 ft. daily was run. The value of the water furnished the two fountains under our rates for the five months was $239.39, or about $575.00 yearly. The streams of water running in these fountains seem small, but they are evidences of the results of a small stream running constantly under pressures like ours, in many localities in excess of 100 lb."

Mr. Christian. J. J. R. Croes, Past-President, Am. Soc. C. E., in his report on "The History, Condition and Needs of the New York City Water Supply and Restriction of Waste of Water," says:

"The quantity of water used for public purposes cannot be accurately measured. No effort has ever been made in New York City to determine this amount, but in a few American and in a large number of foreign cities, particularly in Germany, very careful investigations have been made, which show the average quantity of water thus used to be not far from 5 gal. a day per head of population."

According to the annual report of the Board of Water Commissioners of Yonkers for 1904, the amount used in fountains, bathhouses, and public buildings equaled 1.2 gal. per capita per day, while for street sprinkling there was used 1 gal. per capita per day. To this should be added the water used on account of fires and for construction purposes.

All things considered, it is reasonable to assume that the water used for public purposes in Yonkers will equal 5 gal. per capita per day.

Summarizing, the consumption is found to have been:

For domestic	purposes	20	gal.	per	capita	per day.
" manufacturing	"	26	"	"	"	" "
" public	"	 5	"	"	"	" "
Unaccounted for, waste, etc		36	"	"	"	" "
Total		87	"	"	"	" "

The waste seems to be large, but is undoubtedly accounted for, in a measure, by the pumps registering an unknown quantity more than is actually pumped, by leaks in the mains, loss by meters being out of order, surreptitious connections made back of the meter, etc.

It would seem that much valuable information might be gained, at comparatively small expense, by installing a meter on the large service main, and measuring the exact quantity of water being pumped; then, by districting the city and metering the water passing into and out of each district, it would be possible to detect large leaks, from whatever cause, and in that way reduce the waste to a minimum.

Just west of the Harlem Railroad, and lying both north and south of the line dividing Yonkers from New York, there is a suburban settlement of about 2 000 people. The part north of the city line is known as McLean Heights, and that to the south is called Woodlawn Heights. The social standing of the people, their manner of living, and the style of their houses are similar on both sides

Mr. Christian. of the line. For a number of years McLean Heights and the more elevated section of Woodlawn Heights were supplied with water from the Yonkers Water-Works, that supplied to Woodlawn Heights being passed through a meter as it entered the New York City mains. All the water used in McLean Heights is metered, but there are probably no meters used in Woodlawn Heights.

McLean Heights has no sewers; and in Woodlawn Heights, during the time under discussion, and within the district covered by this inquiry, about 80% of the houses were without sewers.

The speaker has gathered some data on this district, covering the years, 1901, 1902 and 1903, and the following are the results: Eight, one-family, representative houses were selected in McLean Heights, each located on an average of from two to three city lots, seven of them having all improvements, including baths, and most of them having well-kept lawns. The average consumption per capita per day for the three years, for each of the eight houses, ranged from a minimum of 3.5 gal. to a maximum of 40 gal., the average for the eight being 16 gal.

The water received from Yonkers, and metered before entering the New York City mains, was intended for the use of eighty-one houses in Woodlawn Heights, having a population of 400, the average consumption of which, based upon the record obtained from the meter located on the city line, was 54 gal. per capita per day. About 20% of the houses being connected with the sewer, and none of them being metered, it is probable that they used slightly more water than the 16 gal. used in McLean Heights, and, for the purpose of comparison, the speaker has assumed 20 gal. per capita as the probable consumption.

Summarizing, the consumption is found to have been:

For domestic purposes	20 gal. per capita per day.
For public purposes (based upon the same data as assumed for Yonkers)	5 " " " " "
Unaccounted for (mostly in the mains)	29 " " " " "
Total	54 " " " " "

Conclusions.—The records for the past fourteen years seem to show that:

The consumption per capita has not increased.

The waste, or water unaccounted for, has increased.

The metering of every tap has decreased the waste within the houses to a minimum. The waste of 36 gal. per capita in Yonkers, when compared with the estimated waste of 29 gal. per capita in Woodlawn Heights, would seem to show that the loss was probably due, in a large measure, to the same causes, namely, leaks in the mains. The loss for Yonkers should appear to be larger, as it does,

on account of the unknown loss due to the slip of the pumps, and the fact that the mains are probably older and therefore in a poorer state of preservation. Mr. Christian

A goodly portion of the waste from the street mains is undoubtedly due to the high pressures to which most of them are necessarily subjected, varying from 45 to 130 lb. per sq. in. in the center of the town.

In order to locate the waste accurately, it being now, in a measure, a matter of speculation, the large delivery mains should be metered, the city should be districted, and the water entering and leaving each district, through the mains, should also be metered.

The fluctuations in the per capita consumption for manufacturing purposes, which varied from 11 to 35 gal., is accounted for by the closing of some of the factories for many months. One of these factories used more than 300 000 gal. a day. Other factories have sunk wells for their own use.

EDWIN DURYEA, JR., M. AM. SOC. C. E. (by letter).—The writer has nothing to add to Mr. Cooper's statements regarding the use of water at Lick Observatory. The paper was originally a report made by Mr. Cooper, one of the writer's assistant engineers, and seemed to be of sufficient general interest to present to the Society. It covers the subject very fully. Mr. Duryea.

Since then the writer has met with the opposite extreme, which may well be called "An Example of the Illegitimate Use of Water for Domestic Purposes." He is now investigating the water supply system of Bakersfield, Cal., and finds that the quantity of water used there—or rather wasted—is almost beyond belief. Bakersfield is a residence town, near the head of the San Joaquin Valley, in Kern County, with a present population of about 6 800. It has practically no manufactures and no public uses for water; even the streets, principally dirt roads, are oiled instead of watered. The climate is quite warm, and there is a rather large proportion of lawns about the residences. The wasteful use of water on these lawns is the principal cause of the great consumption.

Table 2 shows some of the rates at which the water has been used.

The water is pumped, from a gravel bed underlying the town, by centrifugal pumps acting on wells, 13 in. in diameter, with perforated casings. The pressures carried are usually about 30 or 35 lb. per sq. in., with extremes of, say, 15 to 40 lb. per sq. in., these pressures being at the pumps. There are seven pumps, and all are provided with pressure and suction gauges; and one of the pumps has two automatic recording pressure-gauges in addition. None of the pumps is provided with a water meter.

Two of the pumps have been rated by trial for discharges at different pressures, and these ratings, in conjunction with records

Mr. Duryea. of pressure and pump-hours run, form the basis for the estimated total water pumped. The pumps are operated by electric motors, the charge for power being per pump-hour, and these charges, checked by both the electric power company and the water company, give the pump-hours operated.

TABLE 2.—RATES OF WATER CONSUMPTION AT BAKERSFIELD, CAL.

Year, etc.	WATER PUMPED DAILY PER USER, IN GALLONS.			WATER PUMPED DAILY PER HOUSE CONNECTION, IN GALLONS.		
	Average for the year.	Average for the maximum month.	Average for the minimum month.	Average for the year.	Average for the maximum month.	Average for the minimum month.
1902	374	528 (July)	210 (Dec.)	1 695	2 350 (July)	920 (Dec.)
1903	412	603 (July)	221 (Jan.)	1 790	2 595 (July)	950 (Jan.)
1904	472	719 (June)	298 (Jan.)	2 010	3 090 (June)	1 260 (Jan.)
1905*	440*	629 (Aug.)	338 (Jan.)	1 875*	2 660 (Aug.)	1 450 (Jan.)
Month of June, 1905.	Average for the month, 600	Maximum day, 13th, 678	Minimum day, 1st, 452	Average for the month, 2 530	Maximum day, 13th, 2 850	Minimum day, 1st, 1 930

* Records for only the first eight months of 1905 were available. The figures for 1905 are for the whole year, and are deduced from the relations between the first eight months and the whole year for the preceding years.

The pumps are known to be kept in a uniform condition of efficiency, and the mains to have a minimum amount of leakage. Notwithstanding this, the computed total pumpages are reduced by 12½% to allow for the possible under-performance of the pumps and the leakage in the mains. The quantities in Table 2, therefore, are only 87½% of the rated capacities of the pumps.

The numbers of house connections given are actual, and are from the records of the water company. A careful study was made of the population, by making use of the United States Census, school censuses, registered voters, etc., and the results deduced are believed to be quite reliable. The number of people served by the water system (the water users) was also estimated with much care, and is believed to be very near the true number. The consumptions given in Table 2, therefore, are regarded as reliable and conservative, and as being probably from 5 to 10% less than the actual consumptions.

The principal cause of the excessive consumption is a practically unrestricted use of the water in irrigating the lawns, in many instances the hose being in use during the whole twenty-four hours. It is perhaps unnecessary to add that the town is without water meters, that the water charges are by "flat rates," and that these rates are extremely low.

Mr. Belzner. THEODORE BELZNER, JUN. AM. SOC. C. E. (by letter).—The writer believes that the only way of reducing water waste in tenement

houses is by metering all those which are more than four stories high. Having had considerable experience in the condemnation of buildings in New York City, especially in localities where there is an exceptionally large consumption for domestic purposes, the writer is of the opinion that the waste is due to leaky plumbing fixtures, the overflowing of tanks, caused by the absence of ball cocks, and the carelessness of tenants in allowing faucets to remain open to prevent freezing. Mr. Belzner.

Another cause is defective plumbing design, for instance, in some cases the pipes for hot and cold water are arranged side by side, which results in extravagant waste before the desired hot or cold water is secured. In some of these tenements the faucets on nearly every floor are left open for days.

James Owen, M. Am. Soc. C. E.—There is one point relative to the consumption of water in different communities, which, so far, has received little attention, and that is the relation of the *pro rata* consumption to the death rate. There is such an enormous difference between the 25 gal. per capita daily consumption in Fall River, and the 250 gal. used in Buffalo, that it would be interesting to find out whether the low rate is below normal requirements and detrimental to health; and, as a resultant, the desirability of such economy would be open to great doubt. Of course, the character of the water does have influence on the death rate, and the citing of individual cases would not give correct results, but if it should be found, on tabulating the returns from a number of cities, that a ratio could be found, it might be of great use for future practice. It must be conceded, however, that in almost all public and even private water supplies there is undoubted waste, and it is the object of all hydraulic engineers to stop it. Mr. Owen.

What the waste is in any community depends upon the character of the population, their business, their habits, their material condition, and the cost of the water; and, with the variations, enters the consideration of climate and temperature. In the writer's experience with a suburban locality near New York City, purely residential, and with no factories, the consumption varied from 60 gal. in winter to 100 gal. in summer, but at one time, when the supply was short, the consumption was forcibly reduced to between 20 and 30 gal., with but little demur from the inhabitants, except as to the deprivation of lawn sprinkling.

The question of leakage in a plant is a matter of engineering supervision, as the weak spot in all water-works construction is the joints, both in mains and services. With complete supervision in the original construction, a large proportion of waste will be eliminated.

MEMOIRS OF DECEASED MEMBERS.

STANDISH BARRY BURTON, M. Am. Soc. C. E.*

DIED AUGUST 13TH, 1904.

Standish Barry Burton was born in Herkimer, New York, on March 23d, 1856, of Scotch-Irish parentage. After attending the public schools of his native town and attending college, he took a post-graduate course in Civil Engineering, after which, although but 18 years old, he was engaged in land surveys in the State of New York, under Mr. Amos Freeman, a surveyor of some note at that time. From 1875 to 1877 he was associated with Mr. R. B. Odell, M. E., of Fulton, Oswego County, New York, but in the latter year he obtained his first position with a railroad company, remaining two years with the Atchison, Topeka and Santa Fé Railway. In 1880 he was for a short time with the Atchison and Nebraska Railway, coming thence to Texas, where for nearly two years he held a responsible position in the operating department of the Texas and Pacific Railway, then under construction through Western Texas to El Paso. In 1882 he was offered and accepted a position with the Mexican Central Railway, then building through El Paso, and south through Mexico, from which time until his death he remained in the Republic of Mexico. For seven years he was with the Mexican Central Railway, and during the last five he was Superintendent of the Second Division, and Engineer of Maintenance of Way. When he finally resigned, in 1889, he was Superintendent of the First, Second and Third Divisions. From 1889 until 1893 he was Chief Engineer of both the Monterey Smelting and Refining Company and the Nuevo Leon Smelting and Refining Company, during which time he designed and constructed a large smelting plant now in operation at Monterey, which stands as a monument to his skill and ability in this line. Upon the completion of the Monterey smelter he received the appointment of Chief Engineer and General Manager of the Tehuantepec National Railway, which spans the Isthmus of Tehuantepec from the Atlantic to the Pacific, and it was under his direction that the construction of this road was brought to completion. In spite of the great difficulties of a climate full of malaria, and people unused to labor he succeeded in building a first-class standard railroad, which is to-day becoming a line of the greatest importance, not only to Mexico, but to all countries, and especially the United States, in its commerce with the East. After leaving Tehuantepec, Mr. Burton went

*Memoir prepared by F. H. Dillon, Assoc. M. Am. Soc. C. E.

into mining, in which he was engaged up to the time of his death, and was the General Manager of several important mining companies operating in the Republic of Mexico.

He was a man of great and varied ability, and immense energy. He showed great originality in the many engineering works designed and carried out by him, and was so particular as to details that he always insisted on giving his personal attention to every matter which he had in hand, however small.

He was a man of very amiable character, and beloved by all with whom he came in contact. He was married in San Antonio, Texas, on February 22d, 1887, to Miss Jennie De Ham, who, with four children, survives him. His unfortunate death took place quite suddenly on August 13th, 1904, while he was engaged in examining some mining properties in the State of Tamaulipas, Mexico.

Mr. Burton was elected a Member of the American Society of Civil Engineers on June 1st, 1898.

JOSEPH NORTON GREENE, M. Am. Soc. C. E.*

DIED JULY 26TH, 1904.

Mr. Greene's father, Nathaniel, emigrated from Herkimer County into the wilds of Western New York in 1818, and was one of the pioneer settlers of Chautauqua County, carving a farm and home out of the forests near the present Town of Mayville, at the head of Chautauqua Lake, and there, on January 31st, 1827, the subject of this sketch first saw the light. The family from which he sprang was one of force and character, founded by John Greene, Surgeon, who emigrated from Dorset, England, to Massachusetts in 1635, and in 1642 followed Roger Williams to Rhode Island, where he was one of the original twelve "purchasers," and a leading force in that colony, as were many of his numerous descendants in succeeding years; twice furnishing governors of Rhode Island and several times deputy governors and members of the Colonial and State Legislatures, of Congress and Officers of the Army and Navy, the most noted of these being General Nathanael Greene, of the Continental Army.

The names of several of this family are enrolled as members of this Society, prominent among them being General George Sears Greene, who passed to the great majority in 1899, at the ripe age of ninety-eight, and his sons, George Sears, Junior, and General Francis Vinton Greene, still in active membership.

Educational facilities were scarce in Chautauqua County in the first half of the nineteenth century, and Mr. Greene obtained only the rudiments afforded by the district schools, the benefit of a short year in the Fredonia Academy, and a year or two in teaching, before he began his education as an engineer in the "University of Experience," in 1848, as a rodman on the Michigan Central Railroad. The following year found him Assistant Engineer on the Genesee Valley Canal, until December, when he was appointed Assistant Engineer by the State Canal Board, serving on the State canal work until the middle of 1851, when he was employed as Resident Engineer on the Erie Railroad, serving there in 1851-52. Following this came successive employment on the Albany and Susquehanna, the Atlantic and Great Western, the Northern Central and Missouri Pacific Railroads. From 1863 to 1867 he was engaged in mining and civil engineering in the Copper Country of Northern Michigan, and from 1864 to 1867 he was Chief Engineer of the Portage Lake and Lake Superior Ship Canal, begun by the State of Michigan, but finished by the United States Government.

From 1876 to 1883 he was Chief Engineer and General Manager

* Memoir prepared by L. D. Greene, Esq.

of the Grand Southern Railway, of New Brunswick, 1883 to 1885 Chief Engineer of the Maine Shore Line Railway of Maine, and from 1885 to 1887 Chief Engineer of the New York and Boston Rapid Transit Railroad. He was elected a Member of the American Society of Civil Engineers on October 5th, 1887.

From 1887 to 1888 Mr. Greene was the associate of the late William J. McAlpine, Past-President, Am. Soc. C. E., in designing a proposed underground urban transportation system known as the "Arcade Railway," which was intended to run northward from Bowling Green, underneath Broadway, to Central Park, in New York City. The plans, estimates and full details were thoroughly worked out, but these men were a dozen years ahead of public opinion, many objections were raised, and the projectors were not able to interest the required capital. Theirs was the pioneer work of the recently finished subway.

Mr. Greene spent most of his time in the next dozen years working on railroad development in Maine. In 1900 he was one of the Members who attended the meeting of this Society in London, and afterward he took an extended trip on the Continent.

From 1900 to the time of his death Mr. Greene was interested in the building of electric railways in the country from Boston northward.

He was taken ill on July 21st with dysentery, from which he was recovering on the 26th, when he was seized with an uræmic convulsion and died at 10 P. M. that day, at the Quincy House, Boston, aged 77 years and 6 months. His remains were buried in the family lot at Fredonia, Chautauqua County, New York.

Among his associates in the profession, Mr. Greene was noted for his practical ability as a constructive engineer, and was frequently called in consultation in such matters, and for his persistent energy which often triumphed over seemingly insurmountable obstacles by sheer grit and staying power. A strong friend and a strenuous antagonist, he commanded the respect even of those opposed to him.

On October 25th, 1852, Mr. Greene married Ann, daughter of Daniel Witherel and Hannah (Fenner) Douglass, of Fredonia, New York. She died in 1857, leaving an infant son, now Captain Lewis Douglass Greene, U. S. A. (retired), who was graduated from West Point in 1878. In May, 1865, he married Margaret Lowber, of Medina, New York, by whom he had a daughter, Jessie, now Mrs. Robert P. Comstock, of New York.

RICHARD SOMERS HAYES, M. Am. Soc. C. E.*

DIED MARCH 2D, 1905.

Richard Somers Hayes was born in Philadelphia on October 12th, 1845. He was of a family which had for several generations been actively connected with the Navy and Merchant Marine, and several of his ancestors had commanded vessels with credit, in both services. Aside, however, from a brief term of service with the Philadelphia City Troop, when that organization was on duty during the latter part of the Civil War, Mr. Hayes, from the time of his graduation from the University of Pennsylvania in 1864, devoted himself to engineering, and more particularly to railway work.

Entering Grant's Locomotive Works, at Paterson, New Jersey, in 1864, as an apprentice, he worked there during 1865 and 1866, rising to the position of draftsman, and doing some boiler designing work in the latter year. The intimate knowledge of locomotive design and construction which he obtained there was most useful to him when, a few years later, he came into the management of large railway properties in the Southwest.

During 1867 he was Deputy United States Boiler Inspector for the Northern part of New Jersey.

Beginning in 1868 his strictly civil engineering experience, he was successively: Assistant Engineer on the Erie Railroad, under John Houston, then the Chief Engineer of that road, and was later in charge of the construction of the Weehawken Coal Docks, and, after having been with the American Bridge Works as draftsman for a short time, he opened an office in Paterson as a Civil and Mechanical Engineer. At this time he did considerable work for the various water companies and mills in the vicinity of Paterson.

In 1870, when only twenty-five years of age, Mr. Hayes went to Texas as Chief Engineer and General Manager of the Texas and New Orleans Railroad, then in the hands of a receiver, and partially rebuilt the road during that year. Railroad building was brisk in Texas at this time, and for the next ten years Mr. Hayes led a very active life. Late in 1870 he went to the International Railroad as Principal Assistant Engineer, and, becoming Chief Engineer in May, 1871, he completed 175 miles of that road. Becoming Engineer of Construction for the Eastern half of the Texas and Pacific Railroad, under General J. M. Dodge, in October, 1872, he was appointed Chief Engineer of this road the following fall.

In 1874 Mr. Hayes was appointed Chief Engineer of the International and Great Northern Railway; six months later he became General Manager, and Vice-President in 1875. He became Receiver

* Memoir prepared by H. C. Phillips, Assoc. M. Am. Soc. C. E.

in 1878, and, two years later, President of the road. During these several changes of title Mr. Hayes continued to act as Chief Engineer and, in addition to the work on the International and Great Northern Railway, he completed the Texas and New Orleans Railroad in 1874.

In all, during the ten years, 1870 to 1880, Mr. Hayes acted as Chief or Consulting Engineer on about 1 500 miles of railroad in the Southwest.

During June, 1881, Mr. Hayes was elected Vice-President of the Gould Southwestern System, then comprising the Missouri Pacific; Missouri, Kansas and Texas; Texas and Pacific; St. Louis, Iron Mountain and Southern, and the International and Great Northern Railways. In this position he handled successfully the operation of this system for the next five years, and, in addition, was in 1883 elected First Vice-President of the Wabash Railroad. During this time he made his home in St. Louis.

Late in 1885 Mr. Hayes resigned from the Vice-Presidency of the Gould System, although remaining a Director therein for some time longer, and, after a trip abroad, moved to New York and took up a less active life in that city.

Although a director of various railways and other large properties, and giving more than the usual knowledge and attention to them, Mr. Hayes' desire for more active work led him in 1888 to accept the Presidency of the St. Paul and Duluth Railroad, which he retained until its sale and incorporation with the Northern Pacific in 1900. During his administration he remodelled the grade system and greatly improved the property.

Mr. Hayes was also President of the New York and Northern Railway from the close of its receivership, about 1888, to its sale to the New York Central in 1892.

He was also one of the receivers of the Georgia Central Railroad until its reorganization in 1895.

In 1895, when the reorganization of the Atchison, Topeka and Santa Fé Railway System was undertaken, Mr. Hayes was one of the prominent members of the Reorganization Committee, and was elected a Director of this System in 1895. From that time forward he served on the Executive Committee of the Board of Directors, and took a leading part in shaping the policy of the railroad until ill health forced him to resign from the Executive Committee in 1904, although he still remained a member of the Board until his death.

Mr. Hayes had highly developed that faculty most important to the successful railway man, the ability to master and remember details, while not letting them obscure his view of matters of larger importance. His training and inclination caused him to be always

arrayed with those whose aim was to build up and increase the efficiency of whatever property with which they were dealing, and, to this, much of his success was due. He also had the faculty of inspiring an active loyalty in his subordinates, and was personally very highly esteemed by his business associates.

Fond of outdoor sports, and with that spirit which retains much of its youth, Mr. Hayes numbered among his friends rather more than is usual of the younger people.

Living in recent years much of the time at his country home, Millbrook, Mr. Hayes, with Mrs. Hayes, who survives him, entertained simply but delightfully their large circle of friends.

In 1903 Mr. Hayes made a trip to the German Baths and up the Nile in hopes of shaking off a stubborn illness. In this he was unsuccessful, and, after his return, he was an invalid up to the time of his death, which took place in his New York home on March 2d, 1905.

Mr. Hayes was elected a Member of the American Society of Civil Engineers on September 6th, 1882, and was always interested in the Society's welfare.

CHARLES ANDREWS KNOWLTON, M. Am. Soc. C. E.

Died March 2d, 1903.

Charles Andrews Knowlton was born at Irvington-on-the-Hudson, on December 28th, 1867. He was the only son of Major Charles P. Knowlton, of the Confederate service, and Katherine Andrews, daughter of John Andrews, of Belle Grove Plantation, Iberville Parish, Louisiana. A direct descendant of Colonel Thomas Knowlton, of Revolutionary fame, on his father's side, and of Robert Adams, on his mother's side, he was also a nephew of two brave Confederate generals, Paul Octave Hébert, who was also the war governor of Louisiana, and General Major.

Not being strong, as a child, his early education was directed by tutors at his home, in Louisiana; later, he attended Spring Hill College, Alabama, the Louisiana State University, and the Georgetown College at Washington, D. C. He had intended to study law, but his health requiring an outdoor life, he decided in favor of engineering, and did much surveying along the Gulf Coast near New Orleans.

In 1891 Mr. Knowlton went to Cincinnati as Assistant to Mr. C. C. Chandler, of the Ohio and Mississippi Railway, but, upon the consolidation of that road with the Baltimore and Ohio Southwestern, he established an office of his own in Cincinnati.

In 1895 he left Cincinnati to take charge of a large cotton plantation, returning three years later to assist in the construction of the new Cincinnati Water-Works at California, Ohio. Mr. Knowlton left Cincinnati in 1900 to enter the United States Government service in Cuba as constructing engineer of the Guantanamo Water-Works, and during this period he became a Member of the American Society of Civil Engineers. Upon the completion of the Guantanamo work, which had added not a little to his reputation, he was called to Santiago to make the preliminary surveys and reports upon a new water system for that city. His faithful service and the uniformly high character of his work made his loss a serious one to the department when he left it to become Administrator of the Central Ysabel, a large sugar estate near Guantanamo, Cuba. As manager of the Ysabel he was most successful, his high sense of honor and justice making him loved and respected in a country where Americans, as a rule, were disliked. Mr. Knowlton was so interested in his work, and so ambitious, that he took small heed for himself, and when illness attacked him he was powerless to resist. On March 2d, 1903, he died at Central Ysabel, after an illness of

but four days, leaving a wife and young son to mourn the loss of a devoted husband and father.

A young man, stricken down in the fullness of his youth, he left behind the memory of an earnest life, well spent, well lived.

Mr. Knowlton was elected a Member of the American Society of Civil Engineers on October 20th, 1901.

RICHARD CALVIN McCALLA, M. Am. Soc. C. E.*

DIED JUNE 12TH, 1904.

Richard Calvin McCalla was born on April 20th, 1864, in Chester District, South Carolina.

In September, 1879, he entered the University of Alabama, and received the degree of Bachelor of Arts in 1883. In 1890 the same institution conferred upon him the degree of Civil Engineer.

While a university student, Mr. McCalla filled, during the summer vacations of 1881 and 1882, various positions on the engineering staff of the Knoxville and Ohio Railroad, in East Tennessee; and in 1883, on completing his university course, he accepted the position of Assistant Engineer on the Louisville, New Orleans and Texas Railroad, between Memphis and New Orleans.

From May, 1886, to January, 1887, he held the positions of Leveler and Resident Engineer on the construction of the Kansas City, Memphis and Birmingham Railroad, in Northern Alabama.

From January to March, 1887, he practiced engineering in Birmingham, Alabama.

From March, 1887, to September, 1888, he was Chief of Party and Resident Engineer on the construction of the Tuscaloosa and Northern Railroad in Alabama.

In September, 1888, he entered the civilian service of the Engineer Corps, United States Army, as Assistant Engineer, being associated with Horace Harding in the improvement of the Black Warrior River, Alabama. On the death of Mr. Harding in 1895, Mr. McCalla was placed in charge of all the improvement work on the Tombigbee and Black Warrior Rivers.

A complete history and discussion of this important river improvement work was presented to the American Society of Civil Engineers by Mr. McCalla in Paper No. 941,† September 3d, 1902. This paper is recognized as a valuable contribution to engineering literature on the subject of inland waterways.

In opening the Black Warrior River system to navigation, the United States Government intends to provide an all-water route of 7 ft. depth between the Warrior coal fields and the Gulf Coast. The entire cost of the project is estimated at $10 000 000.

Under Mr. McCalla's supervision, six locks have been completed and the river opened to navigation from a point 282 miles above Mobile to the southern part of the great Warrior Coal Basin. Plans and estimates for nine additional locks and dams have been prepared, and construction work on four of these locks was in progress at the time of his death.

* Memoir prepared by Louis W. Hall, Assoc. M. Am. Soc. C. E.

† *Transactions*, Am. Soc. C. E., Vol. XLIX, p. 212.

In addition to his engineering work, Mr. McCalla was identified with a number of business enterprises in the South, and was a Director of the Merchants National Bank of Tuscaloosa, Alabama.

On June 23d, 1896, he married Mrs. Lula Stringfellow, and is survived by her and their only child, Richard Calvin.

Kind and lovable in his home life, energetic and conscientious in his engineering and business enterprises, a progressive character and a factor for good in the community in which he lived, he held the respect of all with whom he came in contact.

He was elected a Junior of the American Society of Civil Engineers on July 2d, 1890, an Associate Member on September 2d, 1891, and a Member on December 5th, 1894.

GEORGE ANSON MARR, M. Am. Soc. C. E.*

DIED MARCH 24TH, 1905.

George Anson Marr was born in Elgin County, Ontario, on September 9th, 1837, and died at Chicago, Illinois, on March 24th, 1905.

His father, John Saxton Marr, and his mother, Eliza Patton Marr, were born in Nova Scotia, and moved to Ontario at an early age. They were among the pioneers of the section where they located. Mr. Marr received his early education at the County Schools of Elgin County, from which he went to the University of Michigan in 1858. He was graduated from the University in 1862 with the degree of Bachelor of Science, received the degree of Civil Engineer from the same institution in 1863, and the degree of Master of Science in 1867. After graduation he entered the service of the United States as Assistant Engineer, and remained in the service of the Government until the date of his death.

From 1863 until 1877 he was connected with the Survey of the Northern and Northwestern Lakes; after this he was engaged on surveys and investigations on the Missouri and Mississippi Rivers until 1891, when he accepted the position of Superintendent and Assistant Engineer in charge of the U. S. Portage Lake Ship Canals, which position he held at the time of his death.

Mr. Marr was married twice. He left one child, Mrs. Florence Marr Smith, of Aylmer, Ontario. He was a naturalized citizen, and was among the foremost Canadians, who by their energy, ability and integrity have had much to do with making the Middle West what it is to-day.

His professional work was of a high grade, always correct and trustworthy. Age or infirmity had not overtaken him. He was active and energetic up to his last illness. His friends confidently expected many more years of usefulness for him. His character was a lovable one. He was fearless yet considerate, frank yet kind; his integrity was always absolutely unquestioned. His friendship was faithful and unfailing. The number of his friends equaled the number of his acquaintances; all who knew him loved him. By his death our Society has lost a brave, earnest, kindly and manly man.

Mr. Marr was elected a Member of the American Society of Civil Engineers on October 3d, 1883.

* Memoir prepared by J. H. Darling, G. Y. Wisner and E. S. Wheeler, Members, Am. Soc. C. E.

GEORGE CLIFTON WOOLLARD, Jun. Am. Soc. C. E.*

DIED JUNE 3D, 1905.

George Clifton Woollard was born on June 14th, 1879. At the age of eighteen, he started on the Maintenance of Way Corps of the Erie Railroad, and at the early age of twenty was appointed Roadmaster of the Buffalo, St. Marys and Southwestern Railroad. In the same year he was made Engineer of Maintenance of Way of the Clarion River Railroad, and at the age of twenty-two was made Engineer of Maintenance of Way of the Pittsburg, Shawmut and Northern Railroad.

After he left the Pittsburg, Shawmut and Northern Railroad, he went with James Stewart and Company, to work on the Plaquemine Locks, a very heavy piece of work built by them for the Government in Louisiana. This was his first important work, and it is noted as being the longest lock in the United States and as having the highest lift in the world. After that he was on the Passe A L'Outre work at the mouth of the Mississippi River, with the same firm; and later, with the same firm, on the New Orleans Sugar Refinery at New Orleans, said to be one of the tallest buildings in the South.

He then moved to New Jersey, and was appointed Superintendent of the Cedar Grove Reservoir, for the City of Newark, built by Stewart and Abbot. He carried this through practically to completion, and then went back to James Stewart and Company to superintend the construction of the Westinghouse Buildings at East Pittsburg. Subsequent to this, he built a sand plant for James Stewart and Company at Gray Summitt, Missouri. He was then sent by the same firm as Superintendent of a very large plant they were building for the Singer Machine Company at St. Johns, Quebec, consisting of twenty-four buildings, and covering forty acres of ground.

He then severed his connection with James Stewart and Company, and was offered a position by Ambrose B. Stannard, and while with him was employed on the Sewage Dumping Station for the District of Columbia, also on the construction of the New Municipal Building for the District of Columbia, the new building for the Department of Agriculture, and the concrete foundations for the House of Representatives.

Mr. Woollard was an exceptionally bright young man. He was cut off, not even in his prime, but at the early age of twenty-six. For the last four years of his life he was occupied in large contract-

* Memoir prepared by Fred. W. Abbot, M. Am. Soc. C. E., and Edmund I. Bowen, Assoc. Am. Soc. C. E.

ing enterprises, having at all times from 100 to 400 men under his supervision. He combined the unusual qualities of a brilliant engineer and an efficient contractor's superintendent. Had he lived, he would have made a great success in life, and would probably have followed out his natural tendency of being a contractor on a large scale.

In conclusion, it might be added that Mr. Woollard was a most devoted son to his mother and father, and had a loving and kind disposition with all his friends and associates.

He was elected a Junior of the American Society of Civil Engineers, April 3d, 1900.

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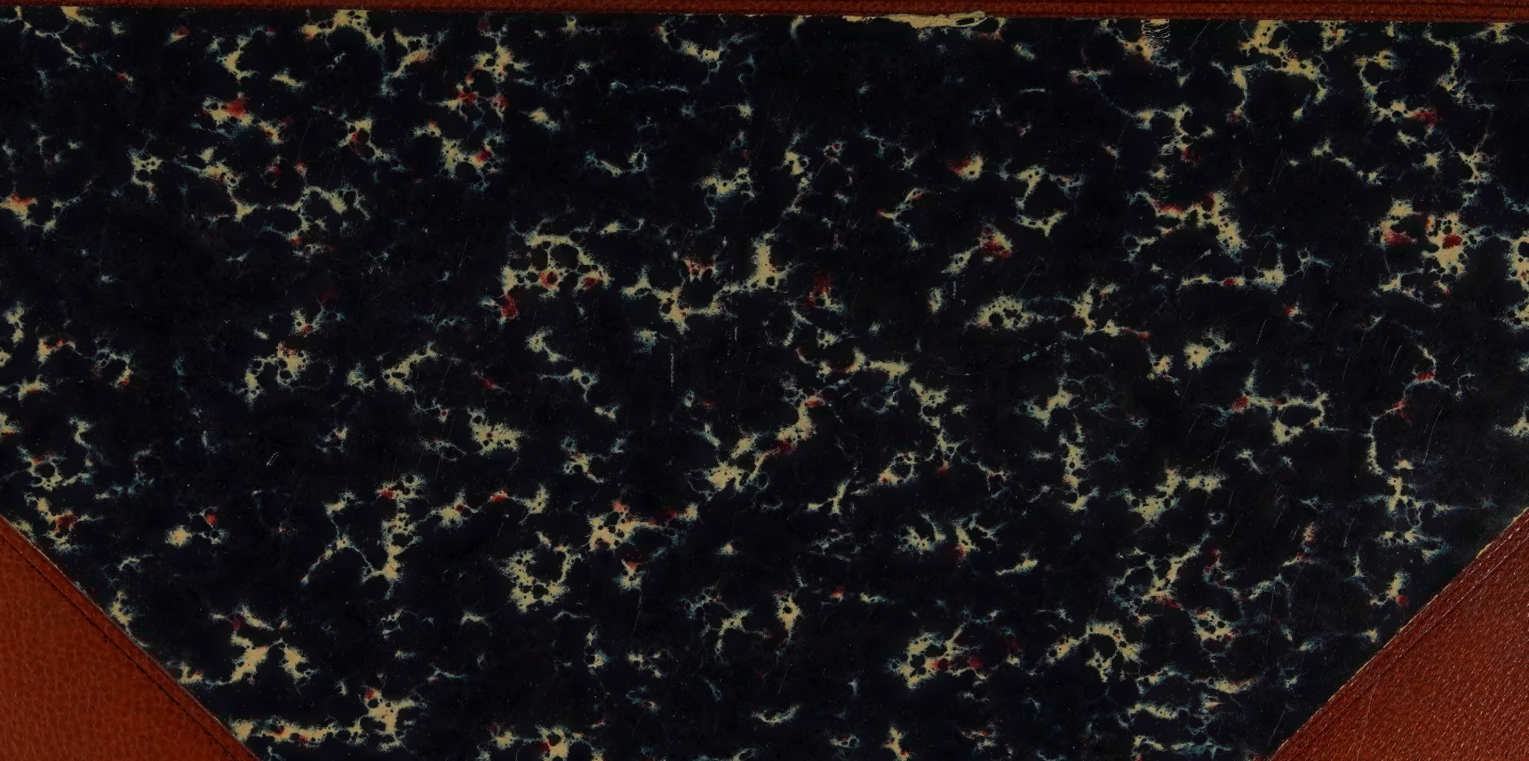
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